Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen
Band: 10 (1950)

Artikel: Dampening effect in suspension bridges
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DOI: https://doi.org/10.5169/seals-10588

Dampening Effect in Suspension Bridges

Dämpfungsvorgang bei Hängebrücken

Les processus d'amortissement dans les ponts suspendus

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In the investigation of the aerodynamic stability of Suspension Bridges the dampening effect of the structure is of great interest. However, only few informations of this effect are available.

The Norwegian Public Road Administration has since 1917 built about one hundred suspension bridges with span length varying between 50 and 230 m. All the bridges have a very slender stiffening girder, rolled beams of I Dip. or I NP section. (I Dip. broad flange, I NP narrow flange I beams). The stiffening girders also forms the cords in the lateral truss.

Of all this bridges only one has given any trouble of importance due to vertical oscillations in strong wind. Several others are reported to have oscillated with small vertical oscillations, without getting any trouble or damages.

The Norwegian Public Road Administration, however, found that it should be made investigations on the aerodynamic stability of the suspension bridges. As a part of this investigations dampening tests were made on suspension bridges in 1949 and 1950. The field tests were superintended by Mr. P. G. Hansson C. E. from the Public Road Administration.

The span and other information of the bridges are given in table I and Fig. 5 and 6.

A problem was to find a method of starting the oscillations. The final choice was to let some young men jump with the same frequency as the natural frequency for the type of oscillation under investigation. The method proved very successful. A lot of vertical and torsional oscillations were built up in this manner. When the amplitude had its maximum value the dancing men made a sudden stand still and the measuring of the oscillations started.

The number of men varied from 10—15 with the bridge. The greatest bridge oscillated — Elverum bridge — was tested with 25 men, and the impulse was sufficient to build up oscillations with an amplitude of $\pm$ 10 cm, and for all the bridges there was no difficulty in starting oscillations in this manner. In many of the tests it was quite clear that the oscillation was mixed with secondary waves. For instance the main wave was of the type 1. node (2 half waves) and a secondary wave was of the type 5 nodes (6. half waves). The interference between the different wave forms give some scattering in the δ values, however, the average value should be quite correct. As will be seen from the figures 1; 2; 3; 4, some tests were successfull with clean oscillations, and some of the tests shows great scattering, a result of the secondary waves.

The figures 1; 2; 3; 4 are representative for the tests.

Table I

Bridge name	Span length l in m	Sag f in m	Width $c-c$ cables in m	Stiffening girder	Bridge.floor
Opstadaaen	72	9	3,67	INP 36	Cont. R. C. slab
Ekernsund	90	10,93	3,05	INP 45	Cont. R. C. slab with asphalt macadam pavement
Haugen	70	9	2,90	INP 38	Cont. R. C. slab
Komnes	100	12	2,85	INP 45	Cont. R. C. slab with asphalt macadam pavement
Kolo	95	9,50	5,35	IDIP 38	Cont. R. C. slab
Hvaara	—	—	—	—	—
Fossum	84	10,50	4,50	IDIM 65	Timber flooring
Tynset	130	14,5	6,60	IDIP 50	Cont. R. C. slab
Elverum	160	17,8	5,95	IDIP 55	Cont. R. C. slab
Atna	150	17,5	3,96	Truss $h=2{,}2$ m	Timber flooring
Kveberg	89	9,0	3,45	INP 36	Timber flooring

The measuring was made in 2 different manners. One instrument measured the vertical deflection and made a record of the amplitude. Two other instruments made a record of the strain in the stiffening beam. The instruments were usually placed on the upper and lower flange of the stiffening beam.

For each wave form there will be proportionality between amplitude of the oscillation and strain in the stiffening beam, and recorded strain measuring may be used for the evaluation of the log decr. δ in the same manner as the recorded amplitude.

The logarithmic decrement δ is evaluated from the formula

$$\delta = \frac{\Delta \eta}{\eta}$$

Where η is the amplitude at any full cycle and $\Delta \eta$ is the decrease in amplitude of one full cycle.

