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Theory of Timber Connections

Theorie der Holzverbindungen

Théorie des assemblages en bois

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Modern timber connections usually consist of a bolt with a toothed dog between the faces of the joint to prevent slipping of the timbers. The function of the connection may be divided into three elementary effects.

1. The dowel effect of the bolt, which depends upon its resistance to bending and the resistance of the wood to crushing.
2. The tensional effect of the bolt, which depends upon its resistance to tension and friction between the abutting surfaces.
3. The effect of the dog, which depends on its form and strength together with the resistance of the wood to crushing.

The elements of the joint were considered as

1. Resistance of the wood to crushing under a dowel.
2. Friction.
3. Effect of the dog.

1. Elements of the joint.

A. Embedding Stress

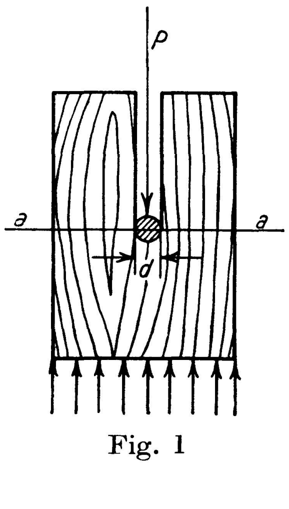

Fig. 1

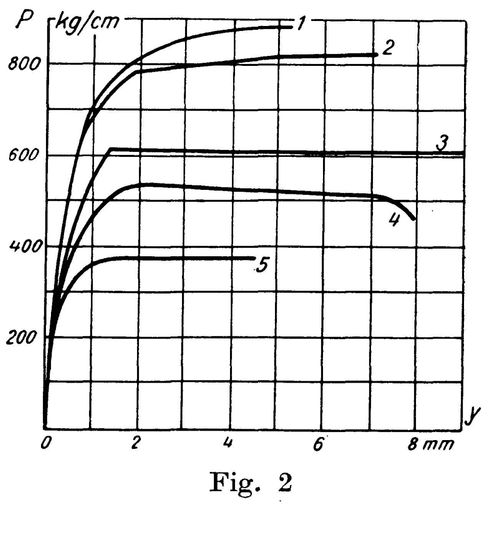

Fig. 2

The experiments illustrated in Fig. 1 yielded the curves for various woods shown in Fig. 2. It will be noted that the curves end in a horizontal line, i.e. a plastic condition. When p is the load per unit length of dowel, the pressure on the bearing surface of the hole is defined as $s_H = \frac{p}{d}$, where d is the diameter of the dowel. Ultimate stress for s_H in relation to the strength of a prism s_C is for 1. Pitchpine, 2. Oak, 3. Larch, 4. Fir and 5. Pine: 0,76; 1,10; 0,96; 0,85; 1,00 respectively.

B. Friction

Three pieces of timber were clamped together (Fig. 3) and the axial force N measured by tensometers. The centre timber was loaded with force P and its slip g in relation to the outer pieces measured. The P/g curves and the variation in force N are shown in Fig. 4. The coefficient of friction varied between 0,4 and 1,4 with an average value of $^2/_3$.

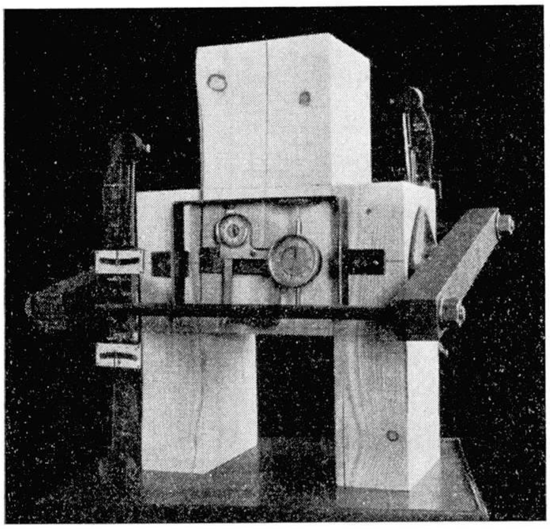

Fig. 3

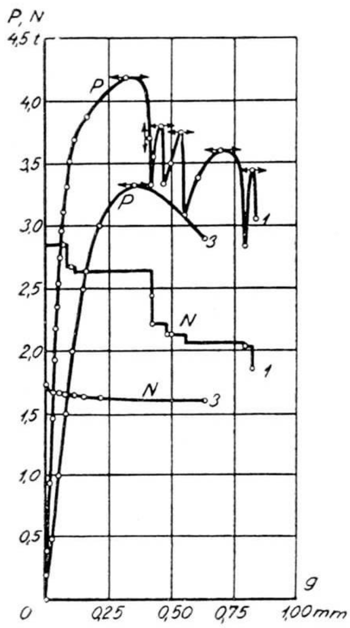

Fig. 4

C. Toothed Dog

The experiments were carried out with a clamp instead of a bolt as in the foregoing, but with the addition of toothed dogs between the abutting surfaces (Fig. 5). At the beginning of the experiment the clamp was but lightly tensioned and the force N at the joints, therefore, of but low value. Resultant curves are shown in Fig. 6. It will be seen that N begins to grow where the curve begins to bend. The increase in load capacity due to the dogs, with a coefficient of friction of $^2/_3$, is

$$G = P_{max} - 2 \cdot \tfrac{2}{3} N .$$

The values found for G divided by the embedded area A of the teeth alone give the stress σ_H, corresponding to embedding stress s_H. The effective embedded area for these toothed dogs is, therefore, the embedded area of the teeth.

The effective embedded area of other dog forms can be determined in the same way.

