Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 14 (1954)

Artikel: Statistical calculation of strength of reinforced concrete beams

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DOI: https://doi.org/10.5169/seals-13941

Statistical Calculation of Strength of Reinforced Concrete Beams

Calcul statistique sur la résistance des poutres en béton armé

Statistisches Berechnungsverfahren für die Festigkeit von Eisenbeton-Trägern

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The deduction of equations for determining the strength of reinforced concrete structures has hitherto been based on the assumption that the quantities entering into these equations, e.g. the strength of concrete and the strength of reinforcement, have definite values (the classic theory of calculation). In principle, this assumption is incorrect since the observed values of these quantities normally exhibit a certain dispersion (scatter, spread), which influences the form of the equations. The present paper is a general study dealing with this effect of the dispersion in the quantities in question.

We introduce the notations listed in what follows.

$\bar{m}, m$ = the mean value and a single observed value of the ultimate moment of the beam.
m_t = the ultimate moment causing tension failure.
$\bar{m}_t, s_t$ = the mean value of, and the standard deviation in, the ultimate moment causing tension failure.
m_c = the ultimate moment causing compression failure.
$\bar{m}_c, s_c$ = the mean value of, and the standard deviation in, the ultimate moment causing compression failure.
σ_s = the strength of reinforcement.
$\bar{\sigma}_s, s_{\sigma s}$ = the mean value of, and the standard deviation in, the strength of reinforcement.
σ_c = the strength of concrete.
$\bar{\sigma}_c, s_{\sigma c}$ = the mean value of, and the standard deviation in, the strength of concrete.
A = the cross-sectional area of reinforcement.
$\bar{A}, s_A$ = the mean value of, and the standard deviation in, the cross-sectional area of reinforcement.

E = modulus of elasticity of reinforcement.
b = the width of the beam.
$\bar{b}$, s_b = the mean value of, and the standard deviation in, the width of the beam.
h = the distance from the centroid of the tension reinforcement to the compression edge of the beam.
$\bar{h}$, s_h = the mean value of, and the standard deviation in, the distance from the centroid of the tension reinforcement to the compression edge of the beam.
f = frequency function (the subscript m_t indicates the frequency function of m_t, etc.).
F = distribution function.
φ = normal frequency function.
Φ = normal distribution function.

If a beam fails in bending, then a distinction may be drawn between two types of failure, viz., tension failure and compression failure. These types of failure are supposed to be clearly defined. We shall now study the actual mean strength of the beam ($=\overline{m}$) with reference to the risk of failure of both these types. For this purpose, it is necessary to integrate the probability of each value of the strength multiplied by the corresponding value of the strength ($=x$). In this calculation, we assume that m_t and m_c are statistically independent of each other. This assumption is only approximately correct since both m_t and m_c are influenced by A, b, h and σ_c. An increase in any of these variables causes an increase both in m_t and m_c. In the case under consideration A influences mainly m_t, whereas b, h and σ_c influence mainly m_c. On account of the error in the approximation $\overline{m}$ calculated from Eq. (1) is slightly too small. The difference between $\overline{m}$ calculated from Eq. (1) and $\overline{m}$ calculated from the classical theory, cf. Eqs. (3a) and (3b), is therefore somewhat larger than the real difference. This error is usually negligible. However, in the case when the variation in h is great compared with the variation in the other variables, the error can become appreciable. This case can be met with when the effective depth of the structural member is small. On the above assumptions, the mean strength of the beam is

$$\begin{aligned} \overline{m} &= \int_0^\infty x\, f_{mt}(x)\,[1 - F_{mc}(x)]\,dx + \int_0^\infty x\, f_{mc}(x)\,[1 - F_{mt}(x)]\,dx = \\ &= \overline{m}_t + \overline{m}_c - \int_0^\infty x\, f_{mt}(x)\, F_{mc}(x)\,dx - \int_0^\infty x\, f_{mc}(x)\, F_{mt}(x)\,dx \end{aligned} \tag{1}$$