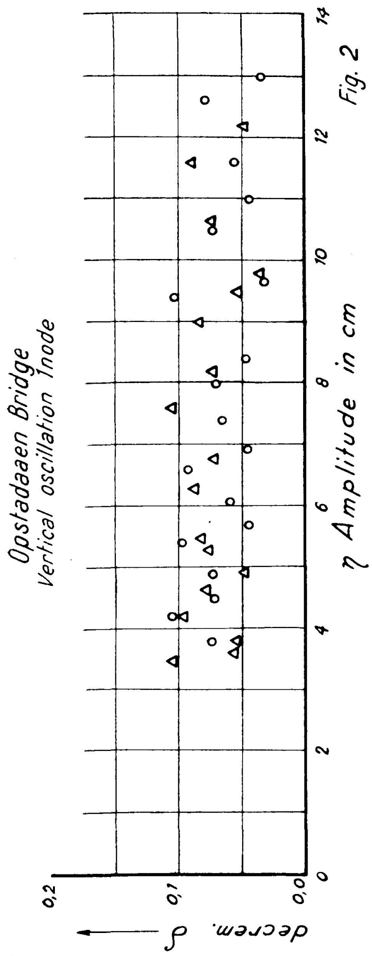


Fig. 2

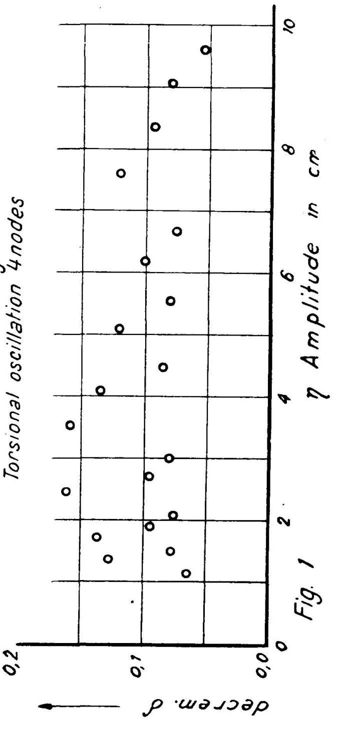


Fig. 1

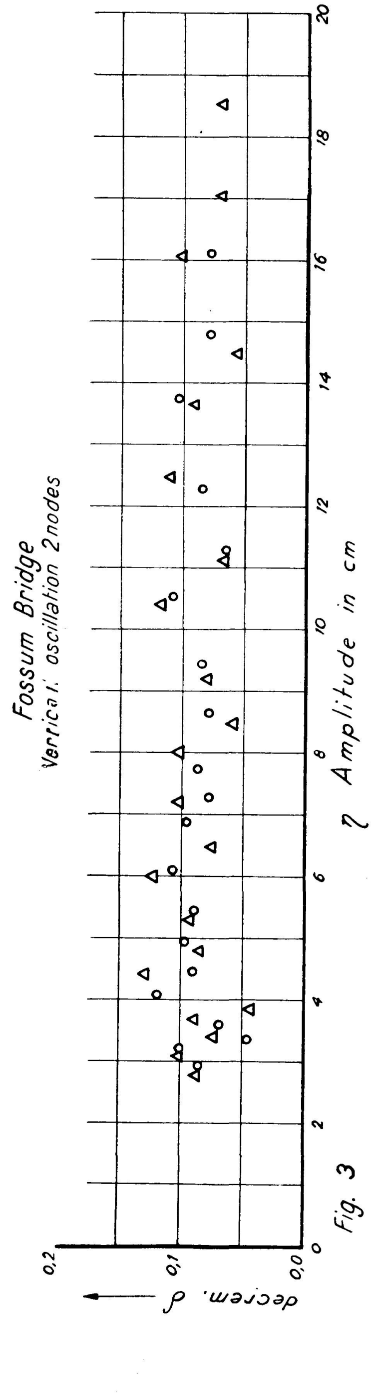


Fig. 3

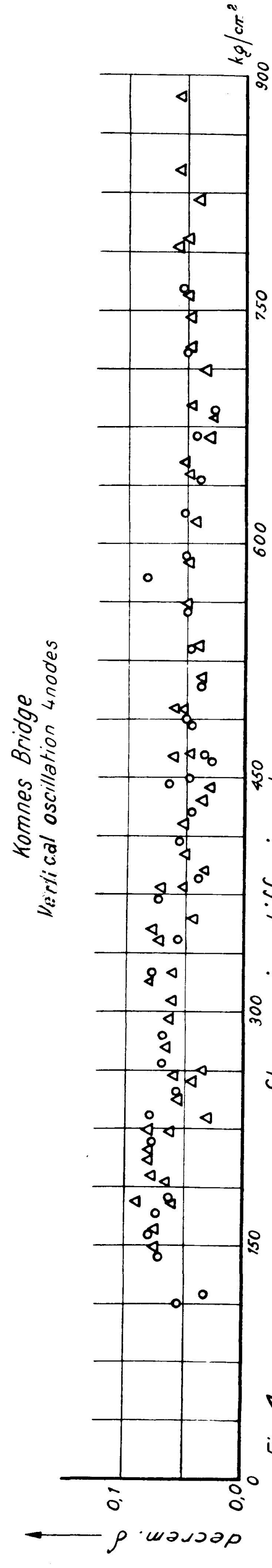


Fig. 4

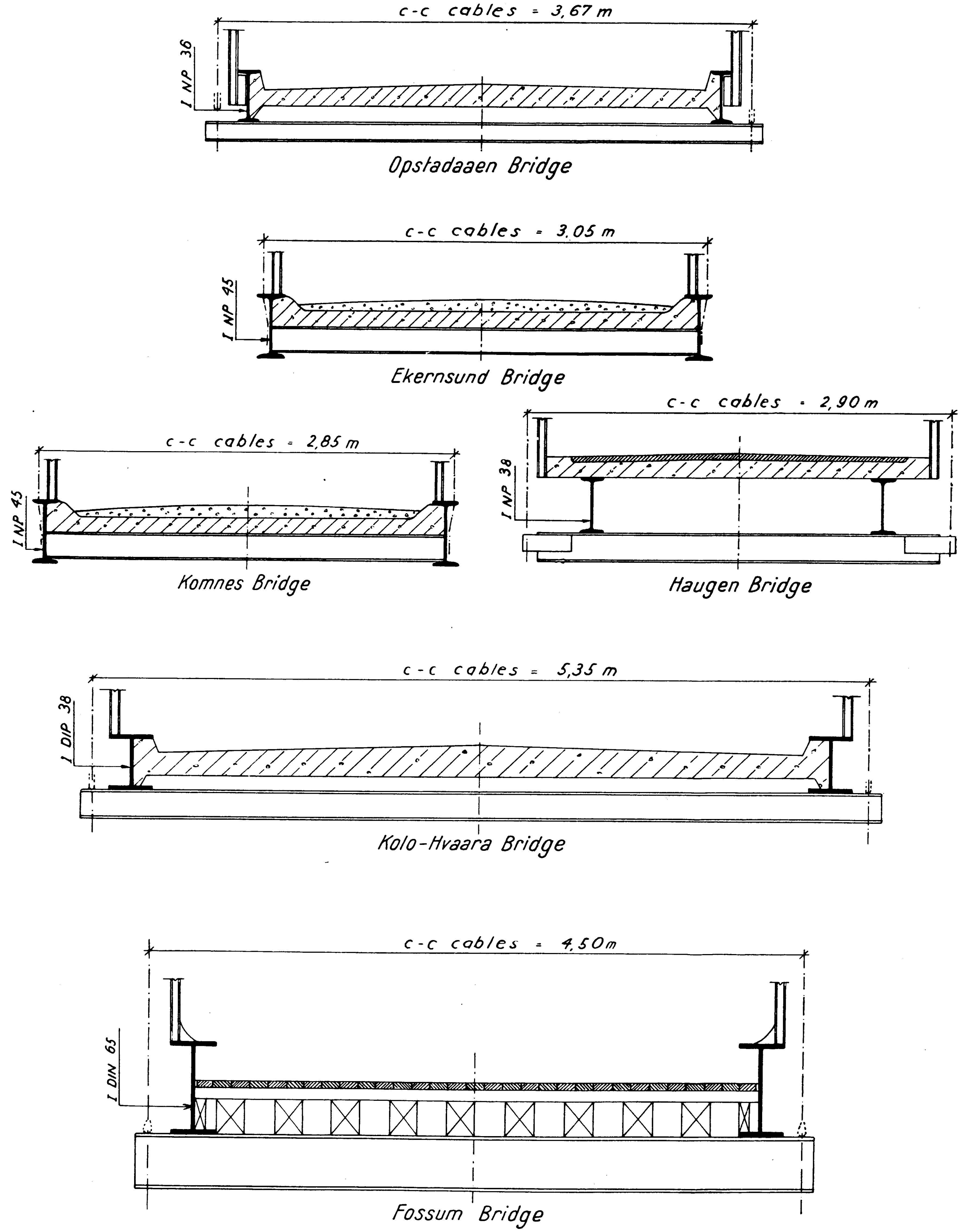

c-c cables = 3,67 m
I NP 36
Opstadaaen Bridge
c-c cables = 3.05 m
I NP 45
Ekernsund Bridge
c-c cables = 2,85 m
I NP 45
Komnes Bridge
c-c cables = 2,90 m
I NP 38
Haugen Bridge
c-c cables = 5,35 m
I DIP 38
Kolo-Hvaara Bridge
c-c cables = 4,50m
I DIN 65
Fossum Bridge