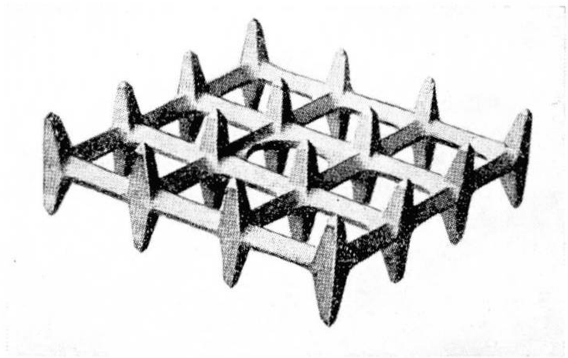

Fig. 5

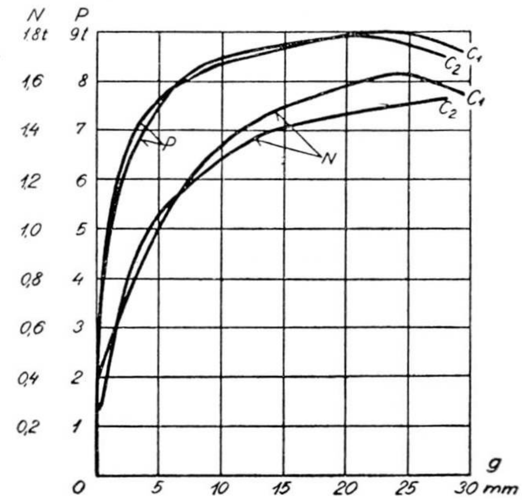

Fig. 6

2. Dowel Connections

The strength of a dowel connection depends partly upon the resistance of the wood to the embedding of the dowel, i.e. s_H and partly upon the resistance of the dowel to bending, i.e. its modulus of rupture s_B. As both the bending of the dowel and the embedding are plastic, the load capacity of a dowel connection can be formulated on this basis.

In a single shear connection with a dowel of sufficient stiffness, the latter will be canted as shown in Fig. 7. With a sufficiently large movement, there will be the pressure $s_H d$ practically over the whole dowel and the pressure distribution, transverse forces Q and moments M shown in Fig. 7 are then obtained.

The transverse force becomes zero twice as far away from the free end of the dowel as the point where the direction of the pressure changes.

The bending moment of the dowel in the joint being zero on account of the antimetry, the equation of equilibrium for the dowel will give

$$P = s_H \, dz$$

$$M_{max} = s_H \, dx^2 = \tfrac{1}{2} \, s_H \, dz^2$$

from which

$$x = z\sqrt{\tfrac{1}{2}}, \quad l = z + 2x = z(1+\sqrt{2}),$$

$$z = l(\sqrt{2}-1) = 0{,}414\, l, \quad x = 0{,}293\, l,$$

$$P = 0{,}414\, s_H \, ld. \tag{1}$$

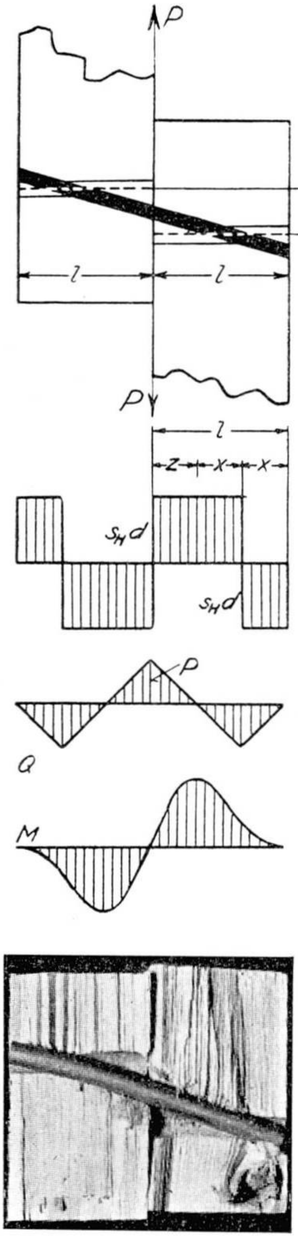

Fig. 7

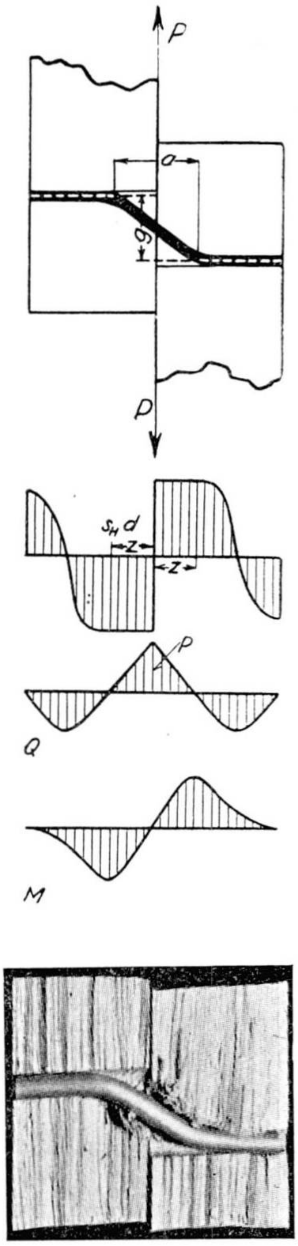

Fig. 8

If the dowel is not sufficiently stiff it will be bent as shown in Fig. 8. In bending, the moment is $\frac{\pi}{32}\, s_B d^3$ and the transverse load 0; between the bends the crushing is so heavy that the pressure is practically $s_H d$. About the pressure outside the bends nothing definite is known, nor is this of any consequence, as the static conditions are formulated only for the length of dowel between the bends.