Eq. (1) differs from the classic method of calculation, in which the dispersion in the various quantities is disregarded. According to this method $\overline{m} = \overline{m}_t$ for $\overline{m}_t < \overline{m}_c$ and $\overline{m} = \overline{m}_c$ for $\overline{m}_c < \overline{m}_t$ (elastic and inelastic theories). In order to estimate the magnitude of the deviation due to this difference, we shall study some cases in what follows. For this purpose, we assume that m_t

and m_c are distributed in accordance with the normal distribution. It is to be observed, however, that the actual distribution of these quantities is not exactly normal. As a rule, this distribution is somewhat negatively skew, cf. ARNE I. JOHNSON, Strength, Safety and Economical Dimensions of Structures, Bulletin No. 22, Swedish State Committee for Building Research (Bulletin No. 12, Division of Building Statics and Structural Engineering, Royal Institute of Technology), Stockholm 1953. On the other hand, the effect produced on the determination of the mean value $\overline{m}$ by a small deviation in the form of the distribution is relatively slight. This case will therefore be disregarded in what follows. Furthermore, in carrying out the integration, the lower limit of the integrals in Eq. (1) is assumed to be $= -\infty$. In normal cases, the effect of this assumption on the final numerical result is completely negligible. This assumption has been made here in order to ensure a formal agreement with the tables available in print.

$$\overline{m} = \overline{m}_t + \overline{m}_c - \overline{m}_t \Phi\left(\frac{\overline{m}_t - \overline{m}_c}{\sqrt{s_t^2 + s_c^2}}\right) - \overline{m}_c \Phi\left(\frac{\overline{m}_c - \overline{m}_t}{\sqrt{s_t^2 + s_c^2}}\right) - \sqrt{s_t^2 + s_c^2}\, \varphi\left(\frac{\overline{m}_c - \overline{m}_t}{\sqrt{s_t^2 + s_c^2}}\right) \tag{2}$$

To facilitate comparisons with calculations made by means of the classic method, Eq. (2) is rewritten in the form

$$\overline{m} = \overline{m}_t - \beta_1 \sqrt{s_t^2 + s_c^2} \tag{3a}$$

$$\overline{m} = \overline{m}_c - \beta_2 \sqrt{s_t^2 + s_c^2} \tag{3b}$$

Moreover, we introduce

$$\overline{m}_c - \overline{m}_t = q \sqrt{s_t^2 + s_c^2} \tag{4}$$

Calculated numerical values of β_1 and β_2 corresponding to various values of q are given in Table 1. As may be seen from Table 1 and Eqs. (3a) and (3b), the deviation of the values of $\overline{m}$ calculated in accordance with the above statistical theory from those computed in conformity with the classic theory increases as the value of q becomes smaller. The former and the latter theories are in agreement only when $s_t = s_c = 0$. The case where $\overline{m}_t = \overline{m}_c$ $(q = 0)$ is of special interest. It corresponds to the case where the beams are said to be provided with "*balanced reinforcement*". In this case, the difference between the statistical and the classic theories reaches its maximum value.

Table 1. Numerical Values of Constants β_1 and β_2 in Eqs. (3a) and (3b)

q	−4	−3	−2	−1	0	1	2	3	4
β_1	(4,00001)	(3,00038)	(2,00849)	(1,08331)	0,39894	0,08331	0,00849	0,00038	0,00001
β_2	0,00001	0,00038	0,00849	0,08331	0,39894	(1,08331)	(2,00849)	(3,00038)	(4,00001)

In the deduction of Eq. (1), only the failure in bending (tension failure or compression failure) has been taken into consideration. At the same time, the failure in shear or the failure of bond can also be taken into account in an analogous manner. If the strength corresponding to each type of failure is statistically independent of the strength referred to any other type of failure, then the strength of a beam related to various types of failure can be expressed in the general form

$$\overline{m} = \Sigma \int_0^\infty f_\nu(x)\, \Pi \,[1 - F_\mu(x)]\, d\,x \tag{5}$$

where the sum is extended over all types of failure, while the product is extended over all types of failure except the ν-th type.

Strength at a Certain Definite Cross Section Submitted to a Moment

The ultimate moment is a function of several variables

$$m = g\,(\sigma_s, \sigma_c, h, b, A, E) \tag{6}$$

Since m is not a linear function of the different variables, its mean value cannot be calculated exactly by inserting the mean values of these variables in Eq. (6). This procedure, though incorrect, is used in the classic method of calculation. In fact, this procedure is approximately correct only on the assumption that m is a nearly linear function of the variables within the greater part of their intervals of variation. An accurate determination of $\overline{m}$ is made in what follows. It is assumed in what follows that the variables are statistically independent of each other. This assumption is normally correct.

