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Shear-Weak Beams on Elastic Foundation

Träger mit geringer Schubsteifigkeit auf elastischer Bettung

Poutres à faible resistance au cisaillement, reposant sur une assise élastique

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The present paper deals with beams subjected to deformations which are caused by shearing forces only. These beams are assumed to be supported on an elastic foundation acting like a fluid. In what follows, expressions will be deduced for their deformations as well as for the moments and the shearing forces. Furthermore, mention will be made of some fields of use for these beams.

I. Fundamental Formulae

Consider the beam shown in Fig. 1! It is submitted at the point $x=0$ to a load P which gives rise to the distributed reaction q. If the shearing force is denoted by Q, we obtain from statics

$$\frac{dQ}{dx} = -q \tag{1}$$

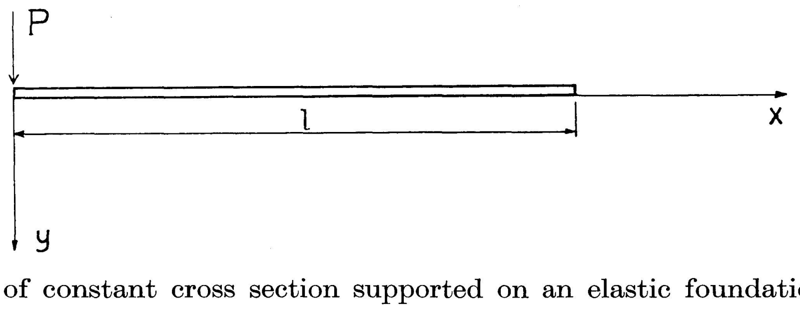


Fig. 1. Beam of constant cross section supported on an elastic foundation acting like a fluid and submitted at one end to a transverse concentrated load

In accordance with the above assumption regarding the foundation, q is directly proportional to the displacement v in the y-direction

$$q = -K\,v \tag{2}$$

From Eqs. (1) and (2) it follows

$$\frac{dQ}{dx} = K v \tag{3}$$

It was assumed that the deformation was caused by shearing forces only. Hence

$$\frac{dv}{dx} = SQ \tag{4}$$

where S is a constant characteristic of each individual beam. From Eqs. (4) and (3) we obtain

$$\frac{d^2 v}{dx^2} = S \frac{dQ}{dx} = SK v$$

whence

$$\frac{d^2 v}{dx^2} - SK v = 0 \tag{5}$$

This differential equation has the solution

$$v = A \sinh \sqrt{SK}\, x + B \cosh \sqrt{SK}\, x \tag{6}$$

Moreover, we have the trivial solution

$$v_1 = \frac{p}{K} \tag{7}$$

where p is the intensity of a uniformly distributed load. From Eq. (6) we get

$$\frac{dv}{dx} = A \sqrt{SK} \cosh \sqrt{SK}\, x + B \sqrt{SK} \sinh \sqrt{SK}\, x \tag{8}$$

whence, by virtue of Eq. (4),

$$Q = A \sqrt{\frac{K}{S}} \cosh \sqrt{SK}\, x + B \sqrt{\frac{K}{S}} \sinh \sqrt{SK}\, x \tag{9}$$

From Eq. (9) we determine the constants A and B

$$\left. \begin{array}{l} x = l \\ Q = 0 \end{array} \right\}$$

$B = -A \coth \sqrt{SK}\, l$, whence

$$Q = A \sqrt{\frac{K}{S}} (\cosh \sqrt{SK}\, x - \coth \sqrt{SK}\, l \sinh \sqrt{SK}\, x)$$

Hence

$$A = \frac{Q_{x=0}}{\sqrt{\frac{K}{S}}} = -P \sqrt{\frac{S}{K}}$$

Therfore, we have

$$v = P\sqrt{\frac{S}{K}}\,(\coth\sqrt{SK}\,l\cosh\sqrt{SK}\,x - \sinh\sqrt{SK}\,x) \tag{10}$$

$$q = -P\sqrt{SK}\,(\coth\sqrt{SK}\,l\cosh\sqrt{SK}\,x - \sinh\sqrt{SK}\,x) \tag{11}$$

$$Q = P\,(\coth\sqrt{SK}\,l\sinh\sqrt{SK}\,x - \cosh\sqrt{SK}\,x) \tag{12}$$

$$M_{x=x_1} = -\int_{x_1}^{l} q\,(x-x_1)\,dx =$$

$$= \frac{P}{\sqrt{SK}}\left(\coth\sqrt{SK}\,l\cosh\sqrt{SK}\,x - \frac{1}{\sinh\sqrt{SK}\,l} - \sinh\sqrt{SK}\,x\right) \tag{13}$$

Observe the following relations!

$x = 0$	$x = l$
$v = P\sqrt{\frac{S}{K}}\coth\sqrt{SK}\,l$	$v = P\sqrt{\frac{S}{K}}\frac{1}{\sinh\sqrt{SK}\,l}$
$\frac{dv}{dx} = -PS \neq 0$	$\frac{dv}{dx} = 0$
$M = \frac{P}{\sqrt{SK}}\frac{\cosh\sqrt{SK}\,l - 1}{\sinh\sqrt{SK}\,l} \neq 0$	$M = 0$

For $\sqrt{SK}\,l = \infty$, we obtain

$x = 0$	$x = l$
$v = P\sqrt{\frac{S}{K}}$	$v = 0$
$\frac{dv}{dx} = -PS$	$\frac{dv}{dx} = 0$
$M = \frac{P}{\sqrt{SK}}$	$M = 0$

Consequently, apart from the uniformly distributed load, the only elementary type of loading that can be realised is the simultaneous action at the point $x = 0$ of P and $M = \frac{P}{\sqrt{SK}}\frac{\cosh\sqrt{SK}\,l - 1}{\sinh\sqrt{SK}\,l}$. Two practicable methods for realising this type of loading will be demonstrated below.