If this piece of dowel is imagined to be cut out and the shear forces added, i.e., M_{max} and $Q = 0$, we obtain

$$P = s_H\, dz$$

$$M_{max} = \frac{1}{2} Pz = \frac{1}{2} \frac{P^2}{s_H d} = s_B \frac{\pi}{32} d^3,$$

which gives

$$P = 0{,}442 \sqrt{s_B s_H}\, d^2. \tag{2}$$

With a double shear connection and a stiff dowel, conditions are as shown in Fig. 9a or b and the resulting formulae are

$$\begin{matrix} P = s_H dm, \\ P = 2 s_H dl, \end{matrix} \quad \text{when} \quad \begin{matrix} m \leqq 2l \\ m \geqq 2l. \end{matrix} \tag{3}$$

A dowel of lesser stiffness bends in the centre timber (Fig. 10).

It will generally yield at two points, and outside these the state in the wood must be plastic, as the movement of rupture is due to the turning of the dowel.

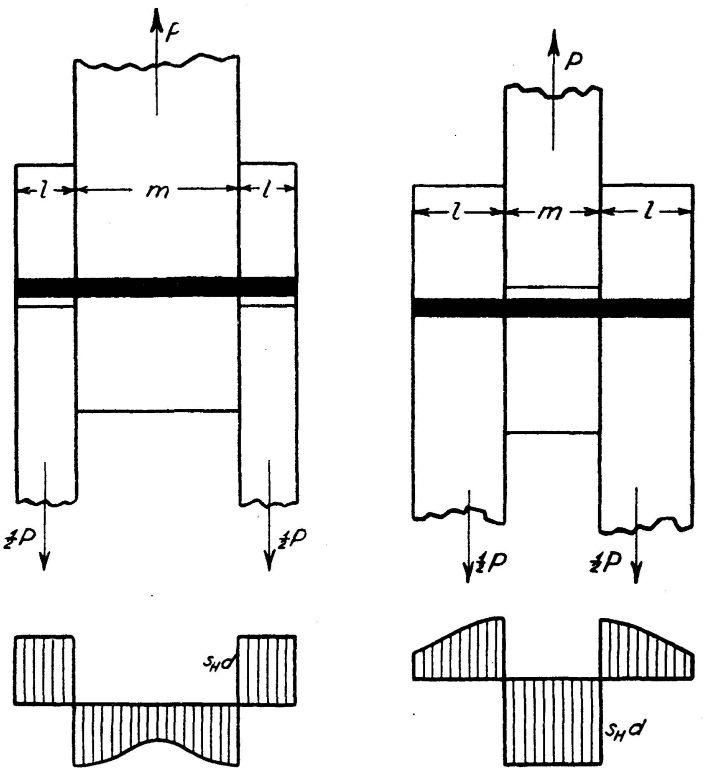

Fig. 9a Fig. 9b

In the strap a pressure distribution corresponding to Fig. 7 is had and in each half of the timber a pressure distribution corresponding to Fig. 8. Q being zero both at M_{max} and M_{min}, these must occur at the same distance z from the joint, as in both cases $s_H dz$ must be equal to $\frac{1}{2} P$. In the strap, $Q = 0$ is, as in Fig. 7, located at a distance twice the one at which the pressure changes its sign, that is, $l = z + 2x$. If the dowel is imagined to be cut at M_{max} under addition of the shear forces M_{max} and $Q = 0$, the equation of moments will be as under, providing that (Fig. 10)

