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DEFLECTION THEORY FOR CONTINUOUS SUSPENSION BRIDGES.

THÉORIE DES DÉFORMATIONS POUR LES PONTS SUSPENDUS CONTINUS.

VERFORMUNGSTHEORIE FÜR DURCHLAUFENDE HÄNGEBRÜCKEN.

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Synopsis.

The more general adoption of the continuous type of suspension bridge, offering advantages of economy and rigidity, has been retarded by the lack of an accurate theory for its analysis. The Deflection Theory for simple-span suspension bridges has been available to the profession for over 40 years; but the corresponding theory for the suspension bridge with continuous stiffening truss has thus far been lacking.

In order to supply this deficiency, the writer has undertaken to develop the Deflection Theory for continuous suspension bridges, with working formulas for practical application.

The resulting analysis, presented in this paper, is a generalized Deflection Theory for suspension bridges, applicable to both continuous and non-continuous types. By simply dropping the recognizable terms due to continuity, the formulas are reduced to those for the two-hinged suspension bridge. Moreover, the general formulas are also found to be applicable to multiple-span suspension bridges, with or without continuity.

In the development of the analysis herein presented, maximum simplicity of formulas and ease of practical application have been governing considerations. Incidentally, new simplifications are here developed and introduced in the working formulas hitherto published for the two-hinged type.

Interdependent functions necessarily introduced in the earlier portions of the theoretical analysis are resolved and eliminated in deducing the final working formulas.

Simplified approximate formulas are also given as alternatives, with their departure from exact values indicated, to facilitate preliminary or approximate evaluation. These approximate formulas also facilitate the interpretation of the expressions or relations represented.

To show the practical workability of the Generalized Theory here developed, this paper includes a numerical example of the application of the formulas to the analysis of the stresses and deflections in a continuous suspension bridge of 800-ft. main span. For the continuous stiffening truss, the Deflection Theory is found to yield an average reduction of 45 percent in the bending moments as previously found by the common Elastic Theory. In comparison with the two-hinged type, the continuous design is found to

be approximately 5 percent more rigid for the same economy, or 5 percent more economical for the same rigidity. For shorter spans, these percentages of superior efficiency would be increased.

1. Introduction.

The common or approximate theory for the stress analysis of stiffened suspension bridges is known as the Elastic Theory. The values of the bending moments and shears yielded by this method are too high, satisfying safety but not economy. The error increases with the flexibility of the structure, the span-length, and the ratio of dead load to live load.

A more exact method of analysis, which takes into account the deformed configuration of the structure, is known as the Deflection Theory. It yields lower stresses and a consequent saving (ranging normally from 20 per cent to 65 per cent) of the metal in the stiffening truss.

The Deflection Theory or "More Exact Theory", as applied to non-continuous suspension bridges, was originated by J. MELAN and was first published by him in 1888 in the second edition of his classic work "Theorie der eisernen Bogenbrücken und der Hängebrücken". (It was republished in 1906 in his third edition, which was translated in 1909 by D. B. STEINMAN in "Theory of Arches and Suspension Bridges", published in 1913. In Professor MELAN's fourth edition, 1925, the Deflection Theory appears again, in amplified form.) The working formulas were amplified by L. S. MOISSEIFF (and independently by the writer) in 1909 to cover the case of suspended side spans, and the theory has since been published in its extended form by F. E. TURNEAURE in "Modern Framed Structures" (1911 and subsequent editions) and by D. B. STEINMAN in "A Practical Treatise on Suspension Bridges", second edition, 1929.

The Deflection Theory, as hitherto developed, is not directly applicable to suspension bridges with continuous stiffening trusses. A number of such structures have been built, including the Rondout Bridge, at Kingston, N. Y., with 705-ft. main span (in 1922), and the General U. S. Grant Bridge over the Ohio River at Portsmouth, Ohio, with 700-ft. main span (in 1927); but the designers have been handicapped by the lack of an accurate method of analysis. Without close proportioning, the full economy of this type of suspension bridge could not be secured.

Continuous stiffening trusses offer greater efficiency than the conventional two-hinged type. This increased efficiency may be in the form of superior economy, or superior rigidity, or both, depending upon the proportions adopted. Comparative designs show that the continuous type is more rigid for the same total quantity of steel, and somewhat more economical for the same specified degree of rigidity. The hingeless type offers incidental advantages in greater efficiency of the continuous lateral truss, in improved and simplified supporting details at the towers, and in reduced variation between minimum and maximum sections. The sole disadvantage arises from the necessity of providing for the necessary expansion movement of the stiffening truss near the anchorages, where the suspenders are short; but this becomes a problem only in the longer structures. For suspension bridges under 1000-ft. main span, the continuous stiffening truss may well be considered the superior type.

