Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 13 (1953)

Artikel: Magnesium alloy structures

Autor: Ghaswala, S.K.

DOI: https://doi.org/10.5169/seals-13197

Magnesium Alloy Structures

Ouvrages en alliages ultra-légers

Bauwerke aus Magnesiumlegierungen

S. K. GHASWALA, Chartered Engineer, Bombay, India

I. Introduction

Metallic magnesium was first produced by Sir HUMPHRY DAVY [1][1]) in 1808, although Nehemiah Grew, an English physician, had, as early as 1695, attempted to learn about its presence, by investigating the mineral contents of the spring waters at Epsom. Although he failed to produce the metal, he isolated magnesium sulphate, generally known as Epsom salt. After DAVY'S work, slightly more efficient methods of production were evolved by BUSSY in 1828, FARADAY in 1833 and by DEVILLE, CARON and ROUSSEAU in 1856/57. In 1863 SONSTADT developed a process in England for small quantity production of magnesium, while investigations were also proceeding on in Germany, which ultimately culminated in the development of an electrolytic method first evolved by BUNSEN in 1852 at Heidelberg [2]. Industrial production of magnesium started towards the close of the last century, while the work of PISTOR and his collaborators later in Germany was of decisive influence in the development of high strength alloys known as "Elektron", which opened up a wide range of uses [3]. No wonder Germany became the world's largest producer of these light alloys and maintained a position of supremacy till nearly the beginning of the second World War. Since 1940, the metacentre of production shifted from Europe to the United States and Canada, which have now become the arsenals for magnesium [4—7].

The magnesium rich alloys developed in Germany between 1908—1920, have in the last three decades taken an important place in modern industry throughout the world. In England these alloys are known as "Elektron" and "Magnuminium", while in the United States they are classified as "Dowmetal" and "Mazlo". The alloys used for structural purposes generally contain alu-

[1]) For references see end of paper.

minium, manganese and zinc as principal constituents. Very recently zirconium has been used as an alloying element to improve the mechanical properties still further. A large variety of alloys in both cast and wrought form are thus available at present for utilisation in engineering applications. A search for extant literature on magnesium alloys in engineering reveals two distinct facets. In the first place, most of the publications on the design aspects pertain to aeronautical engineering in which field, this ultralight metal has been widely used. The second facet shows that although considerable literature has grown and gathered round the subjects on production and fabrication methods, as well as on metallurgy, there is a dearth of published material in the field of structural theory and design, while at the same time its actual applications in that branch of engineering have also been very limited. Only very few publications are at present available on structural engineering design, and even these are scattered in English, American and foreign journals, some of which are not easily accessible [8 to 18].

Table I. Range of Mechanical and Physical Properties of Magnesium Alloys

Density at 20° C for 99.9% purity. . . .	1.738 gm/cm³
Density of alloys	1.76 to 1.87 gm/cm³
Weight	(Average 1.8 for structural work 110 lbs/cft.
Melting temperature (liquidus)	1075 to 1200° F
Boiling point	2025° F
Shrinkage during solidification	4.2 percent
Coeff. of Thermal expansion	
for 68° F to 212° F } for pure magnesium .	$145 \cdot 10^{-7}$ inch/inch/° F
for 68° F to 570° F } for pure magnesium .	$156 \cdot 10^{-7}$ inch/inch/° F
Specific Heat.	0.249 cal/gm/° C
Thermal conductivity at 212° F	0.17 to 0.33 cal/Sec/sq. cm/cm/° C
Latent Heat of Fusion	89 cal/gm
Young's Modulus of Elasticity	$6.5 \cdot 10^{6}$ lbs/sq. inch
Modulus of Rigidity (Shear modulus) . .	$2.4 \cdot 10^{6}$ lbs/sq. inch
Poisson's Ratio	0.34
Ultimate tensile strength	6 to 18 tons/sq.in. (cast) 14 to 26 tons/sq.in. (wrought)
Yield strength (0.2% proof stress). . . .	2 to 10 tons/sq.in. (Cast) 6 to 17 tons/sq.in. (wrought)
Elongation.	1 to 10% in 2 inches (Cast) 5 to 19% in 2 inches (wrought)
Endurance Limit	2.7 to 6.5 tons/sq.in. (Cast) 3.5 to 8 tons/sq.in. (wrought)
Brinell Hardness	33 to 78 (500 kg load and 10 mm ball)
Reflectivity for white light	73 percent
Electrical resistivity at 68° F	5 to 18 michrohms/cc.
Electrical conductivity @ 68° F	9.7 to 34.5 percent of I.A.C.S.

II. Engineering Properties

The structural worth of a new material cannot be assessed without a clear understanding of its engineering properties, as well as a comparison of its strength with the commonly accepted materials of construction. As the properties of different alloys vary with their composition and mode of production, a range of typical properties affecting structural work is given in Table I.

Out of these properties, those which specially characterise magnesium for direct use in structural engineering are density, tensile strength, and elastic modulus.

Magnesium is the lightest of the structural metals used to an appreciable extent at present. Although beryllium has a density practically the same as magnesium, and lithium is only one third in weight, still these metals have not reached a stage of commercial importance and their applications in structural engineering field are practically non existent. In Table II are given the relative weights of a number of structural materials, based on a unit weight of magnesium. It will be seen from this that aluminium is nearly $1\frac{1}{2}$ times heavier than magnesium, while steel is $4\frac{1}{2}$ times as dense. Among the non-metals, timber has practically half the weight, while reinforced concrete is a little less than $1\frac{1}{2}$ times its density.

The tensile strength of magnesium compares favourably from a structural standpoint and is more less akin to that of aluminium alloys. In fact like aluminium alloys, but unlike structural steel magnesium alloys do not exhibit a definite yield point. As such the criteria of proof stress and the ultimate tensile strength assume marked importance in design.

Table II. Relative Weights of Structural Materials

Material	Specific Gravity (gms/cc.)	Relative Weight (Magnesium = 1)
Lithium	0.534	0.29
Magnesium	1.82	1.00
Beryllium	1.82	1.00
Aluminium	2.180	1.54
Titanium	4.50	2.47
Zirconium	6,50	3.57
Zinc die castings	6.70	3.68
Grey cast iron	7.10	3.90
Melleable iron	7.30	4.02
Structural or alloy steel	7.84	4.30
High brass	8.5	4.67
Timber	0.8	0.44
Phenolic Sheet	1.3	0.72
Reinforced Concrete	2.3	1.27

Table III. Modulus of Elasticity and Rigidity of Structural Materials

Material	Mod. of Elasticity lbs/sq. in.	Mod. of Rigidity lbs/sq. in.	Relat. Elast. Mod. (Magnesium = 1)
Magnesium alloys. . .	6.5×10^6	2.4×10^6	1.00
Aluminium alloys. . .	10×10^6	3.85×10^6	1.54
High Brass	15×10^6	5.64×10^6	2.30
Grey Cast Iron	15×10^6	6.0×10^6	2.30
Malleable Iron	25×10^6	10.7×10^6	3.86
Structural steel. . . .	30×10^6	12×10^6	4.55
Timber (maple). . . .	1.6×10^6	—	0.25
Phenolic Sheet	1.0×10^6	—	0.15

The most striking property of magnesium is its very low modulus of elasticity and rigidity as compared with other materials, as listed in Table III. In view of the very low elastic modulus of magnesium, viz. two ninths of steel and two thirds of aluminium, the problems of elastic instability especially in compression members assume considerable significance.

III. Comparison of Magnesium With Other Metals

The various developments in metallurgy and crystal structure of metals have resulted in the production of high strength magnesium alloys, unknown a few decades ago. Generally the wrought alloys are stronger than cast alloys and are quite suited for structural purposes. The tensile strength of such wrought forms varies from 16 to 23 tons/sq. in. and 0.1 % proof stress from 9 to 14 tons/sq. in. The recently developed zirconium alloys show still higher strength and improved mechanical properties.

The high specific tenacity (i. e. ratio of ultimate tensile strength to density) of magnesium alloys results in a large saving in weight. A comparison of various designs under varied loading conditions for sheets and structural shapes indicate that magnesium offers wide possibilities of bringing about material economy if used in a rational manner. Thus there is a saving in weight of nearly 60 percent by replacing steel with magnesium for static loading and where equal strength or stiffness are desired, and 50 per cent for dynamic loading. Under the same conditions and with static loading, an average saving of 20 percent results when aluminium is the replaced material. For structural shapes the savings are more or less the same, the average being slightly higher for steel viz. 65 per cent, and slightly lower for aluminium viz. 18 percent for static loadings. In the case of dynamic loading and rough handling conditions there is no change.