The mean value of the ultimate moment can be obtained from the general expression

$$\overline{m} = \int g\,(\sigma_s, \sigma_c, h, b, A, E)\, f_{\sigma s}(\sigma_s)\, f_{\sigma c}(\sigma_c)\, f_h(h)\, f_b(b)\, f_A(A)\, f_E(E)\, d\,\sigma_s\, d\,\sigma_c\, dh\, db\, dA\, dE \tag{7}$$

The effects produced by the quantities in question on the ultimate moment at a certain definite cross section cannot be considered to be completely investigated. This statement holds true both for tension failure and for compression failure. In the present general study of the effect of the dispersion in these quantities, we shall assume, however, that the equation of the ultimate moment is known. For this purpose, we choose a form of this equation which is in agreement with most of the inelastic theories. At the same time, it is to be noted that the general result would be essentially similar if some other theories, e. g. the elastic theories, were supposed to be valid. In the following study of the ultimate moment, we assume that the beam undergoes either tension failure alone or compression failure alone.

Tension Failure

As regards tension failure, the various inelastic theories are closely in agreement. The ultimate moment of a beam of rectangular cross section can be written

$$m_t = \sigma_s A\, h - \alpha \frac{\sigma_s^2 A^2}{\sigma_c b} \tag{8}$$

where α is a constant which varies from 0,5 to 0,7 in the different theories.

Since m_t is not a linear function of each variable, the mean value $\overline{m}_t$ cannot be calculated exactly by inserting the mean value of each variable in Eq. (8), as has already been pointed out in the above.

If Eq. (8) is expanded in a Taylor series, and is inserted in Eq. (7), then we obtain, after integration,

$$\overline{m}_t = \bar{\sigma}_s \bar{A}\,\bar{h} - \alpha \frac{\bar{\sigma}_s^2 \bar{A}^2}{\bar{\sigma}_c \bar{b}} \left[1 + \left(\frac{s_{\sigma s}}{\bar{\sigma}_s}\right)^2\right] \left[1 + \left(\frac{s_A}{\bar{A}}\right)^2\right]$$
$$\left[1 + \left(\frac{s_{\sigma c}}{\bar{\sigma}_c}\right)^2 - \frac{\mu_{\sigma c}^3}{\bar{\sigma}_c^3} + \frac{\mu_{\sigma c}^4}{\bar{\sigma}_c^4} - + \ldots\right] \left[1 + \left(\frac{s_b}{\bar{b}}\right)^2 - \frac{\mu_b^3}{\bar{b}^3} + \frac{\mu_b^4}{\bar{b}^4} - + \ldots\right] \tag{9}$$

where $\mu_{\sigma c}^{\nu}$ and μ_b^{ν} are the ν-th central moments of σ_c and b respectively.

If, for instance, σ_c and b are assumed to be distributed in accordance with the normal distribution, then a study of the series in the last two expressions in square brackets in Eq. (9) shows that these series first converge relatively rapidly, but after that become divergent $\left(\frac{s_{\sigma c}}{\bar{\sigma}_c} \ll 1,\ \frac{s_b}{\bar{b}} \ll 1\right)$. This is due to the fact that the normal distribution presupposes as a necessary condition $\varphi(\sigma_c \leqq 0) > 0$ and a corresponding inequality in b. On the other hand, neither σ_c nor b can assume values that are less than zero. Moreover, Eq. (8) is not applicable to very small values of σ_c and b $(m_c > m_t > 0)$. On condition that these circumstances are taken into account and that $\varphi(\sigma_c \geqq 2\bar{\sigma}_c)$ and $\varphi(b \geqq 2\bar{b})$ can be disregarded, the above-mentioned series are found to be convergent for applicable distributions. If the expressions in square brackets in Eq. (9) are multiplied and if all powers higher than the second in the bracket are disregarded, then we obtain

$$\overline{m}_t = \bar{\sigma}_s \bar{A}\,\bar{h} - \alpha \frac{\bar{\sigma}_s^2 \bar{A}^2}{\bar{\sigma}_c \bar{b}} \left[1 + \left(\frac{s_{\sigma s}}{\bar{\sigma}_s}\right)^2 + \left(\frac{s_{\sigma c}}{\bar{\sigma}_c}\right)^2 + \left(\frac{s_A}{\bar{A}}\right)^2 + \left(\frac{s_b}{\bar{b}}\right)^2\right] \tag{10}$$

