Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 9 (1949)

Artikel: The effect of holes in circular plates

Autor: Holmberg, ke

DOI: https://doi.org/10.5169/seals-9701

The effect of holes in circular plates

Der Einfluß von Löchern in Kreisplatten

Influence des évidements dans les dalles circulaires

ÅKE HOLMBERG, D.S.C.S.E., Consulting Engineer, Lund. Sweden

A circular plate with radius a and thickness h is placed in a right-handed system of cylinder coordinates r, θ, z, with origin at the centre of the plate and with the $r-\theta$ plane halving the thickness of the plate.

The plate may be subjected to a transverse load, p_z, causing deflections w, positive in the direction of the positive z axis.

The equations of equilibrium are as usual, since small quantities of higher order are neglected and the membrane stresses are not taken into account.

$$\frac{\partial q_r}{\partial r} + \frac{1}{r} q_r + \frac{1}{r} \frac{\partial q_\theta}{\partial \theta} + p_z = 0 \tag{1}$$

$$\frac{\partial m_r}{\partial r} + \frac{1}{r} m_r + \frac{1}{r} \frac{\partial m_{r\theta}}{\partial \theta} - \frac{1}{r} m_\theta - q_r = 0 \tag{2}$$

$$\frac{\partial m_{r\theta}}{\partial r} + \frac{2}{r} m_{r\theta} + \frac{1}{r} \frac{\partial m_\theta}{\partial \theta} - q_\theta = 0 \tag{3}$$

In the above, q signifies the intensity of the shearing forces and m the intensity of the moments.

Equ. (2) may be written

$$\frac{\partial}{\partial r}(r m_r) + \frac{\partial m_{r\theta}}{\partial \theta} - m_\theta - r q_r = 0 \tag{4}$$

The total volume Φ beneath the surfaces m_r and m_θ may be written

$$\Phi = \int_0^{2\pi} \int_0^a m_r \, r \, dr \, d_\theta + \int_0^{2\pi} \int_0^a m_\theta \, r \, dr \, d_\theta \tag{5}$$

Through partial integration there arises from equ. (4) and (5)

$$\Phi = \int_0^{2\pi} \left[\Big[r^2 m_r \Big]_0^a \right] d\theta + \int_0^{2\pi} \int_0^a \frac{\partial m_{r\theta}}{\partial \theta} r \, dr \, d\theta - \int_0^{2\pi} \int_0^a r^2 q_r \, dr \, d\theta \tag{6}$$

The first integral may be written

$$a^2 \int_0^{2\pi} (m_r)_{r=a}\, d\theta$$

and is independent of the qualities of the material of the plate. The second integral may be written

$$\int_0^a r \left[\int_0^{2\pi} \frac{1}{r} \frac{\partial m_{r\theta}}{\partial \theta} r\, d\theta \right] dr$$

where $\frac{1}{r} \frac{\partial m_{re}}{\partial \theta}$ means vertical intensity of forces parallel to the z axis on a circle r. Equ. (1) states that equilibrium of forces exists without this intensity. Thus

$$\int_0^{2\pi} \frac{1}{r} \frac{\partial m_{r\theta}}{\partial \theta} r\, d\theta = 0$$

and the second integral also becomes zero. The third integral may be written

$$- \int_0^a r\, P_r\, dr$$

in which P_r is the total load inside a circle r. Also this integral is independent of the qualities of the material of the plate. The total volume Φ is therefore

$$\Phi = a^2 \int_0^{2\pi} (m_r)_{r=a}\, d\theta + \int_0^a r\, P_r\, dr \tag{7}$$

This is independent of the qualities of the material of the plate, that is to say, also of possible holes, and is a function of the load only as long as the moments do not change signs.

The volume Φ has been calculated as the sum of the volumes under m_r and m_θ. The sum of the moments is, however, at any certain point invariable, wherefore the above-mentioned volume is also the volume under the principal moments.