The first method consists in using two beams connected together at the point $x = 0$ and having the same S and K, their respective lengths being l and l_1, see Fig. 2. At the common point, we have $v = v_1$ and $M = M_1$, whence

$$\left.\begin{aligned} \alpha P\sqrt{\frac{S}{K}}\coth\sqrt{SK}\,l &= (1-\alpha)\,P\sqrt{\frac{S}{K}}\coth\sqrt{SK}\,l_1 \\ \frac{\alpha P}{\sqrt{SK}}\frac{\cosh\sqrt{SK}\,l - 1}{\sinh\sqrt{SK}\,l} &= \frac{(1-\alpha)\,P}{\sqrt{SK}}\frac{\cosh\sqrt{SK}\,l_1 - 1}{\sinh\sqrt{SK}\,l_1} \end{aligned}\right\}$$

These equations yield $l_1 = l$ and $\alpha = \frac{1}{2}$. Hence it follows that, when S and K are the same, only $l_1 = l$ can occur under the given assumptions. When S and K are not the same — a case which is usually of little interest —, we obtain a pair of values of $\frac{l}{l_1}$ and α which is characteristic of each individual combination. This need not be demonstrated.

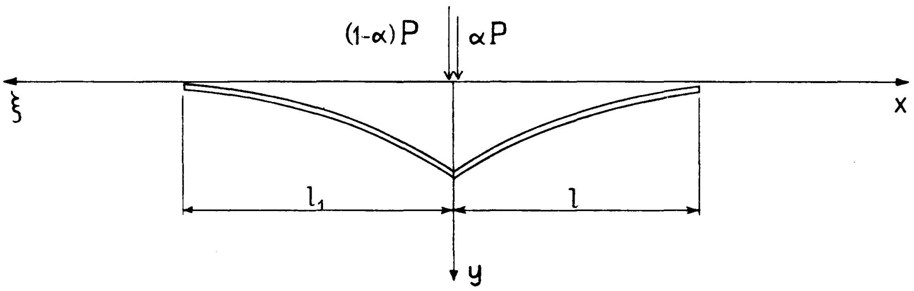


Fig. 2. Two beams differing in length but having the same S and K, connected at the point $x = \xi = 0$, and submitted at that point to a concentrated load P

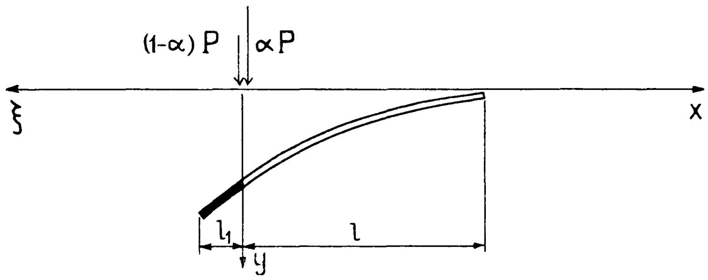


Fig. 3. Two beams, one with S and K, the other perfectly rigid, continuously joined at the point $x = \xi = 0$ and submitted at that point to a concentrated load P

The second method is to join a beam complying with our assumptions to a completely rigid beam, see Fig. 3.

When the joint at $x = 0$ is assumed to be continuous according to Fig. 3, the deflection of the part l_1 is

$$v_\xi = \alpha P \sqrt{\frac{S}{K}} \coth \sqrt{SK}\, l + \alpha P S \xi$$

The moment acting at the point $x = 0$ is

$$M_0 = \int_0^{l_1} K \left(\alpha P \sqrt{\frac{S}{K}} \coth \sqrt{SK}\, l + \alpha P S \xi \right) \xi \, d\xi = \frac{\alpha P}{\sqrt{SK}} \, \frac{\cosh \sqrt{SK}\, l - 1}{\sinh \sqrt{SK}\, l}$$

The equilibrium of the forces yields the equation

$$(1 - \alpha) P = \int_0^{l_1} K \left(\alpha P \sqrt{\frac{S}{K}} \coth \sqrt{SK}\, l + \alpha P S \xi \right) d\xi$$

Using the notations $\sqrt{SK}\,l = \lambda$ and $\sqrt{SK}\,l_1 = \lambda_1$, we obtain, for $\lambda > 0{,}925$,

$$\lambda_1 = \sqrt[3]{\frac{3}{2}\,\frac{\cosh\lambda - 1}{\sinh\lambda} - \frac{1}{8}\coth^3\lambda + \sqrt{\frac{9}{4}\,\frac{(\cosh\lambda - 1)^2}{\sinh^2\lambda} - \frac{3}{8}\,\frac{\cosh\lambda - 1}{\sinh\lambda}\coth^3\lambda} +}$$
$$+ \sqrt[3]{\frac{3}{2}\,\frac{\cosh\lambda - 1}{\sinh\lambda} - \frac{1}{8}\coth^3\lambda - \sqrt{\frac{9}{4}\,\frac{(\cosh\lambda - 1)^2}{\sinh^2\lambda} - \frac{3}{8}\,\frac{\cosh\lambda - 1}{\sinh\lambda}\coth^3\lambda} -}$$
$$- \frac{1}{2}\coth\lambda \qquad (14)$$

$$\alpha = \frac{1}{\lambda_1 \coth\lambda + \frac{\lambda_1}{2} + 1} \qquad (15)$$

The values of $\lambda < 0{,}925$ are scarcely to be taken into consideration. In such cases, the equations are solved as usual. In the whole interval considered above, $0{,}925 < \lambda < \infty$ we have $\lambda_1 \approx 1$.

II. Formulae for Beam of Infinite Length

It has been shown in the foregoing chapter that our extreme assumptions have led to more than extreme consequences. If rigorously applied, the above calculations are almost meaningless. However, the equations show that, even at moderate values of $\sqrt{SK}\,l$, this quantity can to a close approximation be put equal to infinity. Hence it follows that, if the beam is not too short, its central parts can be regarded as a middle cross section of an infinitely long beam, and that all requisite quantities for these parts can be calculated by means of formulae for a beam of infinite length. The end parts shall either be designed as rigid beams, or else shall be treated by the aid of methods which also take into account the deformations due to moments. At the present time, it may be supposed that such calculations for shear-weak beams are mostly made graphically by means of successive approximations, and this procedure is laborious.