In spite of the very low elastic modulus, its low density permits the use of increased material to reduce deflection, without appreciably increasing the

Table IV. Specific Tenacity and Specific Elastic Modulus of Metals

Material	Density	Ultimate Tensile Strength (T/sq. in.)	Modulus of Elasticity (lbs/sq. in.)	Specific Tenacity	Specific elastic modulus
Structural Steel	7.84	28.0	30×10^6	3.58	3.84×10^6
Aluminium alloy	2.80	25.0	10×10^6	8.95	3.57×10^6
Magnesium alloy	1.82	19.0	6.5×10^6	10.40	3.56×10^6
Timber (maple)	0.67	7.0*)	1.6×10^6	10.40	2.2×10^6

*) In the case of timber the value of 7.0 tons/sq. in. is the Modulus of Rupture.

overall weight of the structural component. This aspect becomes more clear from a comparison of the specific tenacity and specific elastic modulus of the three principal metals viz. magnesium, aluminium and steel. The specific tenacity as given above is the ratio of ultimate tensile strength to density of the material and is an indication of it strength/weight characteristic, while the specific elastic modulus is the ratio of modulus of elasticity to density and is an indication of the stiffness/weight ratio of the metal. The values of these quantities for a typical set of aluminium and magnesium alloys are given in Table IV, and a comparison of their merits made in Table V, in which the relative specific tenacity and relative specific elastic modulus of steel and aluminium are compared with magnesium alloys. It is very clear from this table that in strength/weight comparison magnesium is three times superior to steel and slightly more than aluminium; while in stiffness/weight comparison, all the three metals are practically alike, and are nearly one and a half times superior to timber. It is a well-known fact that theoretically the weight of a structural member increases as the first power of depth, the strength increases as the square, and the rigidity or stiffness as the cube. In beams and slender struts, or in other similar components,

Table V. Relative Specific Tenacity and Elastic Modulii of Metals

Material	Relative specific tenacity	Relative specific elastic modulus
Structural Steel	$\frac{1}{2.9}$	1.08
Aluminium alloy	$\frac{1}{1.16}$	1
Magnesium alloy	1	1
Timber (maple)	1	$\frac{1}{1.62}$

where strength is governed by their elastic stability, variations in the cross sectional dimensions have, as a general rule, a greater effect on their strength than have equal variations in the mechanical properties of the material. It is evident from this, that if a comparison is made between two materials of varying densities but more or less similar specific tenacity, the lower density material is more efficient for the type of members described above owing to the greater thickness afforded by such materials, without any increase in the overall weight.

On the other hand, for members in tension, or for short and thick struts, where the problems of elastic instability do not arise, the importance of the variation of strength of materials, outweighs their densities. The specific tenacity rather than the density therefore becomes the guiding criterior of design. In dealing with aluminium structural design the author [19] had evaluated its relative merits with steel by the introduction of the concept of "Criterion of Merit". This concept can also be utilised in the design of magnesium alloy structures, in order to gauge their suitability under various stress conditions. The values of this criterion of merit are given in Table VI numerical values for the ratio of "C" for magnesium to aluminium, to steel, to timber, and to a plastic. This criterion indicates that the member having the maximum value of "C" will be the most efficient from both the stand points of strength and weight. In this particular table when the value of "C" is greater than one, the magnesium alloy member can be taken as superior to its counterpart in that particular condition of stress and vice versa. Thus it can be seen that in the case of pure tension or compression magnesium is superior to aluminium in strength but is on a level with it as far as stiffness is concerned. On the other hand, magnesium is superior to timber and the plastic in pure tension in both strength and stiffness, but is inferior to them when designed on the basis of geometrical similarity. The table also affords a rapid means of comparing one material with another as regards their strength, stiffness and material consumption.

Some theoretical investigations have been carried out on cantilever beams, with a view to assess their relative merits with aluminium alloy members [23]. The comparison is made by plotting the ratio of permitted stresses for aluminium to magnesium against a ratio of equal bending strength, of equal weight and of equal deflection of the two metals. The permitted stress is chosen to indicate not any specific stress, but any value of maximum working stress such as proof stress, for proof stress design or fatigue stress for design of fatigue. The sections considered are round, square and hollow bars, and I, T, [and angles, all of which are assumed to have unit dimensions, with only variable thickness, and to follow a straight line curve of stress/strain within the elastic range. The effects of shear stress are also omitted. Within the limits set forth, and the assumptions made, which of course are not fully justifiable under all design conditions, it is interesting to note the manner of behaviour of these

Table VI. Criterion of Merit "C" for Structural Materials

Stress Condition	Criterion C		Ratio of C for Magnesium to:							
			Aluminium		Steel		Timber		Plastic *)	
	Strength	Stiffness	Strength	Stiffness	Strength	Stiffness	Strength	Stiffness	Strength	Stiffness
Pure tension or Compression or bending with constant depth and variable width	$\frac{F}{\delta}$	$\frac{E}{\delta}$	1.35	1.0	3.45	1.98	1.38	1.78	1.09	1.36
Bending with constant width and variable depth	$\frac{F}{\delta^2}$	$\frac{E}{\delta^3}$	2.08	2.36	14.8	17.4	0.66	0.35	0.87	0.86
Bending with variable depth and width to give geometrically similar section	$\frac{F}{\sqrt{\delta^3}}$	$\frac{E}{\delta^2}$	1.71	1.54	7.13	4.02	0.915	0.79	0.98	1.08

Note: The following values of strength F, Density δ, and Elastic Modulus E, have been assumed.
F for Aluminium 25, for Steel 27.5; for timber 7, for plastic 16, and Magnesium 22, all in tons/sq. inch.
δ for Aluminium 2.80; for Steel 7.84; for timber 0.8; for plastic 1.45; and Magnesium 1.82 all in gms/cubic centimeter.
E for Aluminium $10 \cdot 10^6$; for Steel $30 \cdot 10^6$; for timber $1.6 \cdot 10^6$; for plastic $3.8 \cdot 10^6$ and for magnesium $6.5 \cdot 10^6$, all in lbs/sq. inch.
(The units in the three functions are not corrected to the same values, as C is given only as a ratio and as such, the figures would in any event remain the same.)

*) The plastic, represented, here is the high density craft paper phenol formaldehyde resin having resin content of 28% and formed with a laminating pressure of 2000 lbs/sq. inch.

beams. In fact such a method of comparison, is in the author's opinion far more suitable for preliminary design formulation, as herein the large variety of alloys with different strengths are brought under one cover and their merits evaluated, instead of comparing any individual alloy. The comparison is thus based not on a specific alloy, but on the general permitted stress ratio, thereby enabling the designer to choose the particular alloy best suited for his purpose, within the minimum time.

If Fp denotes the permitted stress ratio of aluminium to magnesium, it is found that for equal weight the bending strength of round or square aluminium beam exceeds that of a magnesium beam only when Fp is greater than 1.9. In the case of channels, tees, tubes and I sections, the strength of aluminium beams exceeds that of magnesium when Fp is about equal to or exceeds 1.4. In general it is observed that the relationship between Fp and the ratio of bending strength of aluminium beam to that of magnesium beam termed M, is given by the expression

$$M = 0.53\ Fp \qquad \text{for round and square bars}$$

and

$$M = 0.7\ Fp \qquad \text{for I, [, and tube sections.}$$

These are straight line curves and are derived for a range of ratio Fp varying from 1.0 to 2.0.

In the case of beams of equal bending strength, it is found that for Fp equal to 1.9 for round and square bars, and about 1.45 for other sections, the weights of aluminium and magnesium beams are the same. However, unlike the previous case, with decreasing values of Fp the aluminium beam becomes heavier than the magnesium beam, and reaches its maximum at Fp equal to one.

Thus when weight and bending strength are considered together, the critical value of Fp should lie in the region 1.9 for round and square bars and 1.5 for other sections, that is in other words the permitted stress in magnesium alloy bars must be greater than approximately half that in the corresponding aluminium alloy, and in other sections of beams of the type described above more than three quarters that in the corresponding aluminium alloy, before it can demonstrate a weight or strength advantage over an aluminium member.