Comparative designs by the Elastic Theory have indicated an economic advantage of 5 to 15 per cent in favor of the continuous type, depending upon the span-length. Comparative designs by the Deflection Theory are needed, however, to make a more conclusive determination.

In past designs of continuous suspension bridges, the deflection corrections have been either neglected or conservatively approximated. With the Deflection Theory for such structures available, more scientifically proportioned designs can be made, and the economic utilization of this bridge type will be facilitated.

2. Fundamental Assumptions.

The Deflection Theory for the analysis of continuous suspension bridges is based on the same assumptions as the corresponding theory for two-hinged suspension bridges, namely:

1. The initial curve of the cable is a parabola. (In practice, the greatest ordinate deviation from a true parabola is seldom as large as ½ per cent.)

2. The initial dead load (W) is carried by the cable (producing the initial horizontal tension H_w) without causing stress in the stiffening truss.

Unlike the Elastic Theory, the Deflection Theory does not assume that the ordinates y of the cable curve remain unaltered upon application of the loading. In other words, the alteration of the lever arms of the cable forces is taken into account. This change in cable ordinates or lever arms makes the initial cable tension H_w significant.

The theory that follows is applicable to either continuous or two-hinged suspension bridges, with or without suspenders in the side spans. The equations are written in their more general form, so as to be directly applicable to continuous suspension bridges; but, upon dropping the terms dependent upon continuity (M_1, M_2, T_1, T_2, ε, and μ), all equations reduce to the simpler formulas for two-hinged suspension bridges.

A symmetrical three-span suspension bridge is assumed. With minor modifications, obvious to the bridge analyst, the formulas can easily be extended to other cases.

The initial loading (W), for which the stiffening truss is unstressed and undeflected, will be called the "dead load". The subsequently applied loading (p), producing stress and deflection, will be called the "live load".

The general notation to be used is shown in Fig. 1. The subscripts (as in I_1, l_1, f_1, x_1, y_1, η_1) are added to distinguish corresponding side-span magnitudes. The abscissa x is always measured from the left end of the main span, and x_1 is always measured from the free end of either side span.

3. Fundamental Equations.

With no suspender forces acting, let M_0 denote the simple-beam bending moment (due to live load) at any section x of the stiffening truss, and let M_1 and M_2 denote the continuous-beam bending moments (due to the live load) at the left and right towers, respectively. Then M′, the continuous-beam bending moment (at the section x) due to live load with no suspender forces acting, is given (Fig. 1 b) by the familiar expressions:

(Main Span:) $$M' = M_0 + \frac{l-x}{l} M_1 + \frac{x}{l} M_2 \qquad (1a)$$

(Side Spans:) $$M' = M_0 + \frac{x_1}{l_1} M_{1,2} \qquad (1b)$$

If the deflections of the cables are neglected, the relieving moment due to live-load cable tension H acting through the suspender forces is given by $H\,y$ for a simple span; but, for continuous spans (Fig. 1 c), the expression for this relieving moment must be modified to $H\,(y-\mu)$, where μ denotes the bending moment, due to c o n t i n u i t y, produced by the suspender forces per unit H. Deducting this relieving moment from M′, we obtain the resultant bending moment M at any section of a continuous stiffening truss, w i t h d e f l e c t i o n s n e g l e c t e d:

$$M = M' - H(y - \mu) \tag{2}$$

This is the b a s i c e q u a t i o n o f t h e E l a s t i c T h e o r y f o r c o n t i n u o u s s u s p e n s i o n b r i d g e s.