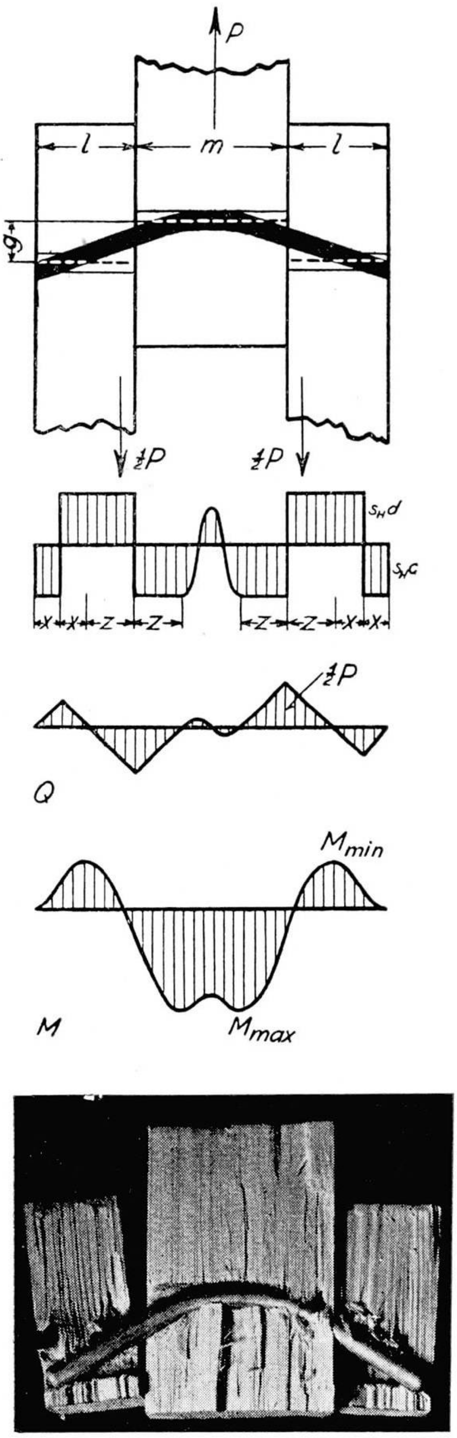

Fig. 10

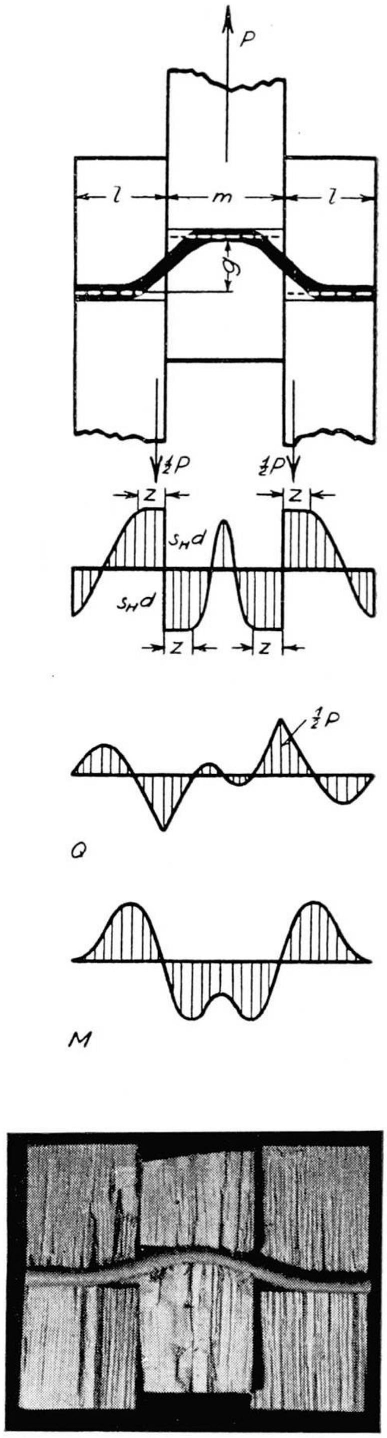

Fig. 11

$$\tfrac{1}{2}\,P = s_H\, dz \quad \text{and} \quad x = \tfrac{1}{2}\,(l-z)\,,$$

$$M_{max} = \frac{\pi}{32}\, s_B\, d^3 = \frac{1}{2}\, Pz - s_H\, dx^2 = s_H\, dz^2 - \frac{1}{4}\, s_H\, d\,(l-z)^2$$

$$z = \frac{l}{3}\left(\sqrt{4 + \frac{3\,\pi}{8}\,\frac{s_B}{s_H}\,\frac{d^2}{l^2}} - 1\right)$$

$$P = \frac{2}{3} s_H dl \left(\sqrt{4 + \frac{3\pi}{8} \frac{s_B}{s_H} \frac{d^2}{l^2}} - 1 \right). \tag{4}$$

This expression is approximated by the following

$$P = \left(\frac{1}{4} s_H l^2 + \frac{3}{5} s_B d^2 \right) \sqrt{\frac{s_H}{s_B}} \tag{4a}$$

with very good accuracy.

A still thinner dowel bends both in the balk and the straps. (Fig. 11)

In the region between the bends where the dowel has turned, the state in the wood must be plastic and the pressure against the dowel therefore $s_H d$. As $Q = 0$ at M_{max} and M_{min}, these must lie at the same distance z from the joint, as the transverse force in the joint must be

$$\tfrac{1}{2} P = s_H dz.$$

Further, we have the equation of moments for the dowel between the bends

$$M_{max} + M_{min} = 2 \cdot \frac{\pi}{32} s_B d^3 = \frac{1}{2} Pz = s_H dz^2,$$

$$z = \sqrt{\frac{\pi}{16} \frac{s_B}{s_H}}\, d = 0{,}442 \sqrt{\frac{s_B}{s_H}}\, d,$$

$$P = 0{,}885 \sqrt{s_B s_H}\, d^2. \tag{5}$$

Experiments showed that in order to avoid splitting of the timber the dimensions given in Fig. 12 must be adhered to.

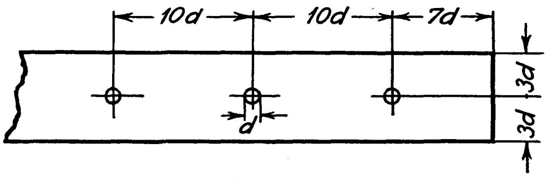

Fig. 12

3. Bolt Connections

The strength of a bolt connection depends partly upon the dowel effect of the bolt and partly upon the friction between the abutting surfaces caused by the tensioning of the bolt. As experience shows that the latter soon disappears because of shrinkage etc., only dowel effect should be reckoned with, likewise the bolt should be a tight fit in the hole. The head and nut of the bolt, together with the washers, caused fixed-end conditions to obtain; the bolt, therefore, bent and yielded here, even if the corresponding dowel would not do so (Fig. 13).

The fixed ends of the bolts will cause yield, as in Fig. 8 or 11 for single and double shear connections, respectively. Yield point P_F is given by the formula for dowels.

$$\begin{aligned} &\text{Single shear:} \quad P_F = 0{,}442 \sqrt{s_B \cdot s_H} \cdot d^2 \\ &\text{Double shear:} \quad P_F = 0{,}885 \sqrt{s_B \cdot s_H} \cdot d^2 \end{aligned} \tag{6}$$

The tension in the bolt being assumed to be zero. Following the yielding, the bolt bends to such an extent that tension is set up and it acts as a string. Friction is then set up between the timbers, and the ultimate load P_{max} for single and double shear connections can be reckoned as

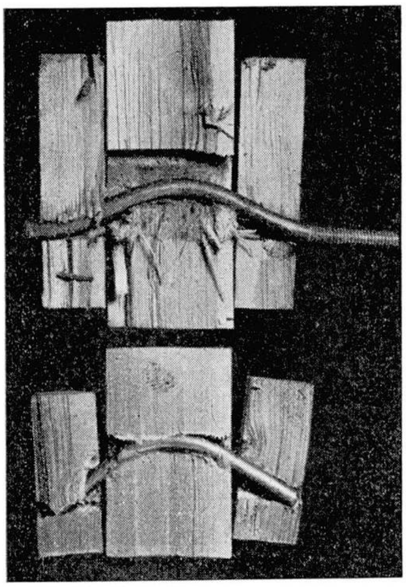

Fig. 13

$$\text{Single shear:} \quad P_{max} = P_F + \mu s_F \frac{\pi}{4} d_1^2$$

$$\text{Double shear:} \quad P_{max} = P_F + \mu s_F \frac{\pi}{4} d_1^2 \qquad (7)$$

where μ is the coefficient of friction wood-on-wood (average value $^2/_3$), s_F the tension yield point, d_1 the shank diameter of the bolt.