In the case of members with equal bending strength, it is found that equal deflections result when the value of Fp is 1.35 for square and round bars and 1.55 for sections. When these values are exceeded, the deflection in aluminium beam increases and when diminished the deflection is reduced. These conditions also exactly hold good in the case of beams of equal stiffness and variable bending strength. A clearer idea of these concepts can be had from Fig. 1, which shows these relationships between weight, stiffness and bending strengths of aluminium and magnesium alloy cantilever beams for I, [and tubes based on stress variation. One of the main advantage, in using magnesium alloy

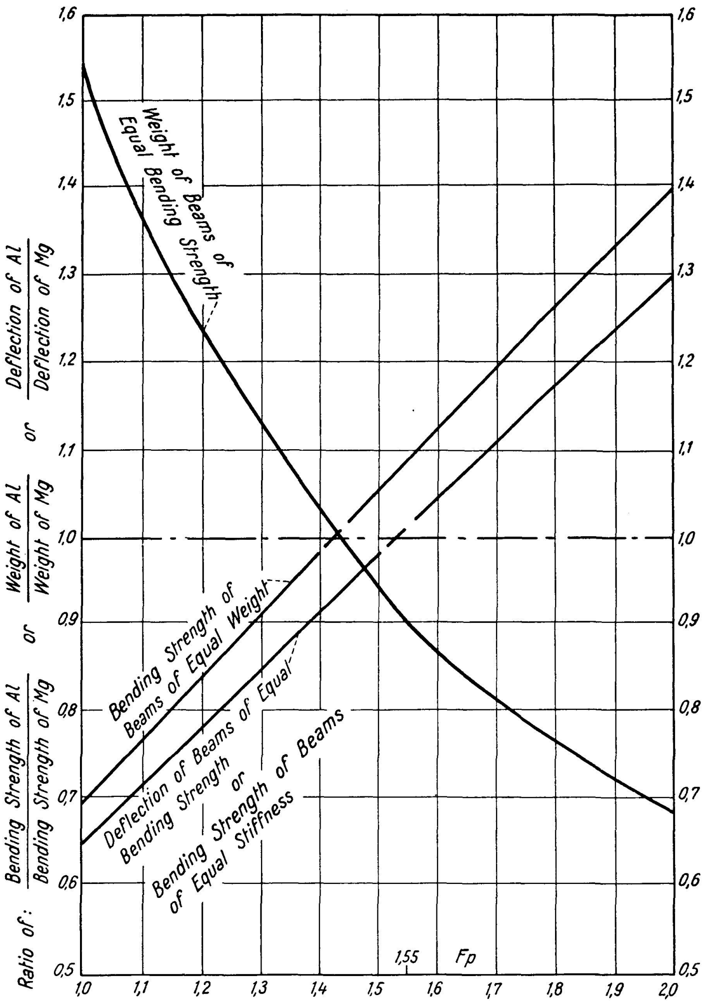

Fig. 1. Comparison of *Al* and *Mg* Cantilever Beams of I, [and Tubes

structures in special types of construction where resilience is of importance, is the ease with which shocks can be absorbed without in any way affecting the strength or stability. According to the laws of elasticity, resilience per unit volume R is given by

$$R = \frac{f^2}{2E}$$

where f is the proof stress and E the modulus of elasticity. Assuming a value of $f = 12$ T/sq. in. for magnesium and 18 T/sq. in. for steel, it can be seen that the proof resilience of magnesium is nearly twice that for steel. Since the proof stress for steel is $1\frac{1}{2}$ times that of magnesium, it can be realised that for a design based on the same factor of safety, the proof resilience of magnesium or its energy to absorb shocks, and deflection, is nearly three times that of steel. This characteristic can be put to good use in beams supporting dance hall floors, where a greater springiness is advantageous for dancers; in diving boards; and in the frames of automobiles and bicycles, where the higher resilience increases the riders sensation of comfort. It must be made clear that although these concepts give a more or less correct picture of the comparative strengths of various materials, in the preliminary stages of design, they cannot be taken for a final appraisal, as the special problems of elastic instability have to be fully solved, each on its own merits. Further the question of cost enters into the economy of the structure and is a factor that is not at all accounted for by the criterion of merit. According to REECE [20] there exists a relation between specific strength of a material to its cost.

If S denotes the load sustrained by one lb. of the material and d, the cost per unit weight of the fabricated product, then $\frac{S}{d}$ represents the specific strength per unit cost. In view of the very wide conditions of stress existing in a member, this simple relation cannot always yield correct results.

Without entering further into the detailed analysis and comparison, the following general aspects arrived at by the author after a number of investigations can be taken as a good guide for design. Based on yield strength criterion, magnesium offers very little advantage in axial tension members, as compared with aluminium and steel. In compression, the advantages of low weight become visible only for members having a slenderness ratio above 60, while below this figure, the light metal exhibits little economy. This is evident from the fact that in the short column range, the value of compressive yield strength forms the guiding criterion; whereas in the long column range the elastic modulus and the moment of inertia play an important part. In bending, various alternate methods of design are available depending on the geometrical proportions and required deflection. Thus for equal depth and strength, a magnesium beam is generally heavier than a high-strength aluminium beam, but lighter than a mild steel beam. If the deflection and depth are maintained the same, the weights of all three beams are more or less equal. A large weight saving results in magnesium beams, when their size is increased to carry the same loads and bear the same deflections as aluminium and steel beams. Instead of increasing the size of the member in all directions, if only the principal side of loading is increased (generally the depth), further weight savings are possible. However, such a procedure tends to produce local instability and has therefore to be carefully considered. On the other hand the disadvantage of the low modulus of elasticity can be compensated by the adoption of continuous beams. The deflection of a beam with fixed ends is only 1/5th of that occurring for pin ends, so that a fixed magnesium beam displays reduced deflection. An intermediate stage between full fixity and pin endedness, appears more desirable, for in addition to reducing deflection, such a partial fixity can produce positive and negative bending moments nearer to each other in magnitudes than with complete fixity. Thus the ideal states should give a positive and negative moment of $\frac{WL}{16}$, as against a value of $\frac{WL}{12}$ for negative bending moment at supports and $\frac{WL}{24}$ for positive bending moment at centre of a fixed beam.

IV. General Design Aspects

If magnesium alloys are to compete with other common structural materials, it is not only necessary to improve their mechanical properties, but it is imperative to evolve designs by the use of which these ultra-light alloys become equal to or even superior to other metals. Very little attempt has been made up to now to evolve newer concepts in designs, and whatever efforts have been made have been entirely in the realms of aeronautics. In this connection the work of Ridder [21] and a few others [22—28] is worthy of close study.

Tension

Members in pure tension are very simple to design and represent the most efficient type of structural components especially when designed in a material of high tensile strength. High tensile steel wire for prestressed girders and for manufacturing stranded cables for long suspension bridges are typical examples of this principle. If P is the allowable load and A the cross sectional area, then the allowable stress in pure tension is given by $\frac{P}{A}$. Magnesium alloys can be designed on the ultimate stress principles (also known as limit design), wherein the permissible load P can be worked out as under:

$$P = \frac{F}{A \cdot S}$$

where F is the ultimate tensile stress, and S is the safety factor, varying from 2.5 to 3. The area of cross section A is the net area, i. e. area obtained after making allowance for rivet holes in the case of riveted construction.

Although not common in steel construction, it is advisable in magnesium like aluminium, to have a limiting slenderness ratio for tension members, to prevent excessive bend or sway. The maximum value of this ratio can be based on the same formulation as in aluminium viz. $\frac{L}{k} = 150 + 0.01\, f$ where f is the lowest net section tensile stress in lbs/sq. in. to which the member will be subjected in practice. Assuming 21 tons/sq. in. to be the ultimate tensile stress for magnesium alloy and a safety factor of 3, then f will be 7 tons/sq. in. and the slenderness ratio will be $\frac{L}{k} = 150 + (0.01)\,(7)\,(2240) = 306.8$. In practice this value is rarely exceeded in normal designs.

Compression

In the present state of knowledge, the design of columns is carried out by a set of expressions established for each individual type of alloy with their individual constants. This expression for magnesium columns loaded axially and sufficiently stable to prevent local failure takes the form

$$P = \frac{A \cdot C}{1 + C\,(L/k)^2/\pi^2\,E}$$

where

P = ultimate load on column in lbs.

A = cross sectional area in square inches.

$\frac{L}{k}$ = slenderness ratio, in which L is the equivalent length of the column i.e. $\frac{L'}{k}$ for fixed ends and L' for pin ends; L' being the actual length.

E = modulus of Elasticity in lbs/sq.″ and C a constant varying according to the alloy used and its compressive yield strength Fy or 0.2 % proof stress. This expression holds good for values of $L/k > 40$.