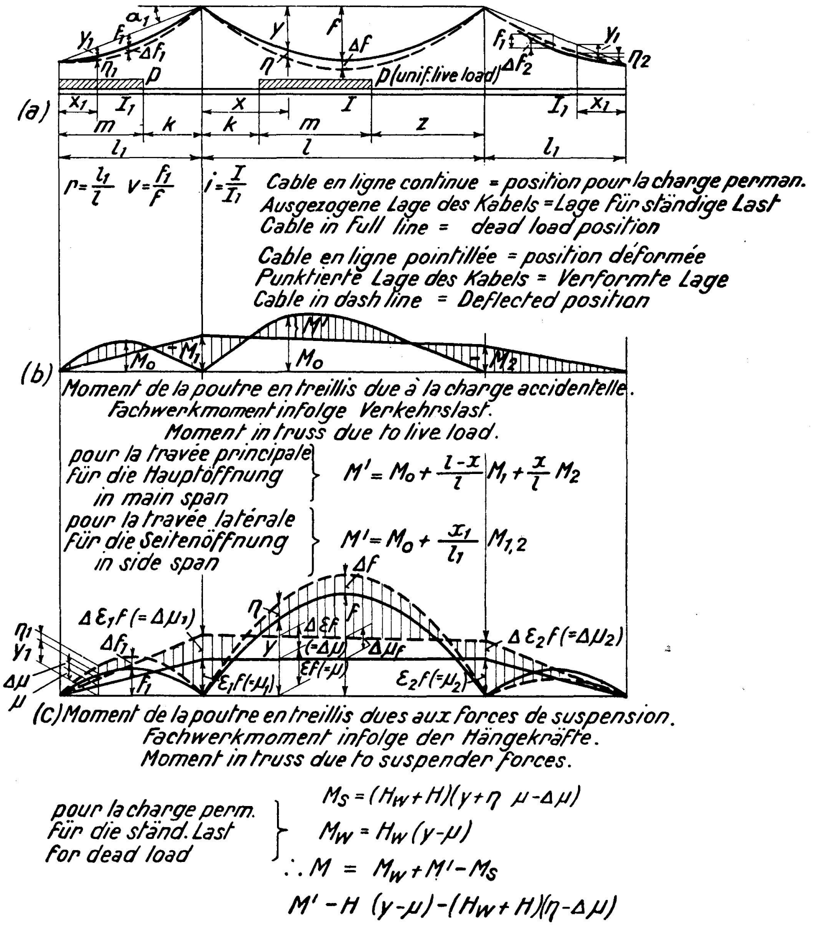


Fig. 1.

In consequence of the deflections η (see Figs. 1 a and 1 c), the cable ordinate y increases to $(y+\eta)$, and μ increases to $(\mu + \Delta\,\mu)$. Accordingly the bending moment M at any section, as given by Eq. (2), is relieved by the additional amount:

$$(H_w + H)(\eta - \Delta\,\mu)$$

and the complete expression for M becomes:

$$M = M' - H(y - \mu) - (H_w + H)(\eta - \Delta\,\mu) \tag{3}$$

This is the b a s i c e q u a t i o n o f t h e D e f l e c t i o n T h e r y f o r c o ntinuous suspension bridges. Another demonstration of this fundamental equation is indicated in Fig. 1 b, and c.

The value of μ is independent of the loading, and may be expressed in terms of the dimensional constants of the structure. Let:

$$i = \frac{I}{I_1}, \qquad r = \frac{l_1}{l}, \qquad v = \frac{f_1}{f} \tag{4}$$

Then, by the theory of continuous beams, for uniform suspender forces acting on a symmetrical three-span structure, the value of μ at each tower will be εf, where

$$\varepsilon = \frac{2 + 2irv}{3 + 2ir} \tag{5}$$

and

$$\varepsilon f = \frac{2f + 2irf_1}{3 + 2ir} \tag{6}$$

Accordingly, the value of μ at any section is given by the expressions:

(Main Span:) $$\mu = \varepsilon f \tag{7a}$$

(Side Spans:) $$\mu = \frac{x_1}{l_1} \cdot \varepsilon f \tag{7b}$$

With the deflection of the spans, let $\varDelta \mu_1$ and $\varDelta \mu_2$ denote the increases in the value of $\mu = \varepsilon f$ at the left and right towers, respectively. Then, at any section of the spans, the value of $\varDelta \mu$, denoting the increase in the continuity moment of the suspender forces per unit H, is given by the expressions:

(Main Span:) $$\varDelta \mu = \frac{l-x}{l} \cdot \varDelta \mu_1 + \frac{x}{l} \cdot \varDelta \mu_2 \qquad \text{(8a}$$

(Side Span:) $$\varDelta \mu = \frac{x_1}{l_1} \cdot \varDelta \mu_{1,2} \tag{8b}$$

(See Fig. 1 c). The quantities $\varDelta \mu_1$ and $\varDelta \mu_2$ will be evaluated in Art. 7, and will be eliminated in Art. 8.

Neglecting the elongation of the suspenders, the truss at any point will have the same deflection η as the cable at that point.