Fig. 5

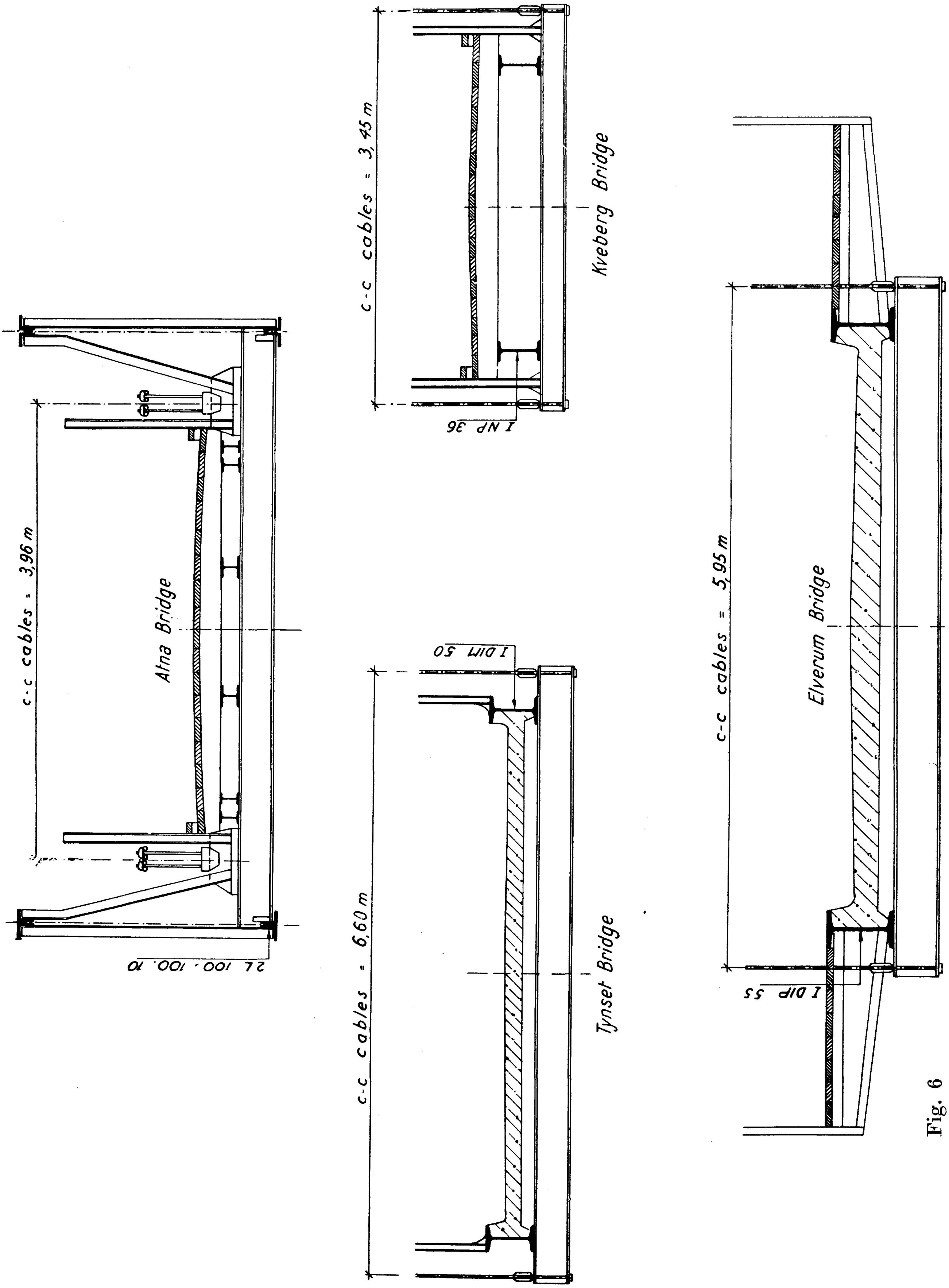

c-c cables = 3,96 m
Atna Bridge
2L 100.100.10
c-c cables = 3,45 m
Kveberg Bridge
I NP 36
I DIM 50
c-c cables = 6,60 m
Tynset Bridge
c-c cables = 5,95 m
Elverum Bridge
I DIP 55

Fig. 6

Table II

Bridge name	Decrement δ for vertical oscillation				Decrement δ for torsional oscillation				Frequencies per sec. vertical oscillation				Frequencies per sec. torsional oscillation			
	Number of nodes				Number of nodes				Number of nodes				Number of nodes			
	1	2	3	4	1	2	2	3	1	2	3	4	1	2	2	3
Opstadaaen	0,075	0,06	—	—	0,10	—	—	—	0,54	0,83	(1,44)	(1,61)	2,05	(2,45)	(3,50)	4,40
Ekernsund	0,15	0,05	—	0,07	0,15	—	0,10	—	0,57	0,79	1,41	2,12	1,43	1,59	2,28	2,94
Haugen	0,125	0,075	0,100	—	0,15	—	—	—	0,75	1,02	2,44	(3,83)	1,85	(2,16)	(3,22)	(4,43)
Komnes	0,085	0,070	—	0,05	—	—	—	—	0,48	0,65	(1,18)	1,67	1,49	(1,59)	2,18	(2,89)
Kolo	0,16	0,08	—	0,06	0,15	—	—	—	0,455	0,67	1,19	1,70	1,59	(2,34)	(4,17)	(3,39)
Hvaara	—	0,055	—	0,06	0,185	0,12	—	—	0,455	0,67	1,18	1,70	1,65	(2,34)	(4,17)	(3,39)
Fossum	0,120	0,080	—	0,08	0,159	0,10	—	—	0,76	{1,21 {1,67	2,08	3,10	1,02	1,61	1,86	1,82
Tynset	0,100	0,045	—	—	0,05	0,05	—	—	0,83	1,18	2,08	—	1,61	1,92	—	—
Elverum	—	0,04	—	0,04	0,08	0,06	—	—	—	0,96	—	1,22	0,85	1,04	—	—
Atna	0,12	0,07	—	—	0,20	0,135	—	—	0,82	0,92	—	—	0,79	1,11	—	—
Kveberg	0,215	0,185	—	0,180	0,30	0,180	—	—	0,55	0,84	—	2,22	0,77	1,22	—	—

Frequencies in ( ) are only calculated. Difference between calculated measured frequencies are max. 6 per cent.

From the strain recorded we find the logarithmic decrement δ from the formula

$$\delta = \frac{\varDelta \sigma}{\sigma}$$

where σ is the stress at any full cycle and $\varDelta \sigma$ the decrease in stress of one full cycle. The stress is evaluated from the strain recorded from the following formula

$$\sigma = E \frac{\varDelta l}{l} = E \epsilon$$

where E is the modulus of elasticity (2100000 kg/cm^2 for steel), l the measuring length (100 cm or 70 cm) and $\varDelta l$ the variation in this length, $\varDelta l$ was recorded with an enlargement of 140 in the strain diagram.

In the table II is given the mean value of δ for each bridge. The different oscillation types were tested 3 to 4 times, and the δ values are the average values from all tests.

As will be seen from the figures of δ, figs. 1, 2, 3, 4 the δ values are scattered a lot especially for small amplitudes. The dampening effect seems to increase a little with very small amplitudes. This have, however, no practical interest as an oscillation may start from a sudden wind blow or a heavy wagon and from the very beginning of the oscillation the amplitudes will be so large that the lower value of δ determine the oscillations.

The tests demonstrated some interesting details about the oscillation.

All oscillations where the movement of the cables in longitudinal direction is of little importance are comparatively easy to obtain. This oscillations are vertical and torsional oscillations of 3, 4, 5 or more half waves (2, 3, 4 or more nodes). Especially 3 half waves or 2 nodes were easy to obtain and with great amplitudes.