Table VII

Compressive Yield strength F_y in lbs. per sq. in.	Value of Constant C
8,000	14,100
10,000	15,700
12,000	17,900
14,000	21,100
16,000	25,500
18,000	31,200
20,000	38,500
22,000	47,000
26,000	68,800
30,000	97,000
34,000	129,000

To indicate these relations, values of the constant C for a typical set of alloys having a compressive yield strength varying from 8000 lbs/sq. in. to 34,000 lbs/sq. in. are given in Table VII.

Curves showing relationship between stress and slenderness ratio for a few typical alloys are shown in Fig. 2. It is observed from Fig. 2 that soon after L/k increases to 60 and beyond, the various curves run very close to each other and converge near a value of $\frac{L}{K}$ of 150. This signifies that the stress in column depends on the value of L/k only and not on the compressive strength of the alloy used. If an average value of these curves is drawn, it can be seen, as worked out by the author, that the ultimate load $P \cdot$ lbs, on a magnesium column irrespective of the alloy used can be given on an average by the following two expressions:

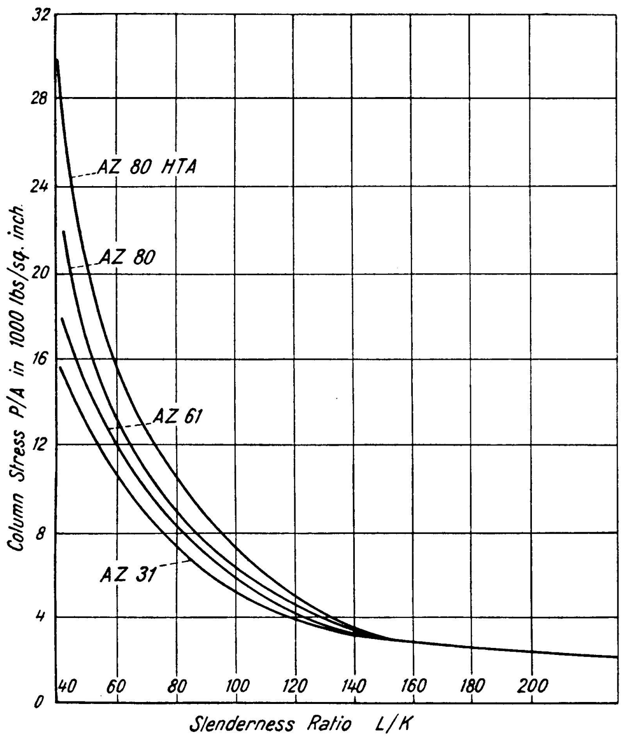

Fig. 2. *Mg* Column Curves

$$P = A\ Fy \text{ lbs for values of } L/k \text{ equal to or less than 40, and}$$

$$P = \frac{\pi^2 E}{(L/k)^2} \text{ for value of } L/k \text{ greater than 40.}$$

These expressions, can be used for preliminary design purposes where Fy is the compressive yield stress in lbs/sq. in., and A is the cross sectional area. By substituting the value of the tangent modulus, instead of the Young's modulus in the expression for E, a still closer approximation results.

Bending

In designing beams, the maximum moment M allowed to be developed is generally based on the maximum yield strength of the alloy. Since the tensile and compressive yield strengths of most of the magnesium alloys are not the same, the value of maximum stress f to be used in the Bernoulli-Euler relation $f = MZ$ where Z is the section modulus, is determined as the average of tensile and compressive yield strengths of the alloy used [22]. This applies to solid sections such as round, squares and rectangles. However in the case of I beams, channels and angles, it has been found that the yield strength in bending is slightly less than the average yield strengths in tension and in compression.

In view of the low elastic modulus of magnesium alloys, and consequent larger deflection, which is nearly four and a half times that of steel structures, it is necessary to know the exact deflection of the member. The expressions generally used in steel design are not exact, but sufficiently accurate for practical use. However, in magnesium design, they may at times be misleading.

From a knowledge of the stress/strain relations it can be shown that the exact deflection δ at mid-span for a simply supported beam of span L, load w lbs per unit length, and modulus of elasticity E lbs/sq.in., and Moment of Inertia I in.[4], is

$$\delta = \frac{5}{384}\,\frac{wL^4}{EI}\left[1+\frac{2\cdot 4\,d^2}{L^2}\,(0.8+0.5\,\mu)\right]$$

where d is the depth of the beam in inches, and μ is the Poisson's ratio.

Taking $\mu = 0.34$ for magnesium, the expression can be simplified to

$$\delta = \frac{5}{384}\,\frac{WL^4}{EI}\left[1+\frac{2.33\,d^2}{L^2}\right]$$

The first term outside brackets is the general expression used in steel design, while the other represents the correction factor, which depends purely on the ratio of beam depth to span. For values $\frac{L}{d}$ between 1 to 5, the value of correction factor is more or less appreciable, varying from 3.33 to 1.095, while for values of $\frac{L}{d}$ equal to and above 10, the correction factor diminishes and has little effect on the deflection. Thus at $\frac{L}{d}=10$, it is 1.0233, at $\frac{L}{d}=20$, it is 1.0058, and as such it is negligible. It can therefore be considered that when the span is less than ten times the depth of beam the common bending equation requires to be corrected. Such cases generally arise in short heavily loaded beams in which deformation is primarily due to shear.

Plastic Bending

In recent years attempts have been made to develop methods of design for fixed and continuous beams and other redundant metal structures which are based on the calculation of the load at which a structure actually collapses as a result of excessive plastic deformation. This inelastic theory of structural design has been now proved to be more logical and rational than the elastic theory, and is especially more suitable for designing members in light metals, which do not exhibit the typical "kinked" stress/strain relation. The inelastic behaviour indicates any type of general mechanical behaviour that is not elastic and covers the theory of anelasticity first propounded by Zener [46], and the various theories of plasticity such as plastic flow, plastic deformation and limit design. The recent works of Baker and his collaborators at Cambridge [19] and of Prager, Neal and Symonds [46—48] in the United States has resulted in the evolution of rational methods of plastic design for several types of framed structures, including fixed and continuous beams. Most of these methods have been developed for steel structures with little analytical or experimental work on light alloys. The author is of the opinion that these methods with suitable changes can be usefully employed in the design of magnesium structures, with considerable economy in material used, and with a full knowledge of the safety of the structure.

Consider a rectangular beam of section modulus Z, stressed elastically in bending to an extreme fibre stress f. The moment M resisted by this section is then, $M = fZ$. Taking Z as $bd^2/6$ where b is the breadth and d the depth of the beam,

$$M = \frac{f b d^2}{6}$$

When the same beam is stressed in bending to full plasticity, the plastic moment of resistance Mp is given by

$$Mp = fy \cdot \frac{b d^2}{2} \cdot \frac{d}{2} = \frac{f y b d^2}{4}$$

where fy is the yield stress.

If the first moment of area is denoted by S, then

$$Mp = f y^S .$$

In this case, by analogy, S is the plastic modulus of the beam and is related to Z, as under,

$$\frac{S}{Z} = \frac{b d^2/4}{b d^2/6} = 1.5$$

This ratio $\frac{S}{Z}$ generally termed the "shape factor" is an important prerequisite in the theory of plastic design, and has got to be evolved for various sections. It is found that for I sections, its average value is 1.15.

To take an actual instance, consider a typical wrought magnesium alloy of the type $= AM\ 35\ H$ having an ultimate tensile strength of 16 tons/sq. in., and an yield strength of 12 tons/sq. in. It is designed to a working stress of say 8 tons/sq. inch. In such a case by the conventional elastic methods of design, no indication of the real margin of safety could be given as two different values are theoretically possible viz.

$$\frac{12}{8} = 1.5 \quad \text{and} \quad \frac{16}{8} = 2 .$$

The plastic theory on the other hand shows that when a beam is designed by conventional methods, the proportionate increase in load that would cause collapse varies considerably according to the form of loading, the cross section of beam or shape factor, and the end-fixity conditions. Based on a working stress of 8 tons/sq. in., an ordinary I section in magnesium, would have an elastic bending moment $M = \frac{W L^2}{8}$ ft. tons, where w is the uniformly distributed load per foot run and L is the span of the beam. The section modulus Z is then given by

$$Z = \frac{M}{f} = \frac{W L^2}{8} \cdot \frac{12}{8} \text{ in.}^3.$$

According to the plastic theory the load that would cause the beam to collapse is say $q w$, and the bending moment at centre at collapse is $\frac{q w L^2}{8}$ ft. tons. The plastic modulus S, in this case is $\frac{Mp}{fy}$.