By the common theory of flexure applied to the truss,

$$\frac{d^2 \eta}{dx^2} = -\frac{M}{EI}$$

substituting Eq. (3), and introducing the symbol

Main Span:) $$c^2 = \frac{H_w + H}{EI} \tag{9a}$$

(Side Spans:) $$c_1^2 = \frac{H_w + H}{EI_1} \tag{9b}$$

we obtain:

$$\frac{d^2 \eta}{dx^2} = c^2 (\eta - \varDelta \mu) - \frac{c^2}{(H_w + H)} (M' - Hy + H\mu)$$

The solution of this differential equation, noting that $\frac{d^2}{dx^2} (\varDelta \mu) = 0$, yields the general formula for deflections or equation of the deflection curve:

$$\eta = \frac{H}{(H_w + H)}\left[C_1 e^{cx} + C_2 e^{-cx} + \left(\frac{M'}{H} - y + \mu\right) - \frac{1}{c^2}\left(\frac{p_x}{H} - \frac{8f}{l^2}\right)\right] + \Delta\mu \quad (10)$$

where p_x is the live load per unit length at the section x. (The terms in the last parentheses are the second derivatives of the terms in the preceding parentheses.)

Substituting Eq. (10) in Eq. (3), and introducing the parameter abbreviation,

$$\varrho = \frac{l^2}{8f}, \qquad \varrho_1 = \frac{l_1^2}{8f_1} \quad (11)$$

we obtain the general formula for M or equation of the M-curve:

$$M = -H\left[C_1 e^{cx} + C_2 e^{-cx} - \frac{1}{c^2}\left(\frac{p_x}{H} - \frac{1}{\varrho}\right)\right] \quad (12)$$

From Eq. (12) we observe that the bending moment M is not simply proportional to the load p that produces it. Eq. (3) also shows that the value of M is affected by the dead load stress H_w in the cable before the application of the live load. In the Deflection Theory, influence lines cannot be used. Stresses producible by combinations of loadings cannot be found by adding algebraically the respective stresses producible by the component loadings.

The general formula for shears V, or equation of the V-curve, is obtained by differentiating Eq. (12), which gives:

$$V = \frac{dM}{dx} = -Hc[C_1 e^{cx} - C_2 e^{-cx}] \quad (13)$$

Differentiating Eq. (13), we obtain an expression for the live load per unit length actually carried by the stiffening truss at any point x:

$$p_x - (s_1 - s_0) = -\frac{d^2 M}{dx^2} = -\frac{dV}{dx} = Hc^2[C_1 e^{cx} + C_2 e^{-cx}] \quad (14)$$

where s_0 and s_1 are the initial and final values, respectively, of the suspender loading (per unit length of span) at the point x, so that $(s_1 - s_0)$ is the live load per unit length actually carried by the suspenders. Eq. (14) shows that the suspender loading $(s_1 - s_0)$ is no longer constant, as in the Elastic Theory, but becomes a variable in the Deflection Theory.

Eqs. (12), (13) and (14) are identical with the corresponding equations for the case of two-hinged suspension bridges.

The constants of integration C_1 and C_2 appearing in the foregoing equations for η, M, V, and s_1 will occur again in the general equation (Eq. 21) for H. They are determined for a given structure from the conditions of loading, as illustrated in Art. 5. Values of C_1 and C_2 for different cases of loading are tabulated in Art. 11.

The foregoing equations for η, M, V, and s_1 (Eqs. 3, 10, 12, 13, 14) are applicable to points in the side spans as well as to those in the main span; all that is necessary is to write the subscript symbols x_1, y_1, l_1, f_1, ϱ_1, I_1, c_1, η_1, instead of the corresponding main span quantities.

An important function, occurring repeatedly in the subsequent equations of the Deflection Theory for continuous suspension bridges, is the total resultant bending moment at either tower, corrected for deflections. This total tower moment will be denoted by T_1 for the left tower and T_2 for the right tower. (See Fig. 4c.) Substituting $y=0$, $\eta=0$, and $M'=M_1$ or M_2 in Eq. (3), and utilizing the expressions given by Eqs. (7) and (9), we obtain:

$$T_1 = M_1 + H(\varepsilon f) + EIc^2(\Delta\mu_1) \qquad (15\text{a})$$

$$T_2 = M_2 + H(\varepsilon f) + EIc^2(\Delta\mu_2) \qquad (15\text{b})$$

These functions T_1 and T_2 are evaluated in Art. 8 and are tabulated, for different loading cases, in Art. 11.