To avoid splitting an end distance of $10\,d$ is now necessary instead of the $7\,d$ with a dowel (Fig. 12).

4. Connections with toothed Dogs

Both P_F and P_{max} are increased by the strength L of the dog and are, therefore, given as

$$\text{Single shear:} \quad P = P_{bolt} + L \qquad (8)$$
$$\text{Double shear:} \quad P = P_{bolt} + 2\,L$$

where L is determined by tests as mentioned in 1 C. To avoid splitting, an end distance of $10\,d$ is necessary.

5. Tests

The abovenamed theory and the derived formulae were verified by tests. Fig. 14 shows some typical load-slip diagrams for the various kinds of connections partly with tensionless bolt, partly with bolt under tension. All the curves apply to 1″ dowel or bolt in 6″ · 6″ timber with 3″ · 6″ straps. The timber was Swedish fir. The dowel was driven in, the diameter of the hole being slightly smaller than that of the dowel. The bolts were tight fits in the holes, which were drilled to the diameter of the bolts.

Dowel and bolts were of the same material. The lowest curve 1 is for the dowel; as this is of mild steel, the curve shows a marked yield point in contrast to the others (Fig. 19) for dowels of compressed steel shafting. The next two curves 2 are for a tensionless and b normally tensioned bolts.

Their P_F is higher on account of the fixed ends at head and nut. On the other hand, the dowel becomes stiffer as it is driven into the hole, while the bolt merely fits tigthly. The two upper curves 3 are for dogs with *a* tensionless and *b* normally tensioned bolts. In both cases g at P_{max} is the same.

With the bolts the slip load P_g and the yield point P_F are pronounced, with the dogs only the yield point P_F. It will also be seen that P_{max} is independent of the distortion due to the tightening-up of the bolt. The slip g at P_{max} is alike for connections with bolts alone and for connections with bolts and dogs.

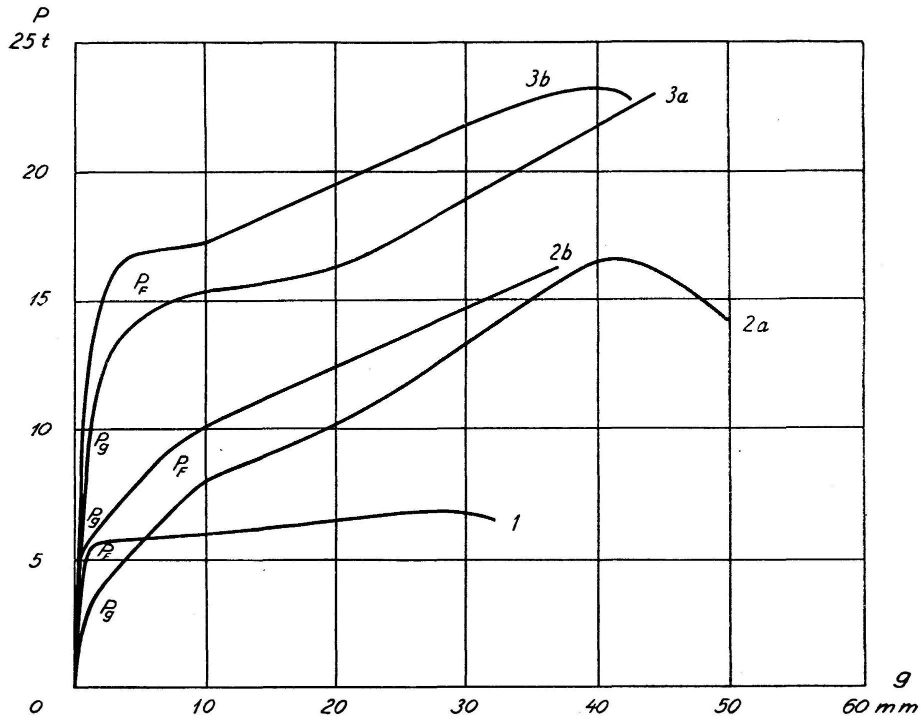

Fig. 14

Tests were made to determine the influence of tensioning of the bolts. Fig. 15 shows the commencement of the curves for $^3/_4''$ bolts in $5'' \cdot 5''$ timber with $2^1/_2''$ straps. The curves from the bottom upwards represent: 1. no tensioning, 2. normal tensioning, 3. tensioning till the threads are mutilated, and 4. tensioning up to a point just short of the mutilated threads. On the curve for the normally tensioned bolt, two characteristic points are found; slip point P_g where the slip between the timbers begins to increase rapidly because friction disappears, and yield point P_F, where the force begins to rise but little because the bolt bends and yields. With the tensionless bolt P_g lies, of course, at zero, as the curve also shows. With the greatest bolt tension, P_g and P_F are less marked and coincide. When the bolt is tightened till the threads are mutilated, P_g and P_F are placed but little higher