Since $Mp = q w L^2/8$ and $fy = 12$,

$$S = \frac{q w L^2/8}{12} \cdot 12 \text{ in.}^3.$$

Since for I sections,

$$Z = \frac{S}{1.15}$$

$$Z = \frac{q w L^2/8}{12} \cdot 12 \cdot \frac{1}{1.15} \text{ in.}^3.$$

Equating the two values of Z obtained from the two designs, we have

$$\frac{W L^2}{8} \cdot \frac{12}{8} = \frac{q w L^2}{12 \cdot 8} \cdot \frac{12}{1.15}$$

$\therefore$ the load factor $q = 1.725$.

The load factor represents the ratio of the collapse load to the working load and should necessarily be more than unity. The value of q is linked up with the effective section modulus, an increase in q producing a diminuition in Z, and thereby in turn producing material economy. Based on the above method, the theory of plastic design can be extended with advantage in the case of fixed and continuous beams, portal frames and arches.

Plates

In general engineering parlance a member having a very small thickness in relation to its other dimensions, is termed a plate or shell, depending on whether it is straight or curved. At present a number of structures in civil engineering employ plate elements, although their largest use has been in the field of aeronautics. The behaviour of these elastic plates under the action of external loads is fairly well understood and predicted, provided their deformations are such that stresses due to stretching in their own planes are small compared with stresses due to bending. If this condition is satisfied the usual linear differential equation of small deflection evolved by LAGRANGE holds good. However, when these conditions do not hold good, and the deflection is appreciable, the Large Deflection Theory of KÁRMÁN is generally employed. The simplest of the elastic buckling case of a flat plate of rectangular section is when it is freely supported on all sides and is subjected to edgewise loading in one direction. In such a case failure takes place by sheet buckling as a whole or through wrinkling into waves and the critical buckling stress f_b is given by

$$f_b = \frac{\pi^2 E}{12(1-\mu^2)} \left[\frac{b\,c}{a} + \frac{a}{b\,c}\right] \left(\frac{t}{b}\right)^2$$

where

E = elastic modulus in lbs/sq. in.
μ = poisson's ratio.
a = length of plate in inches.
b = width of plate in inches.
t = thickness plate in inches.

Generally $x\left(\frac{b\,c}{a} + \frac{a}{b\,c}\right)$ is represented by a constant k, and as such

$$f_b = k \cdot \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2$$

Since for magnesium alloys,

$$E = 6.5 \times 10^6 \quad \text{and} \quad \mu = 0.34,$$

$$f_b = 4.75 \times 10^6 \frac{k\,t^2}{b^2} \text{ lbs/sq. in.}$$

This expression also represents the critical shear stress in the plate, provided the correct values of the constant k are used, which as seen above depend only on the dimensions of the member and the edge fixity conditions.

Without going into the detailed analysis it can be said that for a plate simply supported on all sides $k=4$, for a plate fixed on all sides $k=8$ and for intermediate conditions $k \sim 1$, provided that $\frac{a}{b}$ is greater than or equal to four. Assuming the first condition, for $k=4$, the buckling stress

$$f_b = \frac{4 \times 4.75 \times 10^6}{b^2} \cdot t^2$$

In other words, the dimensions of the plate can be obtained from the relation

$$\frac{b}{t} = \sqrt{\frac{19 \cdot 10^6}{f_b}}$$

If the plate has to be designed such that failure by elastic instability does not take place, the value of yield stress should be put for f_b and the value of $\frac{t}{b}$ obtained.

Shells

The general form of thin shells is represented by two principal curvatures, and a uniform thickness. The limiting case of such a thin shell, where one radius of curvature tends to infinity and the flat elements are all parallel to one axis, gives the case of the ordinary thin-walled cylinder. When both the radii of curvature are infinite, the limiting case of the flate plate arises. In between these limits, falls the shell. The critical buckling stress f_b for a hemispherical shell under a uniform normal stress is given by TSIEN and KÁRMÁN [36] as

$$f_b = \frac{1}{3\sqrt{5(1-\mu^2)}} \cdot \frac{E\,t}{R}$$

Where

$$R = \text{shell radius}$$

and $$t = \text{shell thickness.}$$

It will be observed that this expression differs slightly from the one generally found in all classic texts on elasticity.

Using the standard values of magnesium alloys, viz. $E = 6.51 \times 0^6$ lbs/sq. in. and $\mu = 0.34$, the buckling stress f_b works to

$$f_b = 1.032 \times 10^6 \frac{t}{R} \text{ lbs/sq. in.}$$

In comparison with this, the buckling stress of a steel shell, having

$$E = 30 \cdot 10^6 \text{ lbs/sq. in.} \quad \text{and}$$

$$\mu = 0.3, \quad \text{is given by}$$

$$f_b = 5.28 \cdot 10^6 \frac{t}{R} \text{ lbs/sq. inch.}$$

which is nearly five times that of a magnesium shell of same thickness and radius. Since the value of μ differs very little for metals, an approximate simple equation would be

$$f_b = 0.156 \times E \frac{t}{R} \text{ lbs/sq. in.}$$

A much more exact expression has been worked out by SCHUETTE [38] which is applicable for the determination of stresses in elastic as well as the plastic range. According to him

$$f_b = 0.42\, E\, s \frac{t}{R} \text{ lbs/sq. in.}$$

Where Es is the secant modulus of the magnesium alloy in compression. This formula is valid for shells whose both linear dimensions are more or less equal and for $\frac{R}{t}$ between 86 to 515. Similar relations have also been obtained for thin walled cylindrical tubes in axial compression, the empirical equation developed by KANEMITSU and NOJIMA [37] being the most apt within the specified ranges. The critical axial stress fa is given by

$$f_a = E\left[9\left(\frac{t}{R}\right)^{1.6} + 0.16\left(\frac{t}{L}\right)^{1.3}\right]$$

where L is the length of the cylinder. This expression holds good for values of $\frac{R}{t}$ between 500 and 3000 and $\frac{L}{R}$ between 0.10 to 2.5. For values of $\frac{L}{R}$ greater than 2.5, the value of $\frac{f_a}{E}$ continues as a constant for a given ratio of R/t. For values of $\frac{R}{t}$ less than 500, this equation can be used to find the buckling stress of shells, where L, would represent the rib spacing or the length between which the shell is fixed.

V. Elastic Instability

Magnesium alloys are characterised by low modulus of elasticity as well as low compressive and shear stresses. On the other hand they are available in very thin extrusions, due to their good fabricating properties. These two aspects demand special attention when designing members subjected to shear and compression as well as in bending to some extent, as herein elastic instability plays an important part. Physically such instability conditions arise due to one or more of the three causes. Thus failure of a member may be due to a reduction in the inherent strength of the material at a stress far below the crushing strength; i. e. primary instability. The member or one of its component elements may wrinkle or crumple at a certain lower stress level than the ultimate strength; which is termed local instability. Lastly the section may bend and twist simultaneously at any low stress level, when the member is generally of an unsymmetrical shape. This is termed torsional instability.

Probably no field of elasticity has played as dominating a part as the study of the buckling strength of compression members in metal structures. Ever since that great blind mathematician Leonhard Euler [29] first put up his classic column equation, the subject has gone down in the annals of engineering with a chequered history equalled by few other investigations. Only a very cursory glance is necessary to see the colossal amount of the literature available in this field at present such as the work of Salmon [30], Timoshenko [31, 32], Bleich [33, 34], Jakkula [35], Winter, Chwalla and others, including the extensive bibliography compiled in their works.

Starting from the original concept propounded by Euler, in which the ultimate failure theory was used for designing compression members, the column theory has passed through a wide range of modifications, mainly due to the development of more complex structures, and the evolution of thin-walled material typified by stainless steel, aluminium and magnesium alloys. The use of secant modulus, the tangent modulus, the double (or reduced) modulus and the non-dimensional curves have characterised modern design aspects. The author feels that the whole criterion of design of compression members against elastic instability lies in correctly gauging the value of the modulus of elasticity from the stress/strain diagrams of the alloy used and incorporating this value in the EULER formula or one of its modified concepts. All the three modulii, viz. tangent modulus, reduced modulus and secant modulus are used in aircraft structural design and will have necessarily to be given a greater importance by structural engineers who wish to employ materials of low elastic modulus such as aluminium and magnesium. As is now well known the tangent modulus represents the instantaneous rate of change of stress to strain, while the secant modulus measures the ratio between stress and actual strain at the point in question. They both start functioning above the limit of proportionality, that is in the inelastic range, because within the elastic range their values are identical to the Young's Modulus of Elasticity.