With these new functions, the fundamental equation of the Deflection Theory (Eq. 3) may be written in the form:

(Main Span:) $$M = (M_0 - Hy) - (EIc^2\eta) + \left(\frac{l-x}{l}T_1 + \frac{x}{l}T_2\right) \qquad (3\text{a})$$

(Side Spans:) $$M = (M_0 - Hy_1) - (EIc^2\eta_{1,2}) + \left(\frac{x_1}{l_1}\cdot T_{1,2}\right) \qquad (3\text{b})$$

The first term $(M_0 - Hy)$ is the elementary expression for bending moment; the second term $(EIc^2\cdot\eta)$ is the deflection correction, without continuity; and the last term (containing $T_{1,2}$) is the contribution of continuity.

4. Derivation of the Basic Equation for *H*.

The horizontal cable tension H, due to any live load p (including any supplemental dead load) and temperature change t, all following the condition represented by the initial tension H_w, may be evaluated as follows:

The total virtual work (W_1) done in the vertical displacements η of the suspender loads s_1 and the cable weight g must equal the total virtual work (W_2) done by the cable tension (H_w+H) in stretching the cable. These work quantities, W_1 and W_2, are expressed as the integrated products of the forces and their respective displacements, as follows, using the symbol Σ to denote the summation of similar expressions for all the spans:

$$W_1 = \sum\int_0^l (s_1+g)\,\eta\cdot dx = \sum\frac{8f}{l^2}(H_w+H)\int_0^l \eta\cdot dx, \text{ (approximately)}$$

$$W_2 = \sum(H_w+H)\left[\frac{H}{E_cA_0}\int_0^l\frac{A_0}{A_c}\cdot\frac{ds^3}{dx^2} + \omega\int_0^l\frac{A_0}{A_c}\cdot\frac{ds^2}{dx}\right]$$

$$= (H_w+H)\left[\frac{H}{E_cA_0}\cdot L_s + \omega t\cdot L_t\right]$$

where

$$L_s = \sum\int_0^l\frac{A_0}{A_c}\cdot\frac{ds^3}{dx^2} \qquad (16)$$

and

$$L_t = \sum\int_0^l\frac{A_0}{A_c}\cdot\frac{ds^2}{dx} \qquad (17)$$

In these expressions, A_c denotes the cable section at any point and A_0 denotes the cable section at mid-span.

For a parabolic wire cable, having uniform A_c, we may write with sufficient accuracy:

$$L_s = \sum l(\sec^3\alpha + 8n^2\cdot\sec\alpha), \qquad L_t = \sum l\left(\sec^2\alpha + \frac{16}{3}n^2\right) \qquad (18)$$

where $n=\frac{f}{l}$, and α is the inclination of the closing chord in any span.

Similarly, for a parabolic eyebar cable, assuming A_c varying with the slope secant $\frac{ds}{dx}$, we may write with sufficient accuracy:

$$L_s = \sum l\left(\sec^2 \alpha + \frac{16}{3} n^2\right), \qquad L_t = \sum l\left(\sec \alpha + \frac{8}{3} \cdot \frac{n^2}{\sec \alpha}\right) \tag{19}$$

Equating the expressions given above for W_1 and W_2, we obtain the work equation:

$$\sum \frac{8f}{l^2} \int_0^l \eta \cdot dx = \frac{H}{E_c A_0} \cdot L_s + \omega t \cdot L_t \tag{20}$$

(This is identical with the work equation for two-hinged suspension bridges.)

Substituting the expression for η from Eq. (10), the work equation (20) may be written:

$$\sum K \int_0^l \left[C_1 e^{cx} + C_2 e^{-cx} + \left(\frac{M'}{H} - y + \mu\right) - \frac{1}{c^2}\left(\frac{p_x}{H} - \frac{1}{\varrho}\right) + EIc^2 \Delta\mu \right] dx$$
$$= c^2 \varrho \left(\frac{E}{E_c} \cdot \frac{I}{A_0} \cdot L_s + \frac{EI\omega t}{H} \cdot L_t\right)$$

Solving for H, we obtain the basic H-equation:

$$H = \frac{\sum K \int_0^l \left(M' - \frac{p_x}{c^2}\right) dx - \varrho c^2 EI \omega t L_t + EIc^2 \cdot \Delta\mu_f \cdot \Sigma'(Kl)}{\sum K\left[-\int_0^l (C_1 e^{cx} + C_2 e^{-cx})\, dx + \frac{2}{3} fl - \frac{l}{\varrho c^2}\right] + \varrho c^2 \frac{E}{E_c} \cdot \frac{IL_s}{A_0} - \varepsilon f \cdot \Sigma'(Kl)} \tag{21}$$

where E is the modulus of elasticity of the truss material and E_c that of the cable; ω is the coefficient of temperature expansion; and ϱ is the parameter of the cable parabola, defined by Eq. (11).