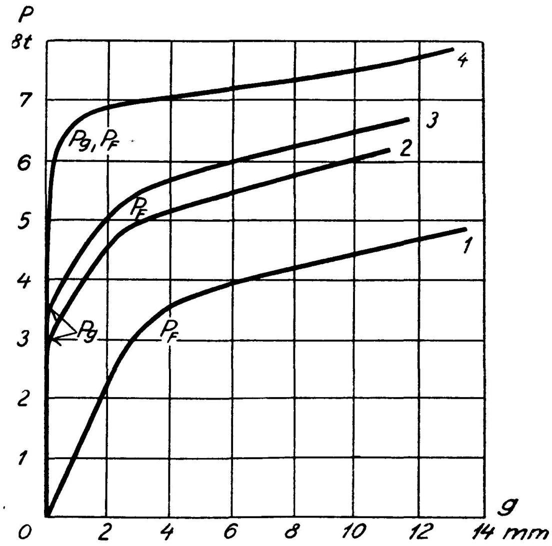

Fig. 15

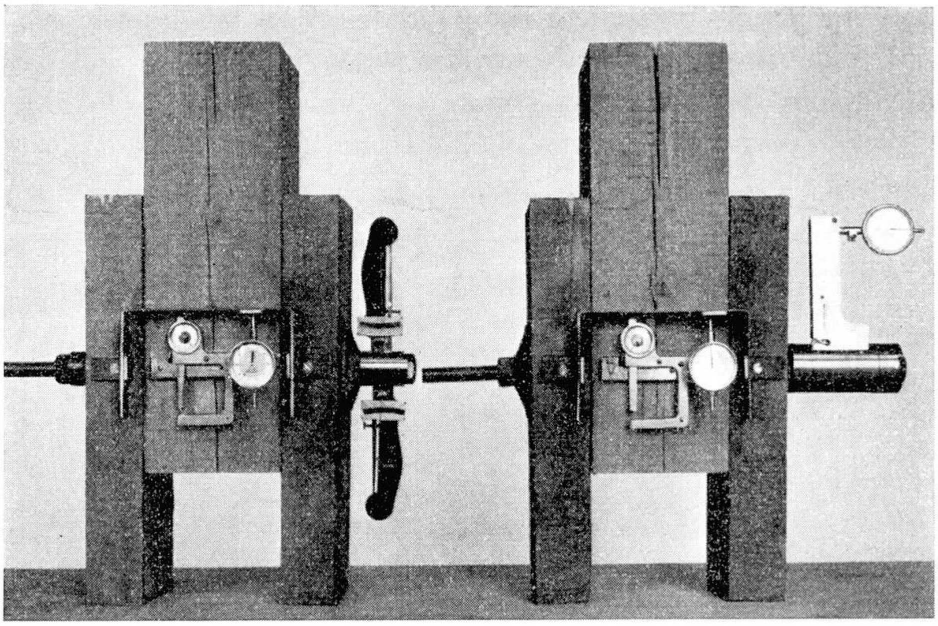

Fig. 16

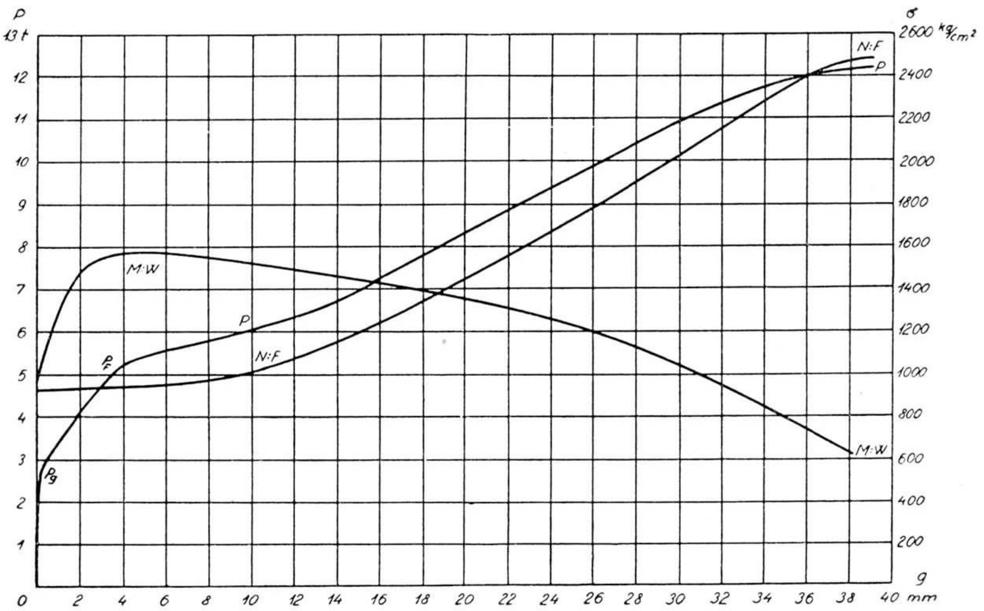

Fig. 17

than with the normally tensioned bolt. The ultimate load P_{max} was the same for all the tests, i.e. unaffected by the initial tension of the bolts, which is also expressed in (7).

These conditions are more closely studied on the experimental arrangement shown in Fig. 16, in which a slotted pipe was placed between the head of the bolt and the washer, permitting measurement of the tension by Huggenberger tensometer (on the left) and Berry tension gauge (on the right). The same illustration also shows how the slip g is measured with a dial-gauge (for larger movements with a Zivy gauge) by means of a ⎽⎾⏋⎽ shaped frame fastened to the straps at the level of the centre line of the bolt. As the measuring apparatus was fixed to the balk on the same line, the gauge length is zero and the elastic compression thereby eliminated from the measured g.

The axial force N on the bolt, and the moment M were determined from the measurements and these are given in Figs. 17 and 18. With the normally tensioned bolt, Fig. 17, P_g was 2750 kg and the coincident N 2220 kg which gives the coefficient of friction $\mu = P_g\colon 2\,N$ of 0,62. This is a suitable value, so the P_g is rightly the slip point. At P_g the effect of the dowel begins and M increases, while N is more or less unaltered. At P_F the bolt yields in consequence of the bending load, and M cannot increase further. Deformation has now become so great that N commences to increase, whereby M decreases and is replaced by a string effect which at P_{max} causes the bolt yield on account of the tension. With the connection having a tensionless bolt, Fig. 18, P_g is at zero and N increases but slightly, both before and after P_F, M can,therefore, increase, also after P_F. As N is quite small at P_F, it will coincide with P calculated by (5).

As a tensioned bolt quickly loses its tension, the latter should be disregarded, and the tests proper were, therefore, carried out with tensionless bolts.