The tangent modulus gives the most conservative value and for safety and reliability can be used in structural engineering. However, it is not quite rational and was therefore superceded by the reduced modulus theory of CONSIDÈRE, ENGESSER and KÁRMÁN. This reduced modulus is a function of the stress and shape of the compression member, and its equation is based on the assumption that the strain on the compression side of the deflected member, is proportional to the tangent modulus of the material at the critical stress, whereas the strain on the tension side is proportional to the Young's modulus of the material. It therefore represents a weighted average of the tangent and Young's modulii and is expressed as $\frac{4EEt}{(\sqrt{E}+\sqrt{Et})^2}$ where Et is the tangent modulus of the material. Thus for a typical magnesium alloy having a compressive stress of 20,000 lbs/sq. in. the value of Et at this stress is 4×10^6 lbs/sq. in. The reduced modulus of the alloy at this stress, would then be

$$\frac{4\times 6.5\times 4\times 10^{12}}{(\sqrt{6.5\times 10^6}+\sqrt{4\times 10^6})^2} = 5.05\times 10^6 \text{ lbs/sq. in.}$$

The reduced modulus is thus more rational and less conservative than the tangent modulus theory.

A study of the secant modulus theory indicates that it is ideally suited for material having a sharp well defined yield point, and because it represents a measure of stress to actual strain, it is far more correct than the other two theories.

Since the light metals and stainless steel do not exhibit a well defined yield limit, the secant theory cannot be applied and expected to give accurate results.

Recently a new school of thought has been instituted by SHANLEY and others, who opine that the failing stress of a compression member should be represented by a modulus which lies somewhere between the tangent and the reduced modulus. The author also shares SHANLEY's opinion that stress analysis of a compression member in the inelastic range having induced or imposed eccentricity, cannot be covered adequately by a single formula. This is due to the fact that such a member contains in reality two functions, viz. stability under axial loading which requires to be solved by the tangent modulus theory and failure by bending which involves the shape of cross section and the shape of the stress/strain curve. SHANLEY'S methods of the use of interaction curves, which when rendered in the non-dimensional form, appear to be probably the best compromise in the present state of knowledge of light metal structures.

Every elastic system under certain loading conditions passes into a state of unstable equilibrium. Because of the fact that metals in general have a higher modulus of elasticity, instability in the elastic range is only possible when there is excessive deformation. However, once the elastic range is crossed, there is a progressive diminution in the elastic modulus, and the entire range of systems which may become unstable under conventional loading conditions is considerably extended. The partial collapse of the internal structure of the material, after entering the inelastic range, accelerates the inception of the critical state of buckling. This aspect becomes considerably pronounced in magnesium alloys, whose elastic modulus as it is, is comparatively very low. The general criterion of elastic instability thus arises in columns in which buckling takes place in the plane of the principal axis, and without any rotation of cross sections. Such examples are observable in symmetrical sections only. When the section has no symmetry or has the principal axis only symmetrical, the problem of torsional instability arises. In such cases, columns tend to bend and twist simultaneously under the action of loads. The analysis of such compressive conditions is based on the assumption that the plane cross sections of the column warp but their geometric shape does not change during buckling, that is there is no distortion or crinkling of the column elements. The determination of torsional rigidity forms the primary task in such problems. It is mainly complicated by the fact that the torsion constant in this expression for torsional rigidity, is not easily obtainable for all but the very common symmetrical sections. Since the efficient use of magnesium members lies in the design of such uncommon type of sections, such as bulb angles and tees, twin webbed I beams and channels and tresselated sections, it is essential to obtain in standard and easily procurable form, the torsion constants of such members. Some work has been carried out in this respect in aluminium alloys and similar work in an extended form appears necessary for magnesium [39]. Without entering into details it may be pointed out here that in comparison with the theory of stability of one piece extruded sections columns, the problem of stability of built up columns of plate sections is much more complex. This is due to the fact that the critical buckling load of a column in the latter case is different from the ultimate load the individual plate elements can carry. Further the disparity between these loads goes on increasing as, the plate thickness and the elastic modulus of their material diminishes as in magnesium. The author feels that the general theory of local instability of thin walled columns is well established both in elastic as well as in the inelastic range. For very thin plates with a low elastic modulus, the concepts of effective width of plates, as used in aircraft design, should prove an asset in the structural field. By introducing this well established concept, only a part of the cross section is considered effective for primary strength of the structure. It therefore becomes quite permissible to stress the plate elements above the buckling stresses for local buckling, as the reduction in the strength of the primary structure has already been allowed for. Although the two modes of design may seem different, there is a common basic principle underlying the methods. Thus if the concept of effective width of plates as applied in aeronautical design were applied to the design of a heavy steel column, it would be found that economy would lay in a section where the effective width of all plates is hundred percent, that is the plates are fully prevented from buckling before the critical load on the entire column is reached. On the other hand for lightly loaded columns of thin sheet metal the effective width principle is much more appropriate to follow. Elastic instability is not only associated with columns or compression members but also manifests itself in beams. If the flexural rigidity of the beam in the plane of the web is much higher than its lateral rigidity, the beam will buckle and collapse long before the bending stresses due to the transverse load reach the ultimate stress. Such instances arise in the case of long beams of small width and large depth, wherein buckling takes place in a plane perpendicular to the minor axis of the section and is combined with a torsional displacement of the section. The problem of lateral buckling of deep beams was first initiated as far back as 1899 and although considerable progress was made in the theoretical analysis since then, experimental investigations on light metal beams was not carried out till 1937. In that year Dumont and Hill [40] published their report on a series of experiments made on rectangular aluminium alloy beams, in which the theory appeared to be in good agreement with experimental facts purely within the elastic range. Very recently Wittrick [41] of Australia has given special attention to this aspect with reference to materials like aluminium and magnesium in which strain hardening occurs. Beyond this, as far the author is aware, no systematic exhaustive work on magnesium alloy beams has appeared so far.

In the case of a narrow rectangular beam, of span L, the critical bending moment Mc for lateral instability is generally given in the form:

$$\frac{1}{L} = \frac{Mc}{\pi}\sqrt{\frac{1}{FT}}$$

where F is the secondary flexural rigidity for bending about the weaker or minor principal axis; and T is the torsional rigidity. In the derivation of this equation, the curvature of the beam due to the primary bending moment is entirely neglected. This is justifiable if A, — the primary flexural rigidity for bending about the major principal axis is extremely large compared with F and T, in other words the ratio of the width of the depth of cross section of the beam is small. More accurate expressions have been obtained by recent investigators, among which the work of Neal [42] is noteworthy. According to him the correct expression would be

$$\frac{1}{L} = \frac{Mc}{\pi}\sqrt{\left(\frac{1}{F} - \frac{1}{A}\right)\left(\frac{1}{T} - \frac{1}{A}\right)}$$

If A is very large, this reduces to the previous expression. Actually when the stresses in a beam pass from the elastic to the plastic range, as would happen in magnesium members, the concepts of elastic and rigidity modulus have to be revised to take into account plastic bending. Here two extreme cases arise. The first is the determination of the lower critical bending moment at which lateral deflection and twist can occur with increasing bending moment; while the second is the determination of the upper critical bending moment at which if no previous lateral deflection had occurred, the beam would buckle laterally under constant bending moment.

If, therefore, the terms A, F, and T are correctly evaluated, NEAL's expression can be used in the plastic range with sufficient accuracy, and thereby made applicable to beams in magnesium alloys. Thus A, should be taken as the overall primary flexural rigidity i.e. the ratio of primary bending moment and primary curvature immediately before lateral deflection occurs. The term F would be defined as the ratio between an increment of secondary bending moment, applied about the minor principal axis and the corresponding increment of secondary curvature. Similarly T, would be the ratio between an increment of torque and the corresponding increment of twist per unit length.

The use of these concepts requires the incorporation of stress and strain functions at various loading points, and are, therefore, not as simple as they appear. Besides the case of beams of purely rectangular cross section are of not much use in engineering practice, except that they serve to indicate the general trend of behaviour of other sections. The problem of the determination of lateral buckling loads for open sections like channels and I-beams, is complicated by the warping of their cross sections under torsion and the consequent increase in the torsional rigidity when the torsion is uniform. These aspects require to be further analysed to arrive at standard and rational design formulations [44].