We shall now compare the mean value of the ultimate moment given by Eq. (10) with the mean value of this moment calculated in conformity with the classic theories. Accordingly, in the latter case, we assume $s_{\sigma s} = 0$, $s_{\sigma c} = 0$, $s_A = 0$ and $s_b = 0$. This comparison will be based on relatively extreme values of these quantities. The value of the ratio $\frac{\bar{\sigma}_s}{\bar{\sigma}_c}$ can vary within the approximate limits from 10 to 40. The factor $\frac{\bar{A}}{\bar{b}\,\bar{h}}$ is supposed to have the greatest value that is obtained when the reinforcement is "balanced". For the extreme values of the ratio $\frac{\bar{\sigma}_s}{\bar{\sigma}_c}$, the respective maximum values of this factor are about 0.07 and 0.012. The constant α is assumed to have its upper limit value, i. e. about 0.7. As a rule, $\frac{s_{\sigma s}}{\bar{\sigma}_s} \leqq 0.10$ and $\frac{s_{\sigma c}}{\bar{\sigma}_c} \leqq 0{,}20$, and we choose these values for the numerical calculations. Furthermore, we assume $s_A = 0$ and $s_b = 0$.

Table 2. Extreme effect of Dispersion in σ_s and σ_c on Mean Value of Ultimate Moment Causing Tension Failure, cf. Eq. (10)

$$\alpha = 0{,}7, \quad \frac{s_{\sigma s}}{\bar{\sigma}_s} = 0{,}10, \quad \frac{s_{\sigma c}}{\bar{\sigma}_c} = 0{,}20$$

$\frac{\bar{\sigma}_s}{\bar{\sigma}_c}$	$\frac{\bar{A}}{\bar{b}\,\bar{h}}$	$\frac{\bar{m}_t}{\bar{\sigma}_s \bar{A}\,\bar{h}}$		$\frac{\bar{m}_{t\text{-stat. theory}}}{\bar{m}_{t\text{-class. theory}}}$
		Calculated from Eq. (10)	Calculated from Classic Theory	
10	0,07	0,486	0,510	0,953
40	0,012	0,647	0,664	0,974

The results of these calculations are reproduced in Table 2. As is seen from this table, for the assumed values of the quantities involved, the maximum difference between the mean value determined in accordance with the statistical theory and the mean value calculated in conformity with the classic theory amounts to about 5 per cent. In ordinary practical calculations, this difference may often be disregarded. On the other hand, this difference is of interest in evaluations of test results, particularly when it is of the same order of magnitude as the differences between some variants of Eq. (8) which are in practical use at the present time.

Compression Failure

The equations of the ultimate moment in compression failure are somewhat different in form in the various inelastic theories. Nevertheless, most of these equations can be written in the common form

$$m_c = \sigma_c b\, h^2 g\,(\sigma_c, A, E) \tag{11}$$

In plastic theories, the function g = const. In deformation theories, g slightly increases as A and E become greater, whereas the relation between this function and the argument σ_c varies in some measure according to the variant of the theory of this type. All the same, to sum up, we can state that a variation in A, E or σ_c causes a relatively slight variation in g. In a general study, it is therefore sufficient to assume $g = \text{const.} = \gamma$. Even though this assumption does not hold true exactly, the resultant error can be assumed to be relatively small. Then we obtain the equation

$$m_c = \gamma \, \sigma_c \, b \, h^2 \tag{12}$$

If this equation is expanded in a Taylor series by analogy with Eq. (8), and if the mean value is calculated from this series, then we get

$$\overline{m}_c = \gamma \, \bar{\sigma}_c \, \bar{b} \, \bar{h}^2 \left[1 + \left(\frac{s_h}{\bar{h}}\right)^2\right] \tag{13}$$

This mean value is slightly higher than that computed in accordance with the classic theories without taking account of the dispersion in the quantities concerned. Since the maximum value of $\frac{s_h}{\bar{h}}$ is probably less than, or approximately equal to, 0,2, the maximum increase in the mean value may be expected to be about 4 per cent.