If a concentrated load is applied in the middle of an infinitely long beam, and if the origin of the coordinate system is located at the centre of the beam, we have

$$v = \frac{P}{2K}\sqrt{SK}\,e^{-\sqrt{SK}\,x} \qquad (16)$$

$$\frac{dv}{dx} = -\frac{P}{2}\,S\,e^{-\sqrt{SK}\,x} \qquad (17)$$

$$q = -\frac{P}{2}\sqrt{SK}\,e^{-\sqrt{SK}\,x} \qquad (18)$$

$$Q = -\frac{P}{2}\,e^{-\sqrt{SK}\,x} \qquad (19)$$

$$M = \frac{P}{2\sqrt{SK}}\,e^{-\sqrt{SK}\,x} \qquad (20)$$

It is to be observed that Eqs. (19) and (20), which express the lines of influence for the shearing force and the moment, are equations of affine curves. This circumstance renders the calculations particularly easy.

The second derivative of the deflection is

$$\frac{d^2 v}{dx^2} = \frac{P}{2} S \sqrt{SK}\, e^{-\sqrt{SK}\, x} = \frac{1}{\rho_Q}$$

where ρ_Q is the radius of curvature.

If Eq. (20) were quite correct, the moment would produce the curvature

$$\frac{M}{EJ} = \frac{P}{2\, EJ \sqrt{SK}} e^{-\sqrt{SK}\, x} = \frac{1}{\rho_M}$$

A necessary condition for the adequacy of this method is $\frac{1}{\rho_Q} \gg \frac{1}{\rho_M}$. Hence

$$S^2 KEJ \gg 1 \tag{21}$$

In the above equation, EJ is the flexural rigidity of the beam.

III. Application to Vierendeel Trusses

A type of structure that is closely in agreement with our assumptions is a Vierendeel truss supported on an elastic foundation. We apply one of the usual assumptions stating that the points of inflection on the chords are

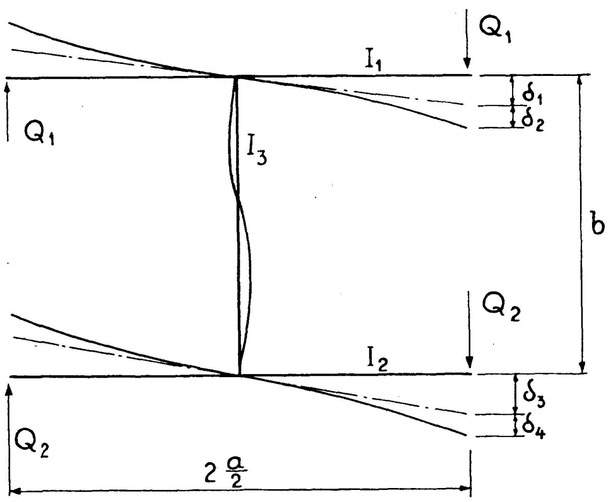


Fig. 4. An element of a Vierendeel truss. Shearing forces and elementary deformations

situated in the middle between the vertical members, whereas these members are not supposed to be perfectly rigid. With reference to Fig. 4, we deduce the constant S and the distribution of the shearing force over the top and bottom chords.

$$\left.\begin{aligned}
\delta_1 &= Q_1 \frac{a^2}{2}\frac{b}{3\,EJ_3} - Q_2 \frac{a^2}{2}\frac{b}{6\,EJ_3} = \frac{a^2 b}{6\,EJ_3}\left(Q_1 - \frac{Q_2}{2}\right)\\
\delta_2 &= Q_1 \frac{a^3}{24\,EJ_1}\\
\delta_3 &= \frac{a^2 b}{6\,EJ_3}\left(Q_2 - \frac{Q_1}{2}\right)\\
\delta_4 &= Q_2 \frac{a^3}{24\,EJ_2}\\
\delta_1 + \delta_2 &= \delta_3 + \delta_4\\
Q_1 + Q_2 &= Q\\
\Theta &= \frac{2(\delta_1+\delta_2)}{a} = SQ
\end{aligned}\right\}$$

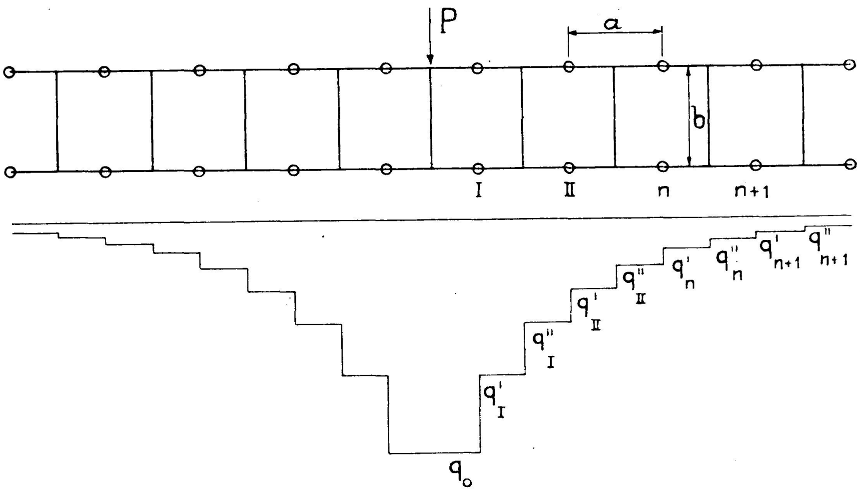


Fig. 5. A Vierendeel truss submitted to a concentrated load and to a discontinuous reaction

$$S = \frac{ab}{12\,EJ_3} + \frac{a^2}{12\,E}\,\frac{3\,b\,J_1 + 3\,b\,J_2 + a\,J_3}{12\,b\,J_1\,J_2 + a\,J_1\,J_3 + a\,J_2\,J_3} \tag{22}$$

$$Q_1 = QJ_1 \frac{6\,b\,J_2 + a\,J_3}{12\,b\,J_1\,J_2 + a\,J_1\,J_3 + a\,J_2\,J_3} \tag{23}$$

$$Q_2 = QJ_2 \frac{6\,b\,J_1 + a\,J_3}{12\,b\,J_1\,J_2 + a\,J_1\,J_3 + a\,J_2\,J_3} \tag{24}$$

For $J_3 = \infty$, Eqs. (23) and (24) are converted into the corresponding equations used in the common theory of Vierendeel trusses.