The summations Σ in Eq. (21) embrace the corresponding expressions for all the spans; the symbol Σ' likewise denotes a summation of corresponding expressions for all the spans, except that the side-span contributions are to be multiplied by ½ before adding them.

The coefficient K occurring in the summations denotes the ratio of $\frac{f}{l^2}$ for any span to $\frac{f}{l^2}$ of the main span. Hence $K = 1$ for the main span; also $K_1 = 1$ for the side spans if they have the same $\frac{f}{l^2}$ as the main span. (Generally the ratio K_1 for suspended side spans is between 1.00 and 1.05, representing the ratio of side-span weight to main-span weight per unit length.) For "unloaded" or "straight" backstays, $K_1 = 0$, and all side-span terms vanish from the summations in Eq. (21). In any case:

$$K = 1, \quad \text{and} \quad K_1 = \frac{f_1/l_1^2}{f/l^2} = \frac{\varrho}{\varrho_1} \tag{22}$$

The symbol $\Delta\mu_f$, introduced in Eq. (21), denotes the value of $\Delta\mu$ at the middle of the main span, or the mean of the respective values of $\Delta\mu$ at the two towers:

$$\Delta\mu_f = \tfrac{1}{2}(\Delta\mu_1 + \Delta\mu_2) \tag{23}$$

It should be noted that two approximations are involved in the foregoing derivation of the H-equation (Eq. 21). In writing the transformed expression for W_1, it is assumed that the suspender and cable loading $(s_1 + g)$ is uniformly distributed over the span; this is not the actual condition, as demonstrated in Eq. (14). (In the Deflection Theory formulas presented by TIMOSHENKO, this assumption is avoided; the effect on the resulting stresses is, however, found to be practically negligible.) The second approximation consists in writing the original cable sag f instead of the augmented cable sag $(f + \Delta f)$ in the expression for W_1. This affects only the terms containing L_s and L_t in the H-formula (Eq. 21), and the effect of this approximation on the value of H does not, in extreme cases, exceed one per cent.

In the H-equation (Eq. 21), the only terms representing the effect of continuity are one in the numerator containing $\Delta \mu_f$ and one in the denominator containing εf. Both of these terms contain the factor $\Sigma'(Kl)$. With these two terms omitted, Eq. (21) becomes identical with the corresponding H-equation for two-hinged suspension bridges. It may be noted that $\Delta \mu_f$ is the augmentation of εf due to the deflections of the spans, or

$$\Delta \mu_f = \Delta (\varepsilon f) \tag{24}$$

5. Evaluation of the Integration Constants.

The constants of integration C_1 and C_2, appearing in the basic Equations (10), (12), (13), (14), and (21), must be determined for each different condition of loading. For each span-segment having a constant value of p and of I, there is a pair of values for C_1 and C_2.

In the treatment that follows it will be assumed, as is usually done for the sake of simplicity, that the moment of inertia I (or I_1) is constant throughout the length of any span under consideration, although it may have different respective values for the three different spans. The error of ignoring the variation of I within a span is found to be practically negligible and on the side of safety. (For greater accuracy, instead of the average value of I for any span, the value of the equivalent uniform I should be used in the computations; this equivalent uniform I may be determined by figuring equal deflections under governing loadings.)

In this article, the integration constants are evaluated for three general cases, covering the division of a span into one, two, and three differently loaded segments, respectively. From the general formulas thus derived, special formulas may be written for a large variety of loading conditions, including all of the loading cases that arise in the usual design computations (Cases I to X, Art. 11).