Effect of Variable Stress Distribution in Longitudinal Direction

If values of the strength of materials obtained from check tests on standard test specimens are to be compared with the strength of beams, then it is necessary, from a statistical point of view, to take into account the dimensions of the beam, the type of loading, and the actual stress distribution in the longitudinal direction of the beam.

It is obvious that the strength is also influenced by other factors, but the present study will be confined to the above-mentioned factors, whose effects are dependent on the dispersion in the quantities concerned. In most cases, these factors are not taken into consideration, and this can in part explain the frequent lack of agreement between experimental and theoretical results. For a general study of these factors, reference is made to Arne I. Johnson, op. cit. p. 77. In the present paper, we shall confine ourselves to a schematic study of a beam submitted to a constant moment.

When a beam is subjected to a constant moment, the stress in the reinforcement will normally not be constant in the longitudinal direction of the beam after the formation of cracks due to bending on the side in tension. This is due to the structural action of the concrete between the cracks on the side in tension. On account of this action, the stress in the reinforcement between the cracks is lower than in the cracks. A corresponding reduction in stress takes place on the side in compression, although this reduction is usually smaller than on the side in tension. The actual stress distribution is intricate, and is normally dependent on the strength of bond between the reinforcement and the concrete, the modulus of rupture (the tensile strength in bending) of the concrete, the method of loading, etc. As a rule, the actual form of the stress distribution cannot be determined exactly.

It is of interest to determine that length of reinforcement subjected to a constant stress which shall have the same strength as the actual reinforcement in the beam. In order to estimate the order of magnitude of this length, it is necessary to assume a certain definite stress distribution in the longitudinal direction of the beam. Various distributions are possible (cf. ARNE I. JOHNSON, Deformations of Reinforced Concrete, I.A.B.S.E., Publication No. XI, Zürich, 1951). Simple explicit solutions are obtained if we assume a linear variation of the stress in the reinforcement from its maximum value σ_{max} in the cracks due to bending on the side in tension to its minimum value σ_{min}. Furthermore, we suppose that the values of σ_{max} in all cracks on the side in tension, just as the values of σ_{min} between all these cracks, are equal. Moreover we assume $\frac{\sigma_{min}}{\sigma_{max}} = \text{const.}$

The distance between the cracks on the side in tension and the position of σ_{min} are of no importance in this connection. The distribution function of the strength of reinforcement is assumed to be of the form

$$F(x) = 1 - e^{-(\rho x)^k}$$

We have chosen this distribution function (which has been introduced into the theory of strength of materials by W. WEIBULL, A statistical theory of strength of materials, IVA, Proc. 151, Stockholm 1939) because it enables us to obtain explicit solutions. On the other hand, it is not certain that this function corresponds exactly to the true distribution. However, as has already been pointed out in the above, a small deviation from the true distribution produces, as a rule, a very slight effect on the result in the determination of mean values. The distribution function of the total strength of the beam under consideration is

$$F_1(\sigma) = 1 - e^{-B} \tag{14}$$

where

$$B = \int_l \ln[1 - F(\sigma_l)]\, dl \tag{15}$$

After that, the mean value is obtained as usual (cf. e.g. Eq. (7)). If the length of the beam is denoted by l and if the beam is assumed to be provided with a *single* reinforcement bar, then this reinforcement bar has the same mean strength as a corresponding *single* reinforcement bar in tension which is submitted to a constant stress and which has the length l_{red}, given by

$$l_{red} = l \, \frac{1}{1 - \frac{\sigma_{min}}{\sigma_{max}}} \, \frac{1}{k+1} \left[1 - \left(\frac{\sigma_{min}}{\sigma_{max}}\right)^k\right] \tag{16}$$

where k is determined from

$$\left(\frac{s_{\sigma s}}{\bar{\sigma}_s}\right)^2 = \frac{\Pi\left(\frac{2}{k}\right)}{\Pi^2\left(\frac{1}{k}\right)} - 1 \tag{17}$$

Table 3. Numerical Values of $\frac{l_{red}}{l}$ *Calculated from Eq.* (16)

$\frac{s_{\sigma s}}{\bar{\sigma}_s}$	$\frac{\sigma min}{\sigma max}$ / k	0	0,25	0,50	0,75	0,9	0,99	1
0,01	128	0,008	0,010	0,016	0,031	0,078	0,564	1
0,02	63	0,016	0,021	0,031	0,063	0,156	0,738	1
0,05	24,8	0,039	0,052	0,078	0,155	0,360	0,860	1
0,10	12,1	0,076	0,102	0,153	0,296	0,550	0,878	1

Some values of l_{red} are given in Table 3. Note that the beam has been assumed to be provided with a *single* reinforcement bar! If the beam is reinforced with several bars, then l_{red} decreases as the number of reinforcement bars becomes greater.