It is obvious, first, that the deformation of the bottom chord of the Vierendeel truss is not exactly in accordance with the theory, so that the distribution of reactions does not comply with the assumptions, and second, that it would be inconvenient to have to use a distribution of reactions expressed by Eq. (18) in detailed calculations. Therefore, with reference to Fig. 5, we shall deduce an expression for that discontinuous reaction intensity with the interval $\frac{a}{2}$ which is equivalent to the reaction intensity given by Eq. (18).

$$\left.\begin{aligned}
q_n' + q_n'' &= -\frac{2}{a}\int_{a(n-\frac{1}{2})}^{a(n+\frac{1}{2})} q\,dx = \frac{P}{a}\,\frac{e^{\sqrt{SK}\frac{a}{2}} - e^{-\sqrt{SK}\frac{a}{2}}}{e^{\sqrt{SK}\,an}} \\
\frac{a^2}{2}\left[q_n'\left(n-\frac{1}{4}\right) + q_n''\left(n+\frac{1}{4}\right)\right] &= -\int_{a(n-\frac{1}{2})}^{a(n+\frac{1}{2})} x\,q\,dx = \\
&= \frac{P}{2}\left[\left(\frac{1}{\sqrt{SK}} + an\right)\frac{e^{\sqrt{SK}\frac{a}{2}} - e^{-\sqrt{SK}\frac{a}{2}}}{e^{\sqrt{SK}\,an}} - \frac{a}{2}\,\frac{e^{\sqrt{SK}\frac{a}{2}} + e^{-\sqrt{SK}\frac{a}{2}}}{e^{\sqrt{SK}\,an}}\right]
\end{aligned}\right\}$$

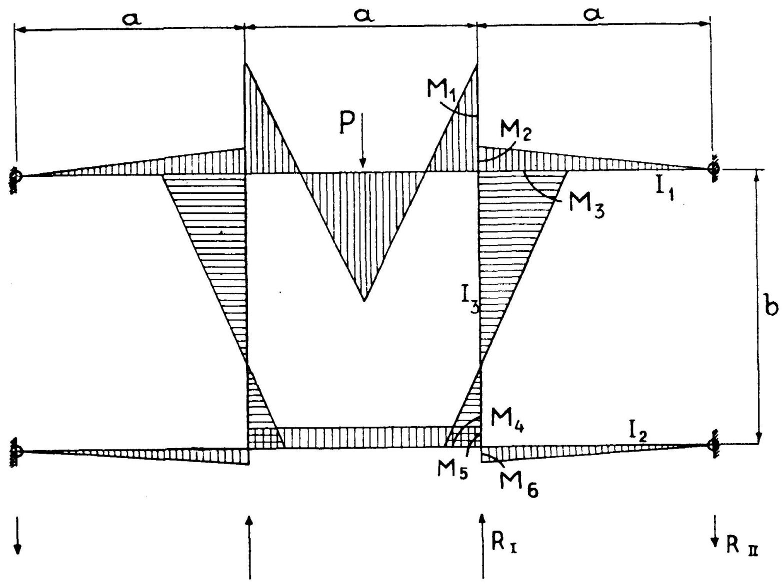


Fig. 6. Secondary system in a Vierendeel truss for determining correction moments when the load is applied in the middle between two vertical members

$$q_n' = \frac{2P}{a} e^{-na\sqrt{SK}}\left[\cosh\frac{a\sqrt{SK}}{2} - \left(\frac{2}{a\sqrt{SK}} - \frac{1}{2}\right)\sinh\frac{a\sqrt{SK}}{2}\right] \tag{25}$$

$$q_n'' = \frac{2P}{a} e^{-na\sqrt{SK}}\left[\left(\frac{2}{a\sqrt{SK}} + \frac{1}{2}\right)\sinh\frac{a\sqrt{SK}}{2} - \cosh\frac{a\sqrt{SK}}{2}\right] \tag{26}$$

$$q_0 = \frac{P}{a}\left(1 - e^{-\frac{a\sqrt{SK}}{2}}\right) = \frac{P}{a}\left(1 - \cosh\frac{a\sqrt{SK}}{2} + \sinh\frac{a\sqrt{SK}}{2}\right) \tag{27}$$

Eqs. (25) to (27), together with Eqs. (19) and (20), yield all those quantities which are required for the design of a Vierendeel truss under the conditions of loading shown in Fig. 5. On the other hand, if the load is applied to the top chord between two vertical members, then assumption concerning the situation of the point of inflection is so manifestly absurd that it is necessary to make a correction. The simplest way to introduce this correction is demonstrated below, see Fig. 6. To begin with, we determine the moments in the system shown in the diagram and the corresponding reactions. After that, these reactions are introduced as loads applied to the vertical members in the principal system. The moments caused in the principal system by these loads are superposed on the moments shown in Fig. 6.

$$M_1 = \frac{Pa}{8}\left(5 + 3\frac{a}{b}\frac{J_3}{J_2} + \frac{15}{4}\frac{b}{a}\frac{J_1}{J_3} + 3\frac{J_1}{J_2}\right)\Phi \tag{28}$$

$$M_2 = \frac{Pa}{8}\left(\frac{15}{4}\frac{b}{a}\frac{J_1}{J_3} + 3\frac{J_1}{J_2}\right)\Phi \tag{29}$$