VI. Applications

The applications of magnesium alloys in major stress carrying members have been very few and far between in the structural engineering field. The most conspicuous application of a major nature was in the design and construction of a bridge built during the last war in Canada. In fact it has been the only large scale structure so far constructed in magnesium [43]. The Ministry of Supply and War Office in London, in collaboration with the Canadian Military Headquarters, decided that Canada, with her development facilities and knowledge of magnesium alloys should go in for a magnesium alloy light infantry assault bridge to be used as a fixed span bridge over ravines and mountain streams and also as a pontoon bridge on sluggish streams. The well known American Dow Chemical Co. of Michigan was then approached, and an agreement was reached whereby that company undertook to fabricate three standard bays of bridge to drawings and specifications of the Directorate of Engineer Development, Dept. of National Defence, Ottawa, Canada. The Fairchild Aircraft Limited of Quebec also played an important part in the design of tools and jigs for fabricating the bridge.

The bridge is 100 ft. long, $2\frac{1}{2}$ ft. wide, and 4 ft. deep, having 25 sections of standard as well as hornbeam bays. The design of the common deck plate girder bridge was followed. The structure is subdivided into standard bays 4 ft. long and 27 in. wide, which dimensions were determined to facilitate man carriage. A span of 100 feet is capable of supporting a live load of 100 lbs. per linear foot, which is approximately one infantry man after another each wearing full battle equipment and well closed up. When two bridges are used abreast, a live load consisting of a jeep towing a 6 pounder gun is capable of crossing the 100 ft. span. The dead load for design purposes is taken as 20 lbs. per linear foot. No addition was made for impact stresses, as it was felt that the combined dead and live load of 120 lbs. covered this up. Had a steel bridge been used, its dead weight would have been over 60 lbs. per linear foot that is three times that of a magnesium section.

The deck is of a hard rolled plate, while a soft alloy is used for the side panels and bracing frames. The sides or webs are 4 ft. long and 4 ft. deep and fold likewise four times into a pack about 1 ft. wide and 4 ft. long each side weighing 27 lbs. Each 4 ft. section is ingeniously fastened together by a chord connector which consists of a hook eccentric and pin, thereby eliminating the use of bolts and nuts. The hook is simple but stirdy in construction and is capable of withstanding a load of over 20,000 lbs.

The bridge is assembled by laying the two sides on end and then fitting the top deck and vertical brace. This is done by aligning and inserting shear lugs found on the underside of the decking, into holes in the chord channel, and locking down with Dzus fasteners by means of a simple screwdriver. The bottom brace is also fitted in the same manner, and the bay is now assembled and may be, turned up into its correct position for joining on to another bay. Assembled bays are hooked together by means of the hook and pin built into the chord channel. Each end of the chord channel has an alternate male and female coupling and a corresponding alternate hook and pin assembly. The hook itself is accentuated by an eccentric, and is in its fully extended position when the handle and the hook are both forward. When two chord channel assemblies are fitted together, the hook is put forward and just clears the pin, the handle is then swung back, at the same time the hook takes up on the pin and the handle is locked back into the chord channel to insure that the hook remains closed. Chord channels are joined in this manner top and bottom. The complete assembly of a standard bay takes only 59 seconds. Cantilever method is adopted for launching the bridge over a dry gap of 100 feet or less. Rollers are placed on the near shore bank seat and the bridge rolled forward.

The whole assembly of this unique bridge is effected by nothing more than a screwdriver to cinch the Dzus fasteners. In fact a screwdriver may even be dispensed with in an emergency as a knife or a dime will clinch the Dzus fasteners. Besides no section of this bridge weighs more than 27 lbs. and hence is capable of being transported with ease in a packaged condition behind a normal man's back.

Among the applications of magnesium of lesser importance, mention may be made of their use in wall and slab forms for reinforced concrete work and such ancillary building equipment as scaffolding, portable ladders, and wheel barrows. Doors and door and window frames have also made use of this ultra light metal. For bridge railings, guard rails, and other semi-stressed components, which have to withstand impact, magnesium in view of its high resilience is very suitable.

VII. Mode of Utilisation

The large scale applications of magnesium in the structural field require a reorientation of design concepts from those commonly understood for steel. It is only when the correct forms are reorganised and "designing in shape" rather than "designing in strength", is given due thought, that magnesium will emerge as a useful structural material. It needs no great imagination to see that in a correctly designed light weight structure, every component of the assembly must be so arranged and sectioned that it is stressed to its maximum permissible limit. By avoiding the inefficient use of material in and around the neighbourhood of neutral zones, and by dispensing with massive solid sections and replacing them by lattice structures and sheet built members, considerable economy can be achieved. The general types of structural components commonly used in aircraft structures, but not known outside that field, also require to be meticulously studied with a view to their adoption according to their suitability. Among such components are the stressed skin construction for roofs, floors and sides of a building; corrugated shell design for roof of large spans; tension field beams for girders carrying high loads, but very light in themselves; sandwich construction for building components such as partitions; light tresselated beams and space frames for bridges and long span structures, hip plate construction, and braced tubular frameworks [19, 45].

VIII. Conclusion

It appears that in the present state of knowledge the following aspects require greater elucidation for achieving some rational concepts of design and bringing magnesium in line with the other accepted materials of engineering construction.

a) A deeper understanding of the "specific stress" and "specific strain" type loading. If it is of the latter type as in magnesium alloys, it is not sufficient to design the member on the usual stress basis but it is necessary to take into account the actual load extension produced in the structure on the appropriate gage length. In other words the actual stress at the specific strain concerned should be obtained from the complete stress/strain diagram of the material and checked back against the design stresses and safety factor.

b) A study of the relationship between the compressive stress/strain curve of magnesium and the strength of an ideal column, to gauge the modification necessary to be incorporated for the actual design of a compression member. At present, the simplest mode of practical design appears to be the use of the classic Eulerean formula incorporating the tangent modulus of elasticity instead of Young's modulus. Deviations from this concept require further investigation especially when large residual stresses are present.

c) Exhaustive investigations into the local buckling of compression members and the evolution of simple and practical expressions for use in engineering design.

d) The determination, in a rationally applicable form, the effect of end restraint of compression members when used as elements of a framework. The problem of the framed column is complicated by the fact that the end restraints and/or applied moments resulting from the action of adjacent framing, do not in general remain constant, during increase of axial load in any particular member. An adjacent tension member will provide increased restraint against sidesway or end rotation of that member in question. On the other hand, an adjacent compression member, will provide decreasing restraint with increasing load, and may even finally reverse its behaviour and apply moment or shear to the column in place of the restraint it previously provided. These considerations necessitate an analysis of the entire set-up of the frame, especially when designed in magnesium alloys, as herein the compression elements require very special scrutiny.

e) The satisfactory development of simple expressions for evaluating the lateral and torsional instability of beams and columns is a prime necessity in the designing of ultra light alloys.

f) Development of new sections based on the principles of form strength, and a complete data on their physical characteristics, including their torsion constants.

Magnesium is a metal of the modern age and like aluminium can come up in the front ranks of the accepted constructional materials, if extended theoretical and practical investigations are carried out. To a comparatively recent date it was relegated only to the domain of aeronautical engineering, with little use in the structural field. Now, with the acceptance of aluminium as a structural material, magnesium will also come out in the limelight if greater attention is given to it and its wide adaptabilities understood. In the structural engineering field, a new theory of light metal structures to be quite different from the existing theory applicable to mild steel design, appears quite necessary, if a rational approach is to be made and aluminium and magnesium are to be used with maximum possible efficiency. Some of the salient features of this new mode of approach are given in this paper in the hope that they will serve as stepping stones to more prolific methods of design and thereby lead to a better understanding of the wide potentialities of magnesium alloys in structural engineering.