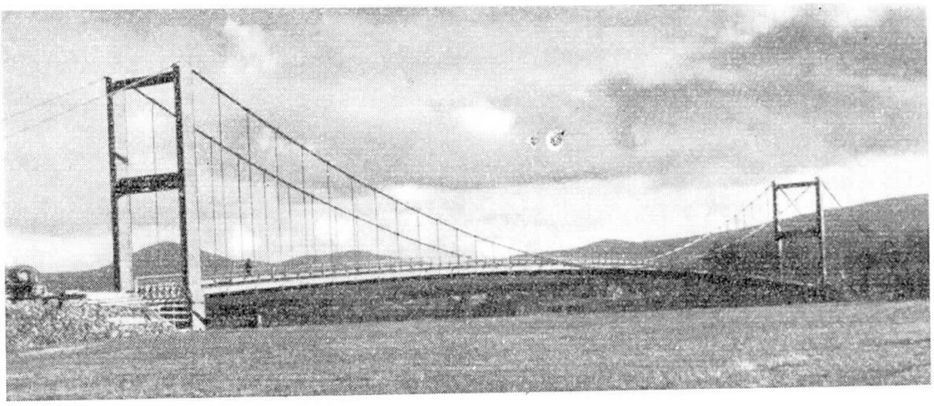
Oscillations with longitudinal movements of the cables will especially be vertical or torsional oscillations with 2 half waves, one node at the middle of the bridge. For this sort of oscillations there will be a considerable dampening effect in the frictional movement of the hangers and the frictional movement on the bearings at each tower.

For some bridges it seemed to be impossible to get oscillations of this form, and the bridges where the decrement δ could be measured showed a somewhat greater value of δ than of other types of oscillation.

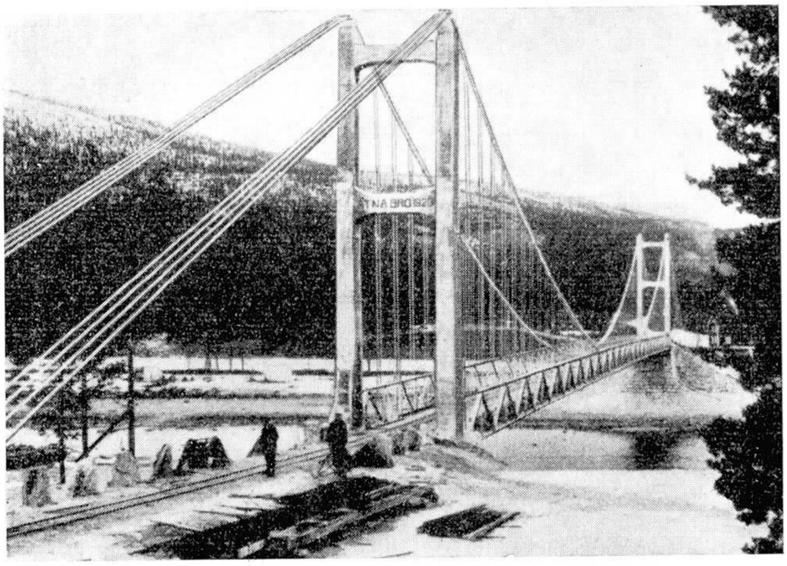
All tests demonstrated that except for the increase in δ with very small amplitudes the δ values could be considered as a constant, having no variation of importance with increasing amplitudes. In the table II are given decrement δ and frequencies measured on all types of oscillations where a clean and stabile oscillation was obtained.

From the table II will be seen that the dampening effect is somewhat larger for torsional oscillations than for vertical oscillations, the main reason for this seems to be the hysteresis dampening in the continuous concrete slab.

This continuous concrete slab have little effect on the frequencies for vertical oscillations. On the other hand the effect of the concrete slab on torsional oscillations is very important and of the same magnitude as the cable tension. The torsional stiffness of the slab will be more dominating for smaller bridges and of lesser importance with increasing span length, as will be seen from the formulas given in the last part of the paper.



Tynset bridge; 130 m span

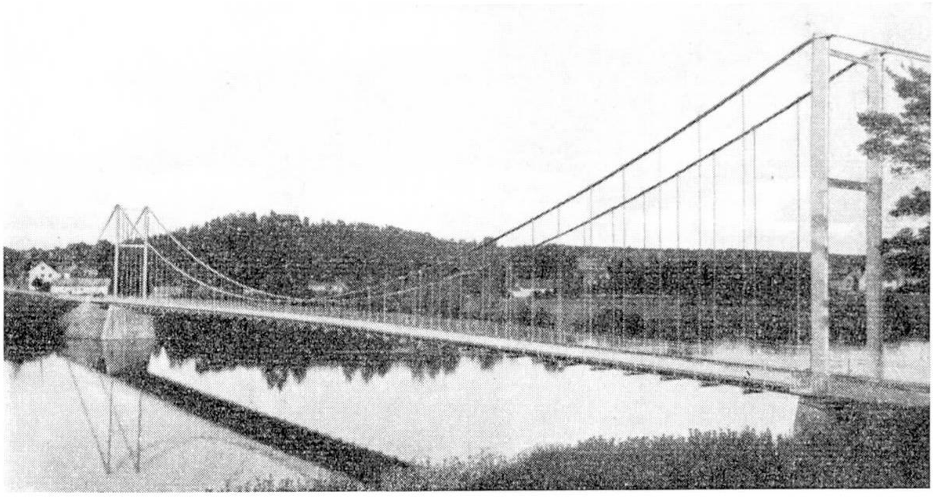


Atna bridge; 150 m span

The torsional stiffness of the slab increase the frequency for torsional oscillations, and with an increase in frequency the critical wind velocity will increase. It seems very reasonable to think that the increased frequency due to the torsional stiffness of the slab is one of the good effects which up to this date have saved the Norwegian suspension bridges from torsional oscillations.

As the effect will decrease with increasing span length, the problem of aerodynamic stability for torsional oscillations must be investigated for bridges of medium or large size or e.g. for span length over 200 m.

From table II one will get some informations of the dampening effect. It is easily seen that the δ value is decreasing with increasing span length, and that bridges with timber flooring have a better dampening than bridges with a continuous concrete slab.