$$M_3 = \frac{Pa}{8}\left(5 + 3\frac{a}{b}\frac{J_3}{J_2}\right)\Phi \tag{30}$$

$$M_4 = \frac{5\,Pa}{16}\Phi \tag{31}$$

$$M_5 = \frac{Pa}{8}\Phi \tag{32}$$

$$M_6 = \frac{3\,Pa}{16}\Phi \tag{33}$$

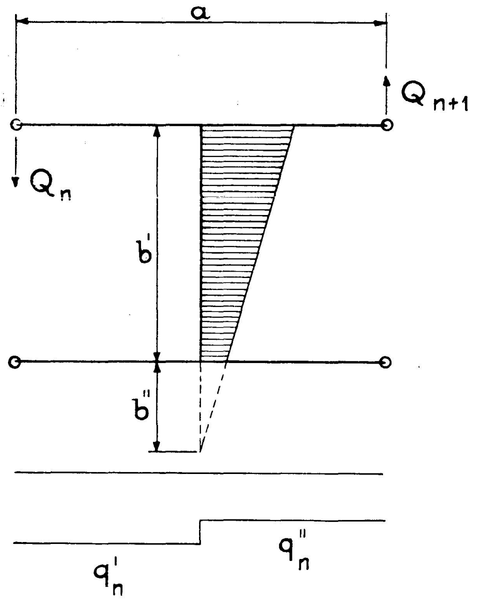
$$R_I = \frac{P}{8}\left(\frac{295}{4}\frac{b}{a}\frac{J_1}{J_3} + 23\frac{J_1}{J_2} + 12\frac{a}{b}\frac{J_3}{J_2} + \frac{37}{2}\right)\Phi \tag{34}$$

$$R_{II} = \frac{P}{8}\left(\frac{15}{4}\frac{b}{a}\frac{J_1}{J_3} + 3\frac{J_1}{J_2} - \frac{3}{2}\right)\Phi \tag{35}$$

$$\Phi = \frac{1}{\frac{35}{2}\frac{b}{a}\frac{J_1}{J_3} + 5\frac{J_1}{J_2} + 3\frac{a}{b}\frac{J_3}{J_2} + 5} \tag{36}$$

Finally, in this connection, we shall study the moments in the vertical members of a Vierendeel truss represented in Fig. 5 for which $J_2 = 0$, so that $Q_1 = Q$. The element under consideration is shown in Fig. 7.

$$\left.\begin{aligned} M_u &= \frac{a^2}{8}(q_n' - q_n'') = \frac{a^2}{8}\frac{2P}{a}e^{-na\sqrt{SK}}\left(2\cosh\frac{a\sqrt{SK}}{2} - \frac{4}{a\sqrt{SK}}\sinh\frac{a\sqrt{SK}}{2}\right. \\ M_0 &= \frac{a}{2}(Q_n + Q_{n+1}) = \frac{a}{2}\frac{P}{2}\left[e^{-(n-\frac{1}{2})a\sqrt{SK}} + e^{-(n+\frac{1}{2})a\sqrt{SK}}\right] \end{aligned}\right\}$$



$$\frac{b''}{b'+b''} = \frac{M_u}{M_0} = 1 - \frac{2}{a\sqrt{SK}} \operatorname{tgh} \frac{a\sqrt{SK}}{2} \qquad (37)$$

It is seen that $\frac{b''}{b'+b''}$ is constant and independent of n. This circumstance will be utilised in the next chapter of this paper.

Fig. 7
Moments in a vertical member of a Vierendeel truss in which the rigidity of the bottom chord is zero

IV. Application to Beam Supported on, and Rigidly Connected to, Columns on Elastic Foundations

Beams which are fixed in columns resting on elastic foundations are always difficult to design, irrespective of the method used for the treatment of this problem. In what follows, it will be shown that the method outlined in the above can in some cases be useful when dealing with such beams. The first example refers to a beam supported on relatively weak columns. In that case, the method in question is not sufficiently accurate since the assumption regarding the positions of the points of inflections is too rough. The second example deals with a beam supported on rigid columns. Under these conditions, the method gives good results.

Example No. 1

The joints are assumed to be located at the bottom edges of the foundations.

$$K = 700 \text{ t/m}^2; \quad E = 2\,100\,000 \text{ t/m}^2$$

The moments of inertia and the cross-sectional areas are

$$J_1 = 0{,}017 \text{ m}^4 \qquad A_1 = 0{,}320 \text{ m}^2$$
$$J_2 = 0 \qquad A_2 = \infty$$
$$J_3 = 0{,}017 \text{ m}^4 \qquad A_3 = 0{,}320 \text{ m}^2$$

To begin with, we determine by the method of trial and error a fictitious value, b', of the height so that $b'+b''$ in accordance with Eq. (37) becomes equal to b in order that the joint should be located as required. In the calculation of S and J we substitute b' for b used in the formulae.

Assume $b' = 3{,}00$ m; $b'' = 3{,}00$ m!

Hence we obtain by means of Eq. (22): $S = 2{,}52 \cdot 10^{-4}$

$$\frac{a\sqrt{SK}}{2} = 1{,}260; \quad \frac{b''}{b'+b''} = 0{,}325; \quad b' = 4{,}05 \text{ m}; \quad b'' = 1{,}95 \text{ m}$$

Assume $b' = 4{,}00$ m; $b'' = 2{,}00$ m!

Hence $S = 3{,}08 \cdot 10^{-4}$; $\frac{a\sqrt{SK}}{2} = 1{,}390$; $b' = 3{,}81$ m; $b'' = 2{,}19$ m

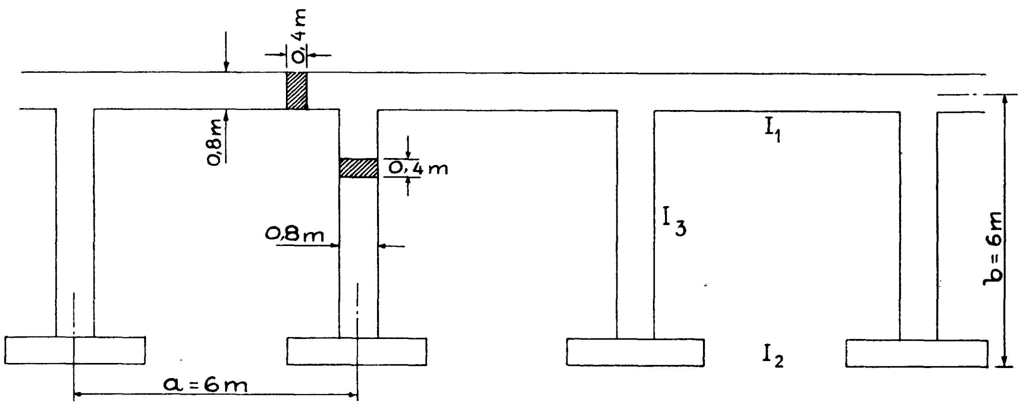


Fig. 8. Beam used in Example No. 1. Dimensions

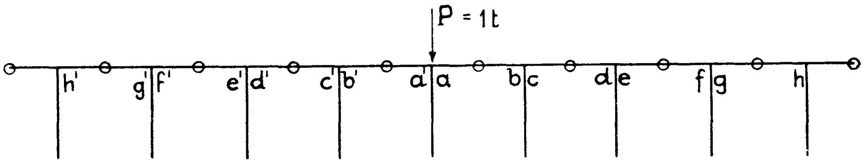


Fig. 9. Beam shown in Fig. 8. Load and notations for joints

Assume $b' = 3{,}80$ m; $b'' = 2{,}20$ m!