From Equ. (7) an expression for the amount of steel in kg. W_s, needed for the reinforcement of a plate of reinforced concrete, can be calculated with the help of some simple assumptions. The lever of internal forces is taken as $\frac{7}{8} h_{eff}$ where h_{eff} is the effective depth and the specific gravity of the steel $= 7{,}85$ kg/dm³. From the above with kg and cm as units:

$$W_s = \frac{7{,}85}{1000} \frac{\Phi}{\frac{7}{8} h_{eff} \cdot \sigma_{Y.P.}}$$

in which $\sigma_{Y.P.}$ is the stress at the yield point.

The calculation has thus been based on the maximum load. With "allowable load" it is calculated instead with "allowable stresses". With 15 % increase for incomplete utilisation and for anchoring, we have

$$W_s = 0{,}0103 \frac{\Phi}{h_{eff} \cdot \sigma_{Y.P.}} \tag{8}$$

In the case of a symmetrical load with moments m_a uniformly distributed along the edge, concentrated load P at the centre, uniformly distributed load p over the whole surface, and a load with the intensity $\bar{P}$ uniformly distributed along a circle concentric with the edge of the plate, and with a radius $b = \beta . a$, we have from equ. (8)

$$W_s = 0{,}0325 \left[\frac{pa^2}{4} + \frac{P}{2\pi} + 2\, m_a + \bar{P} a \beta (1 - \beta^2)\right] \frac{a^2}{h_{eff} \cdot \sigma_{Y.P.}} \tag{9}$$

With the help of equ. (8) or equ. (9) the required reinforcement for a circular plate of reinforced concrete with arbitrary holes can be calculated. It is then, however, a condition that the moments do not change signs. It is thus always possible for a trained designer to distribute the reinforcement bars with tolerable accuracy.

If the moments change signs, the problem becomes more complicated and there is no short cut for solving it. With a none too extreme shape it is yet possible to find a circle with $m_r = 0$. The plate is divided along this circle and each part is treated in accordance with the above. The condition for this is, of course, the well-known circumstance that within wide ranges the moments in a reinforced concrete structure with high stresses are distributed in accordance with the reinforcement.

As regards plates of such a material as steel the problem is partly of a different kind. According to the method, described in my treatise: Tests with Circular Plates (Stockholm, Sweden, 1946), I investigated the distribution of stresses in a circular steel plate with an eccentric hole $\varnothing\ a/2$ with its centre halving the radius of the plate. The plate was subjected to a concentrated load, successively at a number of points $r = a/2$ and at $r = 0$.

The maximum stresses in no case substantially exceeded the maximum stresses in a plate without holes. The central deflections, $w_{r=0}$, increased on the other hand up to 25 %. The plate was practically simply supported.

This investigation which naturally cannot be considered as definite will be published on another occasion.

Summary

The volume Φ beneath m_r and m_θ for a circular plate is calculated and shown to be independent of the material qualities of the plate, i.e. of possible holes also, as long as the moments do not change signs. Arising from this, simple rules are given for the reinforcing of concrete plates.

Zusammenfassung

Das Volumen Φ der m_r und m_θ innerhalb der Ränder wird für eine Kreisplatte berechnet. Es wird bewiesen, daß sie unabhängig von den Materialeigenschaften der Platte ist, d.h. also auch von eventuell vorhandenen Löchern, solange die Momente ihr Vorzeichen nicht wechseln. Gestützt auf diese Erkenntnis werden einfache Regeln für die Armierung von Eisenbetonplatten gegeben.

Résumé

L'auteur calcule le volume Φ correspondant à m_r et m_θ à l'intérieur des bords de la dalle, dans le cas d'une dalle circulaire. Il montre que ce volume est indépendant des caractéristiques du matériau qui constitue la dalle, c'est-à-dire de l'existence éventuelle de trous et évidements, pour autant que les moments ne changent pas de signe. En s'appuyant sur cette notion, il indique des règles simples pour la constitution des armatures des dalles en béton armé.