The length of the test specimens used in check tests on reinforcement bars is often greater than l_{red}, particularly when the beam is reinforced with several bars. This implies that $\bar{\sigma}_s$ in the beam is greater than the value obtained from the check tests. The effect of this circumstance is tantamount to an increase in the ultimate moment of the beam in tension failure. In previous comparisons, the strength of beams loaded to tension failure has often been lower than that obtained from the theories, whereas compression failure tests have given varying results. It is evident that the above corrections relating to tension failure, see Eq. (10), will produce an effect in the opposite direction. On the other hand, the corrections for the reduction in the tensile stress in the reinforcement have often an effect in the direction of the tests.

In this paper, we shall not make any detailed comparisons with test results for the reason, among others, that the form of the basic equation, Eq. (6), is not exactly known. Nevertheless, it is important to take account of the above-mentioned statistical factors in theoretical studies and in comparisons between experimental and theoretical results.

Summary

The strength of reinforced concrete beams is studied in this paper with due regard to the dispersion in the quantities involved, e.g. the strength of concrete and the strength of reinforcement. Attention is directed to the deviations from the classic method of calculation in which the dispersion in these quantities is disregarded.

General equations are deduced for determining the mean strength of a beam, see Eqs. (1), (5), and (7). The difference between the results obtained from the statistical and the classic theories has been calculated in some cases, and is expressed by Eqs. (3a) and (3b) as well as by Eqs. (9), (10) (tension failure) and (13) (compression failure). Finally, a study is made of the effect produced on the strength by the variable stress distribution in the longitudinal direction of the beam (the influence of the concrete in tension between the cracks due to bending on the side in tension).

Résumé

Dans le présent rapport, l'auteur étudie la résistance des poutres en béton armé de façon à tenir compte de la dispersion des quantités en question, par ex. de la résistance du béton et de la résistance des armatures. Il en résulte des écarts de la méthode de calcul classique, dans laquelle on néglige la dispersion de ces quantités.

L'auteur a établi des équations générales pour la détermination de la résistance d'une poutre, voir les équations (1), (5) et (7). La différence entre les résultats obtenus au moyen des théories statistiques et des théories classiques a été calculée dans quelques cas. Cette différence est exprimée par les équations (3a) et (3b) ainsi que par les équations (9), (10) (rupture sous extension) et (13) (rupture sous compression). En outre, l'auteur examine l'effet produit sur la résistance par la distribution variable des contraintes dans le sens longitudinal de la poutre (l'influence du béton soumis à l'extension entre les fissures du côté soumis à l'extension de la poutre fléchie).

Zusammenfassung

Im vorliegenden Bericht untersucht der Verfasser die Festigkeit von Eisenbetonträgern unter Berücksichtigung der Streuung (Dispersion) der zugehörigen Größen, z. B. der Beton- und Bewehrungsfestigkeit. Dabei ergeben sich Abweichungen vom klassischen Berechnungsverfahren, bei dem die Streuung dieser Größen vernachlässigt wird.

Für die Bestimmung der Festigkeit eines Trägers stellt der Verfasser allgemeine Gleichungen auf, siehe Gl. (1), (5) und (7). Der Unterschied zwischen den Ergebnissen, die den statistischen und den klassischen Theorien entsprechen, wurde für einige Fälle berechnet, siehe Gl. (3a) und (3b) sowie Gl. (9), (10) (Zugbruch) und (13) (Druckbruch). Schließlich behandelt der Bericht die Wirkung, welche von der veränderlichen Spannungsverteilung in der Längsrichtung des Trägers auf die Festigkeit ausgeübt wird (d. h. den Einfluß des auf Zug beanspruchten Betons zwischen Rissen auf der Zugseite des auf Biegung beanspruchten Trägers).