References

1. Davy H., Phil. Trans. Roy. Soc. Vol. 1, 1808, p. 333.
2. Perry J., "The Light Metals Industry", Longmans, N. Y. Pp. 128, 1947.
3. Zeit. Elektrochem. Vol. 42, p. 437, 1936.
4. "Magnesium in 1938". Min. Inl. (Lond.) 4 March 1939, p. 171.
5. Ball. C. J. P., "The Progress of Magnesium and Its Alloys in Britain", 1924—1945, Metallurgia, Aug. 1942, p. 153.
6. — "A Century of Magnesium, 1852—1952", Mag. Rev. and Abst. Apr. 1952, p. 147.
7. Metal Industry Handbook and Directory, 1952, Louis Cassier & Co. Ltd., London, Pp. 448, 1952.
8. Designing with Magnesium, 1945, Amer. Mag. Corp., Ohio (Contains a good bibliography of 84 papers).
9. Mazlo Magnesium Data, 1944. Amer. Mag. Corp. Ohio.
10. Metals Handbook (Section on Magnesium Alloys). Amer. Soc. for Metals, Ohio.
11. S. A. E. Handbook, Soc. of Automotive Engrs., New York.
12. Werkstoff Magnesium, Ver. Deu. Ing. Verlag Berlin, 1939 (Ridder's article on "The Design of Components in Magnesium Alloys").
13. Magnesium, a Handbook, 1923. Amer. Mag. Corp. New York.
14. "Structural Applications on Steel and Light. Weight Alloys". A symposium, Amer. Soc. Au. Engr. Trans. Vol. 102, p. 1179, 1937.
15. Pidgeon L. M., Mathes J. C., Woldman N. E., Winkler J. V. and Loose W. S., "Magnesium", a chapter on "Magnesium Structural Design", Amer. Soc. for Met., Ohio, 1946.
16. Beck A., „Magnesium und seine Legierungen". English translation, by F. A. Hughes & Co., Ltd., and Magnesium Elektron Ltd., London, Pp. 512, 1943.
17. Dowmetal Magnesium Alloy Data Book, The Dvn Chemical Co., U.S.A. 1944.
18. Pannell E. V., "Magnesium, Its Production and Use", Pitman, N. Y. 1944.
19. Ghaswala S. K., "Basic Concepts of The Structural Theory of Aluminium Alloys", Paper read at the 4th Congr. of I.A.B.S.T.E., at Cambridge, August 1952. Prelim. Pub. 1952, p. 571.

20. Recce P. O., "Timber Construction", a paper in the Symposium, "Relative Economics of Prestressed Concrete compared with other forms of Construction", Proc. Int. Civil. Engrs., Part I, July 1952, p. 400.
21. Ridder E. J. de, "The Design of Components in Magnesium Alloys". Werkstoff Magnesium, Ver. deu. Ing. Verlag, Berlin, 1939.
22. Rappleyea F. A. and Eastman E. J., "Flexural Strength in the Plastic Range of Rectangular Magnesium Extrusions", Int. Aero. Sciences, Oct. 1944, p. 373.
23. "A Comparison of some Properties of Beams in Magnesium and Aluminium Alloys", Mag. Rev. and Abst., Oct. 1943, p. 107.
24. Gerard G., "Secant Modulus Method for Determining Plate Instability above the Proportional Limit", Int. Aero. Sciences, Jan. 1946, p. 38.
25. Eastman E. J., McDonald J. C. and Moore A. A., "The Relation of Stress to Strain in Magnesium Base Alloys", Int. Aero Sciences, July 1945.
26. Schuette E. H., "Hyperbolic Column Formulas for Magnesium Alloy Extrusions", Int. Aero. Sciences, Sept. 1948, p. 523.
27. Mortenson C. H., "The Bending Modulus of Rupture of Round Magnesium Tubing". Int. Aero. Sciences, Nov. 1948, p. 661.
28. Schuette E. H., "Column Curves for Magnesium-Alloy Sheet", Int. Aero. Sciences, May 1949, p. 301.
29. Euler L., "De Curuis elasticis", Lausanne and Geneva, 1744.
30. Salmon E. H., "Columns", Oxford Tech. Pub. London 1921.
31. Timoshenko S., "Theory of Elastic Instability", McGraw Hill, N. Y. 1936.
32. — "Theory of Plates and Shells". McGraw Hill, N. Y. 1940.
33. Bleich F., "Buckling Strength of Metal Structures", McGraw Hill, N. Y. 1952.
34. — „Stahlhochbauten", 2 Vols, Verlag Springer, Berlin 1933.
35. Jakkula A. A. and Stephenson H. K., "Steel Columns, A Survey and Appraisal of Past Works", Bullt., Agric. and Mechn. College of Texas, No. 91, 1949. (Gives over 600 references on column design).
36. Karman Th. von and Tsien H. S., "The Buckling of Thin Cylindrical Shells under Axial Compression", Int. Aero. Sc. June 1941, p. 303.
37. Kanemitsu S. and Nojima H., "Axial Compression Tests of Thin Circular Cylinders", Thesis, Calif. Inst. of Techn., 1939.
38. Schuette E. H., "Buckling of Curved (Magnesium) Sheets in Compression and Its Relation to the Secant Modulus", Int. Aero. Sc., Jan. 1948, p. 18.
39. Cullimore M. S. G. and Pugsley A. G., "The Torsion of Alluminium Alloy Structural Members", Alum. Dev. Assoc. Res. Repl. No. 9, July 1951, 60 pages, London.
40. Dumont C. and Hill H. N., NACA. Tech. Note 601, 1937.
41. Wittrick W. H., "Lateral Instability of Rectangular Beams of Strain — Hardening Material, Under Uniform Bending Stress", Int. Aero. Sc., Dec. 1952, p. 835.
42. Neal B. G., Phil. Tran. Roy. Soc. A, Vol. 242, 1950, p. 197.
43. Ghaswala S. K., "Assault Bridge in Magnesium", Light Metals, Sept. 1947, p. 434.
44. Papers presented at the First U.S. National Congress of Applied Mechanics. A.S.M.E., N. Y., June 11 to 16, 1951.
45. Pippard A. J. S., "Studies in Elastic Structures", Arnold, Lond., 1952.
46. Zener C., "Elasticity and Anelasticity of Metals", Univ. of Chicago Press, 1948.
47. Prager W. and Hodge P. G., "Theory of Perfectly Plastic Solids", Wiley, N. Y. 1951.
48. Murnaghan F. D., "Finite Deformation of An Elastic Solid", Wiley 1951.
49. Westergaard H. M., "Theory of Elasticity and Plasticity", Wiley, 1952.

Summary

The paper gives a brief historical introduction on magnesium alloys, which are the lightest of all structural metals. In view of their very limited applications in structural engineering, the physical and mechanical properties of the alloys are described, and their comparison with the common engineering materials made, in order to evaluate their intrinsic worth.

An indication is then given of the salient points of design, covering such topics as tension and compression; elastic and plastic bending; and plates and shells. The problems of elastic instability, which assume considerable importance in magnesium alloys, are then discussed and emphasis laid on the various modes of elastic failure.

The paper concludes with a description of the first magnesium alloy bridge built in Canada, and the mode of utilisation of these ultra-light alloys in structural engineering. The various aspects requiring further elucidation for achieving a rational form of design are summarised, in order to appraise the latent potentialities of magnesium alloy structures.

Résumé

L'auteur brosse un tableau d'ensemble de l'évolution des emplois des alliages de magnésium, métaux de construction les plus légers dont nous disposions. Ces alliages ne sont d'ailleurs employés qu'extrêmement rarement en construction; c'est pourquoi l'auteur, après avoir exposé leurs caractéristiques physiques et mécaniques, les compare à celles des autres métaux de construction.

Il donne ensuite des indications sur les valeurs et caractéristiques essentielles pour les calculs, comme le comportement en traction, compression, flexion dans les domaines élastique et plastique, sous forme de plaques et de parois minces. Il attire tout particulièrement l'attention sur l'instabilité élastique et sur les déficiences qui en résultent.

Il termine par la description du premier pont en alliages de magnésium, construit au Canada et indique les modalités d'emploi de ces métaux ultralégers en construction. Diverses questions nécessitent une étude plus poussée, si l'on veut pouvoir tirer intégralement parti des possibilités qu'offrent les alliages de magnésium en faisant appel à des méthodes économiques de calcul.

Zusammenfassung

Der Aufsatz vermittelt einen kurzen Überblick über die Entwicklung auf dem Gebiet der Magnesiumlegierungen, unserer leichtesten Baumetalle. Da diese im Bauwesen nur äußerst selten zur Anwendung gelangen, werden zuerst einmal ihre physikalischen und Festigkeitseigenschaften erörtert, und mit denjenigen anderer Baumetalle verglichen.

Es folgen Angaben über die für die Berechnung wichtigen Kenngrößen und Eigenschaften, wie das Verhalten bei Zug und Druck, unter Biegung im elastischen und plastischen Bereich, als Platte und Schale. Ferner wird der elastischen Unstabilität und dem Versagen unter einer solchen Beanspruchung ganz besondere Aufmerksamkeit geschenkt.

Die Abhandlung schließt mit einer Beschreibung der ersten, in Kanada gebauten Brücke aus Magnesiumlegierungen und der Verwendungsart dieser ultra-leichten Metalle im Bauwesen allgemein. Verschiedene Fragen bedürfen noch weiterer Abklärung, bevor man anhand wirtschaftlicher Berechnungsmethoden die verborgenen Möglichkeiten einer Bauweise in Magnesiumlegierungen ausschöpfen kann.