Hence $S = 2{,}97 \cdot 10^{-4}$; $\frac{a\sqrt{SK}}{2} = 1{,}365$; $b' = 3{,}85$ m; $b'' = 2{,}15$ m

From this we obtain $S = 3{,}00 \cdot 10^{-4}$; $\sqrt{SK} = 0{,}458 \text{ m}^{-1}$.

$J = 0{,}017 + 0{,}320 \cdot 3{,}85^2 = 4{,}75 \text{ m}^4$

$S^2 K E J = 628 \gg 1$ cf. the inequality (21)!

Eq. (19) yields

$$Q = -\frac{P}{2} e^{-\sqrt{SK}\,x} = -0{,}5\,P e^{-0{,}458\,x}$$

From Eq. (20) we get

$$N = -\frac{M}{b'} = -\frac{P}{2\,b'\sqrt{SK}} e^{-\sqrt{SK}\,x} = 0{,}569\,Q$$

$$Q_{ab} = -\ 0{,}1264\ \text{t} \qquad N_{ab} = -\ 0{,}0718\ \text{t}$$
$$Q_{cd} = -\ 0{,}0081\ \text{t} \qquad N_{cd} = -\ 0{,}0046\ \text{t}$$
$$Q_{ef} = -\ 0{,}0005\ \text{t} \qquad N_{ef} = -\ 0{,}0003\ \text{t}$$
$$Q_{gh} = -\ 0{,}0000\ \text{t} \qquad N_{gh} = -\ 0{,}0000\ \text{t}$$

$$M_a = M_b = -Q_{ab} \cdot \frac{a}{2} = 0{,}3791\ \text{tm}$$
$$M_c = M_d = 0{,}0243\ \text{tm}$$
$$M_e = M_f = 0{,}0015\ \text{tm}$$
$$M_g = M_h = 0{,}0001\ \text{tm}$$

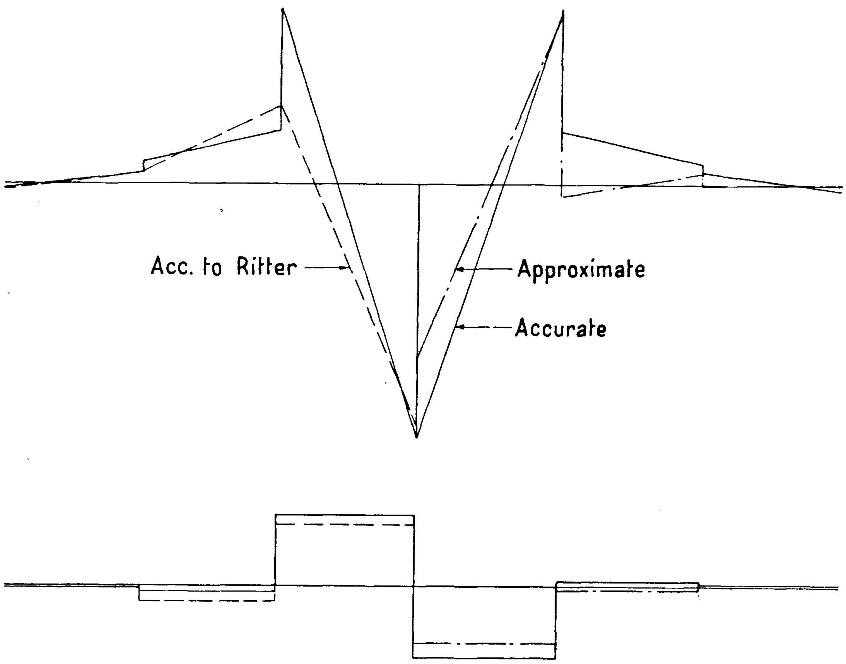


Fig. 10. Moments and shearing forces acting on the beam shown in Figs. 8 and 9

For comparison, the tables and diagrams given below show three groups of values of the moments and the shearing forces, viz., first, the values calculated by means of the above-mentioned method, second, the accurate values computed with hard work and toil from the theory of elasticity, and third, the corresponding values for completely weak columns calculated by the aid of Ritter's coefficients (see Anwendungen der graphischen Statik, 3. Teil, Zurich 1900).

	M_a	M_b	M_c	M_d	M_e	M_f
Approximate	+ 0,379	+ 0,379	+ 0,024	+ 0,024	+ 0,002	+ 0,002
Accurate	+ 0,556	+ 0,385	− 0,115	+ 0,044	− 0,025	− 0,008
Acc. to Ritter	+ 0,528	+ 0,172	− 0,172	+ 0,024	− 0,024	− 0,012

	R_{ab}	R_{cd}	R_{ef}
Approximate	$-0{,}126$	$-0{,}008$	$-0{,}001$
Accurate	$-0{,}157$	$+0{,}012$	$+0{,}006$
Acc. to Ritter	$-0{,}132$	$+0{,}038$	$+0{,}004$

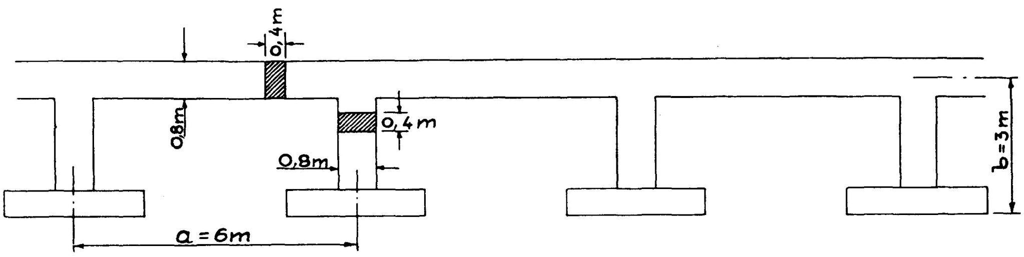


Fig. 11. Beam used in Example No. 2. Dimensions

Example No. 2

All other dimensions as in Fig. 8.
The foundations are assumed to be clamped.