Finally, the numerical agreement between the calculated and the observed values of P_g, P_F and P_{max} as well as the quantities x and z in Figs. 7, 8, 10 and 11 was very good – cf. the test report (Bulletin No. 10 from the Structural Research Laboratory of the Technical University of Copenhagen).

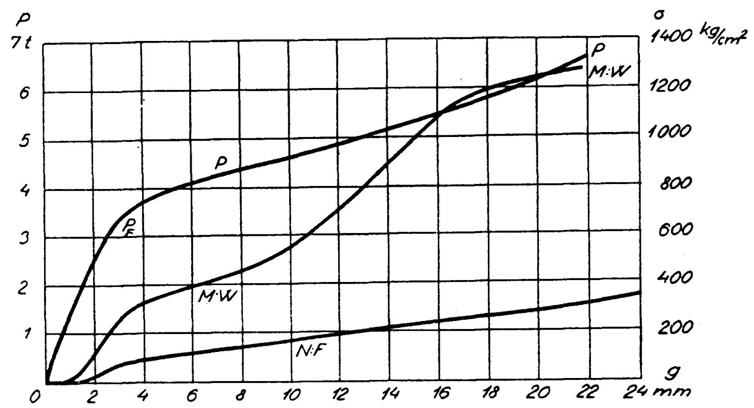

Fig. 18

6. Slip

Having dealt with the strength of the timber connections under discussion, attention will now be turned to the question of stiffness.

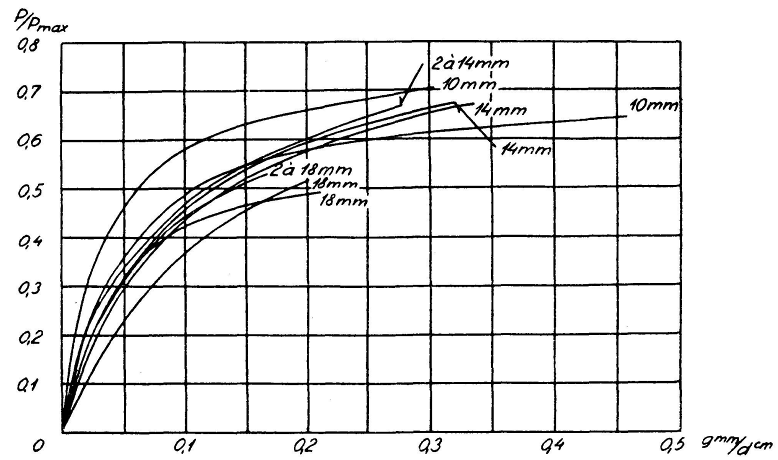

Fig. 19

Dowels

By plotting the test results with $P \div P_{max}$ and $g \div d$ as coordinates, curves are obtained which gather very well about a mean curve (Fig. 19)

$$\frac{g}{d} = \frac{1}{50} \cdot \frac{P}{P_{max}} + \frac{2}{5} \left(\frac{P}{P_{max}} \right)^2; \qquad P/P_{max} \leqq 0{,}5. \tag{9}$$

With $P \div P_F$ and $g \div d$ as co-ordinates, curves are obtained for bolts in Fig. 20 and for dogs in Fig. 21 which gather very well about the following mean curves:

Bolt alone: $$\frac{g}{d} = 1{,}25 \frac{P}{P_F}. \tag{10}$$

Bolt and dogs: $$\frac{g}{d} = \frac{1}{20} \frac{P}{P_F} + \left(\frac{P}{P_F} \right)^2. \tag{11}$$

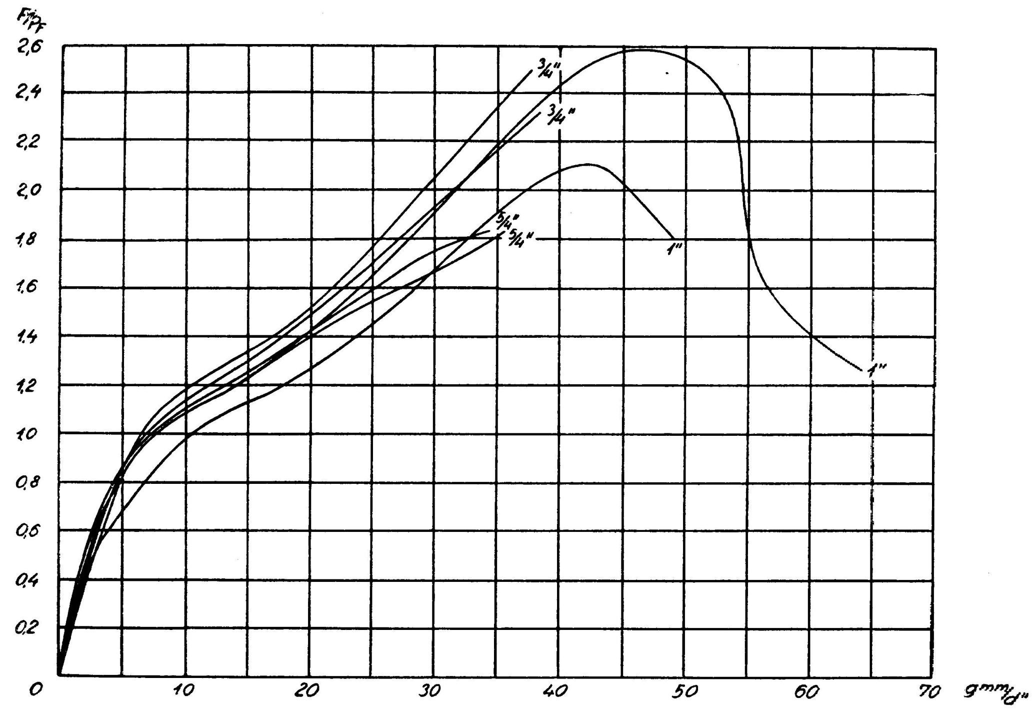

Fig. 20

7. Working Load

The strength being dependent on two different materials, conditions are similar to those obtaining in reinforced concrete. As the methods of calculation with working stresses are still those generally used despite their obvious shortcomings, the formulae for the working loads of the timber connections will have to be indicated at the working stresses of the materials. It now appears that when the ultimate stresses in the formulae (1)–(8) are replaced with the corresponding working stresses, we obtain both a suitable margin against yiel-