One Loading Segment. — For the case of the main span fully loaded with a uniform applied load p, and assuming constant moment of inertia I, the quantities C_1 and C_2 are obtained from the two known conditions that, for $x = 0$ and $x = l$, $\eta = 0$ in Eq. (10) or $M = T_1$ and T_2 respectively in Eq. (12). Substituting these values and solving the resulting two independent equations, we find:

$$C_1 = \frac{1}{(e^{cl}+1)} \cdot \frac{p}{Hc^2} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl}-e^{-cl})}\,\frac{(T_1 - T_2)}{H} \tag{25a}$$

$$C_2 = \frac{1}{(e^{-cl}+1)} \cdot \frac{p}{Hc^2} - \frac{1}{(e^{-cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) - \frac{1}{(e^{cl}-e^{-cl})}\,\frac{(T_1 - T_2)}{H} \tag{25b}$$

$$(C_1 + C_2) = \frac{p}{Hc^2} - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) \tag{25c}$$

These are the values recorded for Case II, Art. 11.

It may be noted that the expression for C_2 may be written from the accompanying expression for C_1 by simply changing the sign of c wherever it occurs in an exponent. This rule holds true for all loading cases; it is a neccessary consequence of the symmetrical occurence of C_1 and C_2 as the coefficients of e^{cx} and e^{-cx}, respectively, in Eqs. (10), (12), and (13). It will therefore suffice to give the expressions for C_1 for the loading cases that follow, without writing out the parallel expressions for C_2.

For either side span fully loaded, the following corresponding expressions are similarly obtained, provided x_1 is always measured from the free end of the span:

$$C_1 = \frac{1}{(e^{c_1 l_1}+1)} \cdot \frac{p}{H c_1^2} - \frac{1}{(e^{c_1 l_1}+1)} \cdot \frac{1}{\varrho_1 c_1^2} - \frac{1}{(e^{c_1 l_1} - e^{-c_1 l_1})} \cdot \frac{T_{1,2}}{H} \tag{26a}$$

$$C_1 + C_2 = \frac{p}{H c_1^2} - \frac{1}{\varrho_1 c_1^2} \tag{26b}$$

These are the values recorded for Case VIII, Art. 11.

It may be noted that the formulas for C_1 and C_2 in a side span may always be written from the corresponding formulas for C_1 and C_2 in the main span, by simply writing c_1, l_1, and ϱ_1 instead of c_1, l_1 and ϱ and substituting zero for T_1 (bending moment at origin of x) and T_1 or T_2 for T_2 (bending moment at other end of span).

The first term in each of the formulas for C_1 and C_2 is an expression containing p and varies for different loading conditions; the remaining terms are repeated unchanged in the respective C_1 and C_2 formulas for all loading conditions.

For any span fully unloaded, substitute $p = 0$ in the formulas for C_1 and C_2 for that span. This yields the expressions recorded for Cases I and VII.

Two Loading Segments. — For the case of a partial loading of the main span (Case III, Art. 11), with a uniform load p per unit length extending a distance k from the left end of the span, the constants C_1 and C_2 for the loaded segment (k) and the constants C_3 and C_4 for the unloaded segment ($m = l - k$) are obtained from the four known conditions that the moment and shear at the right end B of the loaded segment must be equal respectively to those at the left end B of the unloaded segment, and that $\eta = 0$ (or $M = T_{1,2}$) at each end of the span. Substituting these relations in Eqs. (10), (12) and (13), and solving the resulting four independent equations, we find:

$$C_1 = \frac{p}{2Hc^2} \frac{(e^{cm} + e^{-cm} - 2e^{-cl})}{(e^{cl} - e^{-cl})} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl} - e^{-cl})} \frac{(T_1 - T_2)}{H} \tag{27a}$$

$$C_2 = -C_1 + \frac{p}{Hc^2} - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) \tag{27b}$$

$$C_3 = \frac{p e^{-cl}}{2Hc^2} \frac{(e^{ck} + e^{-ck} - 2)}{(e^{cl} - e^{-cl})} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl} - e^{-cl})} \frac{(T_1 - T_2)}{H} \tag{28a}$$

$$C_4 = -C_3 - \frac{p}{2Hc^2}(e^{ck} + e^{-ck} - 2) - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) \tag{28b}$$

The foregoing values of C_1 and C_2 are recorded for Case III in Art. 11.

To calculate the deflection, moment, or shear at any point in the loaded segment of the span, the foregoing values of C_1 and C_2 must be substituted in Eq. (10), (12), or (13), respectively. To calculate the corresponding values in the unloaded segment, the values of C_3 and C_4 replace the general constants in the same basic equations; and since the segment is unloaded, the value of p_x is taken as zero in Eq. (10) or (12), though p retains its value in Eqs. (28).

As a check upon the foregoing formulas, Eq. (27) for the loaded segment may be reduced, by substituting $m = 0$, to Eqs. (25) for the span fully loaded. Similarly by substituting $k = 0$ or $p = 0$ in Eqs. (28), the integration constants may be obtained for the span fully unloaded (Case I).