$$K = 700\ \mathrm{t/m^2};\quad E = 2\,100\,000\ \mathrm{t/m^2}$$

$$J_1 = 0{,}017\ \mathrm{m^4} \qquad A_1 = 0{,}320\ \mathrm{m^2}$$

$$J_2 = 0 \qquad A_2 = \infty$$

$$J_3 = 0{,}017\ \mathrm{m^4} \qquad A_3 = 0{,}320\ \mathrm{m^2}$$

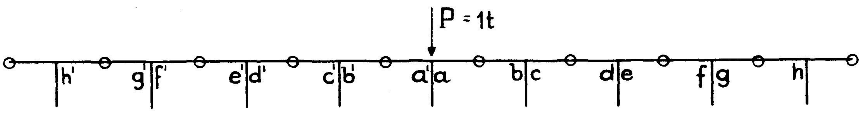


Fig. 12. Beam shown in Fig. 11. Load and notations for joints

We determine b' so that $b' = b''$ becomes equal to $2\,\mathrm{m} = \frac{2}{3}\,b$, which corresponds to foundations clamped.

Assume $b' = 1{,}00$ m; $b'' = 1{,}00$ m!

Hence $S = 1{,}40 \cdot 10^{-4}$; $\dfrac{a\sqrt{SK}}{2} = 0{,}940$; $b' = 1{,}564$ m; $b'' = 0{,}436$ m.

Assume $b' = 1{,}50$ m; $b'' = 0{,}5$ m!

Hence $S = 1{,}68 \cdot 10^{-4}$; $\dfrac{a\sqrt{SK}}{2} = 1{,}028$; $b' = 1{,}508$ m; $b'' = 0{,}492$ m

And from this $S = 1{,}68 \cdot 10^{-4}$; $\sqrt{SK} = 0{,}344\ \mathrm{m^{-1}}$;

$$J = 0{,}017 + 0{,}320 \cdot 1{,}50^2 = 0{,}737\ \mathrm{m^4}$$

$$S^2 K E J = 30{,}6 \gg 1$$

$$Q = -0{,}5\, P e^{-0{,}344\, x};\quad N = 1{,}940\, Q$$

$Q_{ab} = -\ 0{,}1745\ \text{t}$ $\qquad N_{ab} = -\ 0{,}3440\ \text{t}$

$Q_{cd} = -\ 0{,}0225\ \text{t}$ $\qquad N_{cd} = -\ 0{,}0436\ \text{t}$

$Q_{ef} = -\ 0{,}0029\ \text{t}$ $\qquad N_{ef} = -\ 0{,}0055\ \text{t}$

$Q_{gh} = -\ 0{,}0004\ \text{t}$ $\qquad N_{gh} = -\ 0{,}0007\ \text{t}$

$$M_a = M_b = 0{,}5324\ \text{tm}$$
$$M_c = M_d = 0{,}0676\ \text{tm}$$
$$M_e = M_f = 0{,}0086\ \text{tm}$$
$$M_g = M_h = 0{,}0011\ \text{tm}$$

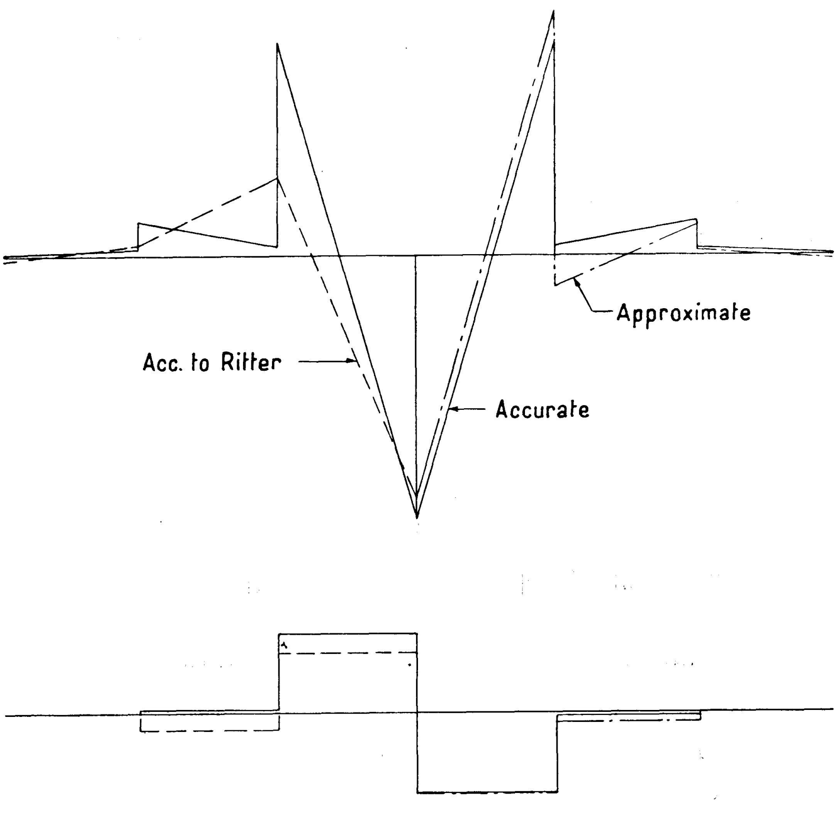


Fig. 13. Moments and shearing forces acting on the beam shown in Figs. 11 and 12

	M_a	M_b	M_c	M_d	M_e	M_f
Approximate	+0,532	+0,532	+0,068	+0,068	+0,009	+0,009
Accurate	+0,578	+0,467	−0,021	+0,075	−0,013	+0,002
Acc. to Ritter	+0,528	+0,172	−0,172	+0,024	−0,024	−0,012

	R_{ab}	R_{cd}	R_{ef}
Approximate	−0,177	−0,023	−0,003
Accurate	−0,174	−0,009	+0,002
Acc. to Ritter	−0,132	+0,038	+0,004

The examples speak for themselves. In the second example, the agreement between the values of the two greatest moments was completely satisfactory, considering the accuracy with which the assumptions can be formulated. In the first example, the agreement was less close. The difference between these two examples is brought out best in Fig. 14 which represents the rigidities.