Elverum bridge; 160 m span

The bridges Tynset and Elverum are the best representants of recent Norwegian suspension bridges, and for bridges of this type the following decrements δ are on the safe side. Vertical oscillations with 1. node $\delta = 0.08$. For all other vertical oscillations $\delta = 0.04$.

For all types of torsional oscillations $\delta = 0.05$.

Suspension bridges with a stiffening truss is only represented by Atna bridge, as there exists only few bridges of this type in Norway. From the table II will be seen that the decrement δ will be approximately the same for Fossum, Atna and Kveberg bridges, and it seems reasonable to conclude that the better dampening of Atna bridge is mainly the cause of the timber floor.

Calculation of natural frequencies for suspension bridges

For oscillations without any dampening the amplitude at any point may be found from a series

$$\eta = \left(\sum a_n \sin n \pi \frac{x}{l} \right) \sin \omega t, \tag{1}$$

where η is the deflection, l the span, x the abscissa, $\omega = 2 \pi N$ and N is the frequency (cycles pr. sec.), t is the time.

Oscillations with 3, 7, 11 nodes (4, 8, 12 half waves) will have a frequency

$$\omega_n = 2\pi N = \sqrt{\alpha n^4 + \beta n^2}\,; \quad n = 4, 8, 12 \tag{2}$$

and the oscillation will have the form

$$\eta_n = a_n \sin n\pi \frac{x}{l} \sin \omega_n t\,; \quad n = 4, 8, 12. \tag{3}$$

Oscillations with (1) 3, 5 — half waves or (0), 2, 4 . . . nodes will have frequencies which satisfies the equation

$$\sum_{1,3,5..} \frac{1}{n^2\{\omega^2 - \alpha n^4 - \beta n^2\}} - \lambda = 0\,; \quad n = 1, 3, 5; \quad \omega = 2\pi N \tag{4}$$

ω and N values which satisfy the equation are found from a graphical representation of the equation for different values of ω.

Oscillations with 1 half wave (o nodes) will not occur on bridges with no suspended side spans, however, it may occur 2 types of oscillations with 3 half waves. Of practical interest is mainly the one with the lesser value of the frequency N, this oscillation will have its maximum amplitude at the middle of the span length.

The form of the oscillation is found from

$$\eta = \left(\sum_{1,3,5} a_n \sin n\pi \frac{x}{l}\right) \sin \omega t\,; \quad n = 1, 3, 5 \ldots \tag{5}$$

where ω has values found from equation (4), and a_n from the expression

$$a_n = \frac{k}{n\{\omega^2 - \alpha n^4 - \beta n^2\}}\,; \quad n = 1, 3, 5 \ldots \tag{6}$$

k is an arbitrary constant.

Oscillations with 2, 6, 10 half waves (1, 5, 9, . . . nodes) will have frequencies which satisify the equation

$$\sum_{2,6,10..} \frac{1}{n^2\{\omega^2 - \alpha n^4 - \beta n^2\}} - \kappa = 0\,; \quad n = 2, 6, 10 \ldots \tag{7}$$

The equation is solved in the same manner as equation (4). However, the effect of the inclination of the hangers will have little influence on oscillations of 6, 10 . . . half waves and equation (7) may be replaced by:

$$\omega_2 = 2\pi N_2 \approx \sqrt{2^4\alpha + 2^2\beta + \frac{1}{2^2\kappa}} \tag{7a}$$
$$\text{when } \kappa \gtrless 0\,.$$

N_2 is the frequency of 2 half waves. The equation must be solved with a trial and error proccess as ω_2 is a part of the factor κ.

$$\omega_{6,10} = 2\pi N_{6,10} \approx \sqrt{\alpha n^4 + \beta n^2}\,; \quad n = 6, 10.. \tag{7b}$$

For oscillations with 6, 10,... half waves. Eq. (7b) is as will be seen identical with eq. (2).

The form of the oscillations is found from

$$\eta = \left(\sum_n a_n \sin n\pi \frac{x}{l}\right) \sin \omega t; \quad n = 2, 6, 10.. \tag{8}$$

where

$$a_n = \frac{k}{n(\omega^2 - \alpha n^4 - \beta n^2)}; \quad n = 2, 6, 10.. \tag{9}$$

However, the wave which is characteristic for the oscillation will be so dominating that only one term may be used and formula (8) replaced with

$$\eta = a_n \sin n\pi \frac{x}{l} \sin \omega t; \quad n = 2, 6, 10 \ldots \tag{8a}$$

The coefficients α, β, λ and κ may be found from the following expressions.
Vertical oscillations:

$$\left.\begin{aligned} \alpha_v &= \frac{\pi^4 JE}{l^4 w} g; \quad \beta_v = \frac{\pi^2}{l^2} \frac{H\left(1+4\left(\frac{f}{l}\right)^2\right)}{w} g; \quad \lambda_v = \frac{\pi^2 l^3}{8^3 f^2} \frac{w \cdot L_s}{g \cdot E_c A_c} \\ \kappa_v &= \frac{1}{4}\lambda \cdot \left\{1 + \left(\frac{2}{A_1} - \frac{4A_2}{A_1 l} \frac{g}{w_s \omega^2}\right) \frac{E_c A_c}{L_s}\right\} \end{aligned}\right\} \tag{10}$$