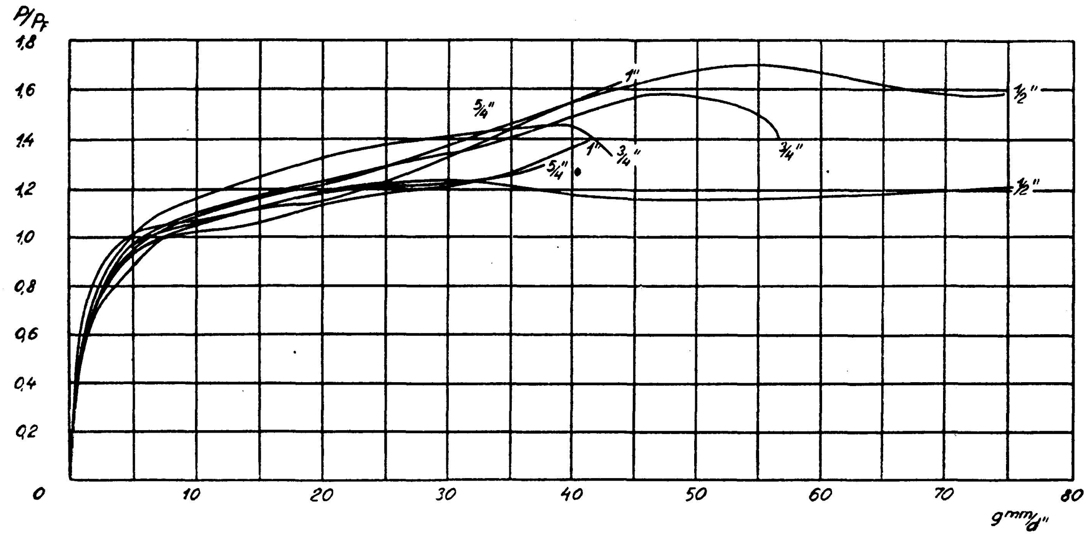

Fig. 21

ding and rupture and suitable small slips at the working load. For dowels we thus get the slip

$$g = 0{,}04\, d \tag{12}$$

at the working load.

For ordinary bolts in fir we get the following working loads in tons:

$$\begin{aligned} \text{Single shear:}\quad & P = 1{,}5\, d^2 \\ \text{Double shear:}\quad & P = 3{,}0\, d^2, \end{aligned} \tag{13}$$

where d is the diameter in inches. The corresponding slip is

$$g^{cm} = {}^1/_4\, d'', \tag{14}$$

or, when g and d are figured in the same measure:

$$g = 0{,}1\, d\,, \tag{14a}$$

that is, considerably more than with the dowels.

For bolts with the type of dogs used in the tests we get in tons:

$$\begin{aligned} \text{Single shear:}\quad & P = 3{,}4\, d^2 \\ \text{Double shear:}\quad & P = 6{,}8\, d^2 \end{aligned} \tag{15}$$

when d is in inches. The corresponding slip is

$$g^{cm} = 0{,}11\, d'', \tag{16}$$

or, when g and d are figured in the same measure:

$$g = 0{,}044\, d\,, \tag{16a}$$

that is, about the same as with dowels.

It should be observed expressly that these formulae assume that:

1. The dowels are driven, the holes being drilled with a drill the diameter of which is slightly smaller than that of the dowels.
2. The bolts are tight fits in the holes, which are to be drilled with a drill having the same diameter as the bolt.

Summary

Assuming plasticity in the wood and in the dowels or bolts formulae are derived for the yield point and the ultimate load of connections with dowels, with bolts and with bolts and toothed dogs. Both the assumptions and the formulae are verified by tests. The tests also give expressions for the slip, the dimensionless magnitudes slip ÷ diameter and load ÷ yield point being connected by simple formulae. Finally formulae for the working load and the corresponding slip are given for the practical applications.

Zusammenfassung

Unter der Voraussetzung, daß sich das Holz und die Dübel oder Bolzen im plastischen Zustand befinden, werden Formeln abgeleitet für die Streckgrenze und die Bruchlast von Verbindungen mit Dübeln, Bolzen und gezahnten Verbindern mit Bolzen. Die Voraussetzungen und die Formeln wurden durch Versuche bestätigt. Die Versuche ergaben auch Werte für die Gleitung, wobei die dimensionslosen Größen

$$\frac{\text{Gleitung}}{\text{Durchmesser}} \quad \text{und} \quad \frac{\text{Last}}{\text{Streckgrenze}}$$

durch einfache Formeln ausgedrückt werden konnten. Schließlich werden noch zur praktischen Anwendung der Theorie Werte angegeben für die zulässige Belastung und die dabei auftretende Gleitung.

Résumé

En admettant que le bois et les goujons ou boulons se trouvent à l'état plastique, l'auteur établit des formules permettant de calculer la limite d'écoulement et la charge de rupture des assemblages goujonnés, boulonnés ou réalisés avec pièces d'assemblage dentelées et boulons. Hypothèses et formules ont fait l'objet de vérifications expérimentales. Les essais ont également fourni des résultats concernant le glissement et les grandeurs non dimensionnelles que constituent les rapports

$$\frac{\text{Glissement}}{\text{Diamètre}} \quad \text{et} \quad \frac{\text{Charge}}{\text{Limite d'écoulement}}$$

ont pu être exprimées à l'aide de formules simples. Enfin, l'auteur donne également des valeurs concernant la charge admissible et le glissement sous ces charges permettant l'application pratique de la théorie.