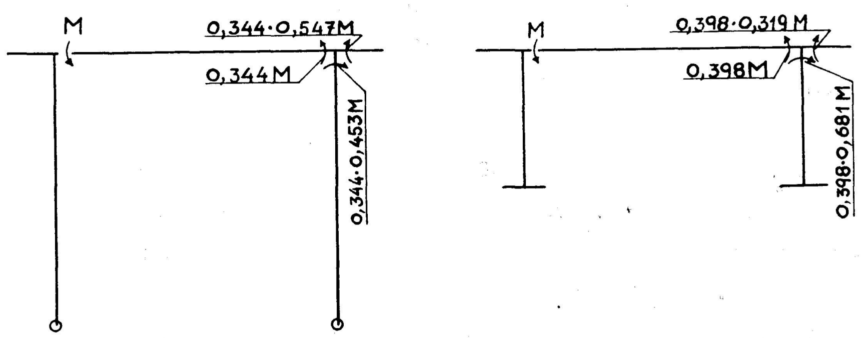


Fig. 14. Left: Rigidity of the structure used in Example No. 1. Right: Rigidity of the structure used in Example No. 2

V. Final Remarks

In what follows, some comments are made on the conditions which must be fulfilled in order that the use of the method advanced in this paper may be advantageous. To begin with, reference is made to the inequality (21) which stipulates $S^2 K E J \gg 1$. The quantitative interpretation of this condition is uncertain. Nevertheless, the second example in the preceding chapter shows that $S^2 K E J = 30$ provides an ample margin of safety. Another uncertain point is the magnitude of K. A foundation that is strictly in conformity with our assumption is never met with under practical conditions. In many cases, an elastic, isotropic, and homogeneous semi-infinite body would be a better approximation. However, it has not been possible to find any transformation, such as Biot's, for beams deformed by moments. When the foundation consists of slabs separated from one another, as in the examples adduced in the foregoing chapter, the problem is simplified since these slabs can mostly be assumed to be pressed down individually, without affecting the adjacent slabs.

In the application of the method to Vierendeel trusses, just as always in the approximate treatment of such structures, it is necessary to pay attention to the basic assumption that the points of inflection of the chords are situated in the middle between the vertical members. This assumption usually presupposes that the vertical members are perfectly rigid. If the practical conditions differ too much from this presupposition, the calculation will be deprived of its necessary basis. This is illustrated by the first example in the preceding chapter.

Summary

This paper deals with beams deformed by shearing forces only, in accordance with Eq. (4), and supported on an elastic foundation acting like a fluid in conformity with Eq. (2). Expressions for the moments, shearing forces, and deformations are deduced in the elementary cases of loading which result in equilibrium. The Author establishes a relation that must exist between the flexural rigidity, the rigidity in shear, and the modulus of foundation in order that the formulae should be applicable.

It is shown that the method advanced in this paper can conveniently be applied to elastically supported Vierendeel trusses. All necessary formulae are deduced for these trusses. A special study is made of the case where the rigidity of the bottom chord supported by the foundation is zero. Moreover, it is demonstrated that the method can sometimes be used as a suitable approximation in the design of continuous beams supported by, and rigidity connected to, columns on elastic foundations. The applicability and the limitations of the method are illustrated by two examples.

Zusammenfassung

In der vorliegenden Arbeit werden Träger untersucht, die nach Gleichung 4 nur durch Querkräfte deformiert werden und die gemäß Gleichung 2 auf einer flüssigkeitsähnlichen, elastischen Bettung aufliegen. Ausdrücke für Momente, Querkräfte und Verformungen wurden für die elementaren Lastfälle, die die Gleichgewichtsbedingungen erfüllen, abgeleitet. Der Autor stellt eine Beziehung her, die zwischen der Biegungssteifigkeit, der Schubsteifigkeit und dem Bettungsmodul bestehen muß, damit die Formeln anwendbar sind.

Es wird gezeigt, daß die in der vorliegenden Arbeit erläuterte Methode ohne weiteres für elastisch gelagerte Vierendeel-Träger angewendet werden kann. Alle für diesen Fall notwendigen Formeln werden abgeleitet. Im besondern wurde der Fall untersucht, bei dem die Steifigkeit des auf der Fundation ruhenden Untergurtes Null ist. Es wird außerdem gezeigt, daß die Methode manchmal auch als gute Näherung für die Berechnung von durchlaufenden Balken mit biegesteif angeschlossenen, elastisch senkbaren Stützen verwendet werden kann. Die Anwendungsmöglichkeit und der Gültigkeitsbereich der Methode werden an zwei Beispielen erläutert.

Résumé

L'auteur étudie les poutres qui ne sont déformées que par des efforts tranchants, suivant l'équation (4) et qui portent sur une assise élastique se comportant à la manière d'un fluide, suivant l'équation (2). Dans les cas

élémentaires de mise en charge correspondant aux conditions d'équilibre, il établit des expressions concernant les moments, les efforts tranchants et les déformations. Il indique une relation qui doit exister entre la résistance à la flexion, la résistance au cisaillement et le module de compressibilité élastique, pour que les formules ci-dessus soient applicables.

L'auteur montre que la méthode proposée peut être opportunément et directement appliquée aux poutres Vierendeel supportées élastiquement; toutes les formules nécessaires à ce cas sont établies. Il étudie spécialement le cas où la rigidité de la membrure inférieure, reposant sur la fondation, est nulle, et montre en outre que cette méthode peut souvent être utilisée en bonne approximation pour le calcul des poutres continues supportées par des appuis rigides portant eux-mêmes sur des assises élastiques. Deux exemples mettent en évidence les possibilités et les limites d'application de la méthode exposée.

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