For torsional oscillations:

$$\left.\begin{aligned} \alpha_T &= \frac{\pi^4 (\sum JE \cdot r^2)}{l^4 M}; \quad \beta_T = \frac{\pi^2 \left(H b_c^2 \left(1+4\left(\frac{f}{l}\right)^2\right) + 2\sum G J_T\right)}{l^2 \cdot 2M} \\ \lambda_T &= \frac{\pi^2 l^3}{8^3 f^2} \frac{2 \cdot L_s M}{E_c A_c b_c^2}; \quad \kappa_T = \frac{1}{4}\lambda_T \left(1 + \frac{2}{A_1} \frac{E_c A_c}{L_s}\right) \end{aligned}\right\} \tag{11}$$

In the equations (10) and (11) we have

$$\left.\begin{aligned} A_1 &= \frac{w_s l}{2h(l/2)} \left(\frac{1+C}{B} \operatorname{arc.tg.} B - C\right) \\ A_2 &= \frac{w_s l}{2h(l/2)} \frac{\operatorname{arc.tg.} B}{B} \\ B &= \sqrt{\frac{h(0) - h(l/2)}{h(l/2)}}; \quad C = \frac{1}{B^2} \end{aligned}\right\} \tag{12}$$

Nomenclature for (10), (11) and (12)

l = length of suspended span.
f = sag of suspended span.
b_c = width center to center of cables.
g = acceleration of gravity.

$h(l/2)$ = length of suspenders at the middle of the span.
$h(0)$ = length of suspenders at the towers ($\approx$ distance between roadway and cable saddles).
w = dead load per unit length on one cable (assumed constant).
w_s = dead load per unit length suspended in the hangers on one cable (assumed constant)
A_c = Area of one cable.
E = modulus of elasticity of truss material.
E_c = modulus of elasticity of cable material.
G = modulus of elasticity in shear.
H = horizontal component of cable tension for dead load.
J = moment of inertia of one stiffening girder about its horizontal axis. (For bridges with continuous concrete slab the effect of the slab is included in JE.)
J_T = torsional moment of inertia.
$\sum JE\,r^2$ = sum of bending rigidity multiplied by the square of distance from its longitudinal torsional axis. The sum is taken over the whole cross section.
$\sum GJ_T$ = sum of torsional rigidity of the cross section of the bridge.
L_s = $\int \frac{ds^3}{dx^2}$ for entire length of cable.
M = $\frac{1}{g}\int r^2\,dw$ = the mass moment of inertia of the cross section of the bridge about its longitudinal axis of rotation. (Since the motion of the cable is essentially vertical, only its horizontal distance from the axis of rotation is used.) For bridges of the type used in Norway the rotation axis will go through the center of gravity of the cross section.
N = frequency.
ω = circulare frequency = $2\pi N$.

The formulas given above are deduced with the following simplifications:

1. The cable curve in the suspended span is a parabola.
2. The bending and torsional rigidity is constant over the span length.
3. The oscillation amplitudes η are small compared with the span length l and the distance between center – center cables b_c.
4. The effect of the inclination of the hangers are represented by an approximate correction. (Publications 8. Vol. 1947. A. Selberg: Suspension bridges with....)
5. Rodes differential equation for suspension bridges is replaced by Melans differential equation corrected with the term $H\left(1+4\left(\frac{f}{l}\right)^2\right)$ instead of H as given by Melans equation.

The above simplifications are for the points 1—3 the same as commonly used in suspension bridge analysis. The point 4 and 5 represents an improvement compared with the common analysis. The formulas deduced above give very accurate values for all bridges tested, and as there exist some uncertainty about the shear modul G for the concrete slabs the correlation with the measured values are satisfying.

When the first investigations on the vertical oscillations in Fyksesund bridge were started in 1944 it was built a model to investigate the vertical oscillations. This model has been used for control of the formulas above and a correlation within 2 per cent was found for all types of oscillations investigated.

The formulas represented above may very easily be extended to include all types of oscillations in suspension bridges with 2, 3 or more spans and with different span length. Especially the common suspension bridge type with one main span and 2 equal side spans give very simple formulas.

The curious problem that a suspension bridge, oscillating vertically with 2 half waves, gets one frequency if the oscillation is built up by impulses in the span and an other different frequency if the impulse is working longitudinally at the end of the stiffening truss may easily be declared with the formulas. The difference in frequency is easily demonstrated on a model.

For a bridge with no dampening the frequencies will be the same, however, if there is any dampening the inclination of the hangers will be different for the two cases. In the formulas (1)—(12) the coefficient κ will be different. The horizontal component of cable tension at left tower is called $H+\Delta H$ at right tower $H-\Delta H$. $2\,\Delta H$ is the resultant of the inclination of all the hangers for one cable. Then we have with oscillations without any dampening

$$\Delta H = \frac{\pi\, l^2 w}{8^2 f g} \cdot \frac{1}{\kappa} \sum \frac{a_n}{n} \sin \omega t; \qquad n = 2, 6, 10 \ldots \tag{13}$$

or as 2 half waves is dominating:

$$\Delta H = \frac{\pi\, l^2 w}{g^2 f g} \, \frac{1}{\kappa} \, \frac{a_2}{2} \sin \omega_2 t; \qquad \omega_2 = 2\,\pi\, N_2 \tag{13a}$$

Nomenclature see above. With damped oscillations started and continued with impulse at the end of the stiffening girder, this impulse will take all the frictional work in bearings etc. and on the top increase the longitudinal movement a bit. This will increase ΔH and decrease the coefficient κ. As will be seen from Eq. (7a) a decrease of κ will mean an increase in frequency as measured on a model.

On the other hand if the oscillation is started and continued with impulse vertically in the span, the frictional forces in bearings etc. will reduce the longitudinal movements, reduce the inclination of the hangers and reduce the resulting force ΔH. This means κ is increased and on the other hand the frequency decreased. It is possible to use this difference in frequencies to find the friction in bearings etc.

Dampening effect on model tests

From the formulas given above we may find the frequencies for simple harmonic oscillations with no dampening effect. This frequencies are called N and $2\pi N = \omega$. Oscillations with a dampening effect have a frequency N_δ and $2\pi N_\delta = \omega_\delta$. N_δ may be found from the following formula:

$$\omega_\delta = \sqrt{\omega^2 - \delta^2} \quad \text{or} \quad N_\delta = \sqrt{N^2 - \left(\frac{\delta}{2\pi}\right)^2} \tag{14}$$

where δ is the logaritmic decrement.

From the table II it will be seen that δ have values varying from 0.25 to 0.04 and the effect of dampening on the frequency will be $\approx$ Zero or $N \approx N_\delta$.

Suspension bridges oscillating in a strong wind will have a dampening and on the other hand an impulse which builds up the oscillations. With the small values of δ a relatively small impulse will build up catastrophic oscillations and neither the dampening nor the impulse will produce any important change in frequencies.

Model tests may in other words be made with models having frequencies relating to the natural oscillations in the bridge.

Testing a full model or a section model of a suspension bridge in a wind channel, we find that the wind flow is approximately independent of Reynolds number and the following formulas may be deduced. Relation between wind velocity, frequencies

$$\frac{V}{N\,b} = \frac{V_m}{N_m\,b_m}$$

V = wind velocity, N = frequency and b = a characteristic length (for instance roadway width) in the bridge. The index m marks the same for the model.

The length reducing factor is $\frac{b}{b_m} = K$ and the formula above will be

$$\frac{V_m}{N_m} = \frac{1}{K} \cdot \frac{V}{N} \tag{15}$$

which give one relation between wind velocity and frequency for the model.

The wind flow will give the bridge an increased kinetic energy per unit length which approximately can be introduced with the formula

$$F\,(\eta\,b)\,V^2$$

and for the model

$$F(\eta_m b_m) V_m^2 \approx \frac{1}{K^2} F(\eta b) V_m^2$$

On the other hand will the structural dampening decrease the energy E in free oscillations with an amount

$$E\, 2\,\delta = 2\, E_1 N^2 \eta^2 \delta$$

where E_1 is the energy pr. unit length of the bridge in an oscillation with $N = 1$ and $\eta = 1$.

$$\text{Vertical oscillation} \quad E_1 = 2\pi^2 \frac{w}{g}$$

$$\text{Torsional oscillation} \quad E_1 = 8\pi^2 \frac{M}{b_c^2}$$

w, g, M and b_c are defined in the nomenclature for the equations (10)—(12).

For the model the decrease in energy will be

$$E_m \cdot 2\,\delta_m = 2\, E_{1m} N_m^2 \eta_m^2 \delta_m = 2\, E_{1m} N_m^2 \frac{\eta^2}{K^2} \delta_m$$

The conditions for oscillation in bridge and model will be the same when:

$$\frac{F(\eta b) V^2}{\frac{1}{K^2} F(\eta b) V_m^2} = \frac{2\, E_1 N^2 \eta^2 \delta}{\frac{1}{K^2} 2\, E_{1m} N_m^2 \eta^2 \delta_m}$$

$$\delta_m = \delta \frac{E_1}{E_{1m}} \left(\frac{V_m}{V}\right)^2 \left(\frac{N}{N_m}\right)^2 \qquad (16)$$

Introducing the expression for E_1 and V_m from eq. (15):

Vertical oscillation

$$\delta_m = \delta \frac{w}{w_m} \frac{1}{K^2} \qquad (17a)$$

Torsional oscillation

$$\delta_m = \delta \frac{M}{M_m} \frac{1}{K^4} \qquad (17b)$$

where K is the length reducing factor, w is weight pr. length unit and M mass moment of inertia pr. length unit for the cross section about its longitudinal axis of rotation.

When possible the model shall have a dampening δ_m as given in eq. (17), and it is very important that δ_m is independent of the reactions in bearings etc. and all form of frictional dampening should be avoided.

From table II will be seen that all oscillations with 2 or more nodes will have approximately the same dampening effect. A consequence of this fact will be that the catastrophic oscillations will be vertical or torsional with one or two nodes. Oscillations with more than 2 nodes will have a higher frequency and subsequently a higher critical wind velocity than an oscillation with 2 nodes. Model tests should be limited to this types of oscillations.

Summary

The dampening effect of vertical and torsional oscillations are tested on small and middle size suspension bridges. The result is given in Table II. As will be seen the decrement δ is very low on suspension bridges.

Formulas for calculating the natural frequencies on suspension bridges are given in (1)—(17).

Zusammenfassung

Der Dämpfungsvorgang bei vertikalen und Verdrehungsschwingungen wird bei kleineren und mittleren Hängebrücken untersucht. Das Ergebnis ist in Tabelle II dargestellt. Es zeigt sich, dass der Dämpfungsfaktor δ sehr gering ist.

Die Gleichungen (1) bis (17) geben Formeln zur Berechnung der Eigenschwingungen von Hängebrücken.

Résumé

L'auteur étudie les processus d'amortissement des oscillations verticales et de torsion dans les ponts suspendus de petite et de moyenne importance. Les résultats obtenus font l'objet d'un tableau. On constate que le coefficient d'amortissement δ est très faible.

Les équations (1) à (17) constituent des formules pour le calcul des oscillations propres des ponts suspendus.

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