For the case of partial loading of a side-span, corresponding formulas for the integration constants are similarly obtained, or they may easily be written from Eqs. (27) and (28). See Cases IX and X, Art. 11.

Three Loading Segments. — For other loading conditions, the integration constants are determined by a procedure similar to that followed in the cases represented by Eqs. (25) to (28). If the main span is divided into three segments ($k + m + z = l$) having different uniform loads p_k, p_m and p_z, respectively, the three corresponding pairs of integration constants are obtained from the six known conditions that M and V at the right end of the first segment must be equal to M and V at the left end of the second segment, that the same two equalities hold at the junction of the second and third segments, and that $\eta = 0$ (or $M = T_{1,2}$) at each end of the span. Upon substituting these relations in Eqs. (10), (12), and (13), the solution of the resulting six independent equations yields the following values of the three pairs of integration constants:

$$C_1 = \frac{1}{2Hc^2}\left\{\frac{(p_k - p_m)\left[e^{c(l-k)} + e^{-c(l-k)}\right] + (p_m - p_z)(e^{cz} + e^{-cz}) - 2p_k e^{-cl} + 2p_z}{(e^{cl} - e^{-cl})}\right\} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl} - e^{-cl})}\frac{(T_1 - T_2)}{H} \tag{29a}$$

$$C_2 = -C_1 + \frac{p_k}{Hc^2} - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) \tag{29b}$$

$$C_3 = \frac{1}{2Hc^2}\left\{\frac{(p_k - p_m)(e^{ck} + e^{-ck})e^{-cl} + (p_m - p_z)(e^{cz} + e^{-cz}) - 2p_k e^{-cl} + 2p_z}{(e^{cl} - e^{-cl})}\right\} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl} - e^{-cl})}\frac{(T_1 - T_2)}{H} \tag{30a}$$

$$C_4 = -C_3 - \frac{1}{2Hc^2}\left[(p_k - p_m)(e^{ck} + e^{-ck}) - 2p_k\right] - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) \tag{30b}$$

$$C_5 = \frac{1}{2Hc^2}\left\{\frac{(p_k - p_m)(e^{ck} + e^{-ck})e^{-cl} + (p_m - p_z)\left[e^{c(l-z)} + e^{c(l-z)}\right]e^{-cl} - 2p_k e^{-cl} + 2p_z}{(e^{cl} - e^{-cl})}\right\} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl} - e^{-cl})}\frac{(T_1 - T_2)}{H} \tag{31a}$$

$$C_6 = -C_5 - \frac{1}{2Hc^2}\left\{(p_k - p_m)(e^{ck} + e^{-ck}) + (p_m - p_z)\left[e^{c(l-z)} + e^{-c(l-z)}\right] - 2p_k\right\} - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) \tag{31b}$$

The three-segment loading condition represented by Eqs. (29) to (31) may be regarded as a general case from which the more usual loading conditions may be evaluated by simple substitution. Thus, upon substituting appropriate values 0, k, m, or l for the lengths k, m, or z, and 0 or p for the loads p_k, p_m, p_z, Eqs. (29) to (31) reduce to the simpler cases recorded in Eqs. (25), (27) and (28), and in Cases I to VI, Art. 11.

The expressions for the constants C for side-span conditions are similarly obtained, or are written from the main span constants by substitution of the side-span magnitudes, and are recorded in Cases VII to X, Art. 11.

It should be noted that unsymmetrically loaded spans are not reversible left to right without altering the values of the integration constants (unless the origin of x is also reversed). That is because the integration constants occur in Eqs. (10), (12), and (13), in which x is assumed measured from the left end of the span (main span or left side span) and which represent the unsymmetrical graphs of η, M, and V respectively. It is for this reason, for instance, that the values of C given by Eqs. (28) for a right-hand unloaded segment cannot be applied to Case IV (Art. 11) representing a left-hand unloaded segment. Eqs. (28) and the corresponding side-span formulas would properly be applicable to the unloaded segments of Cases III and IX; they would also be applicable to Case IV and the unloaded segment of Case X if x were measured from the other end of the span.

It should also be noted that the expressions for the integration constants C, for any loading condition in a span, are unaffected by the loading conditions in the other spans.

Upon substituting zero for T_1 and T_2 (representing continuity), the formulas for C_1 and C_2 will reduce to the corresponding formulas for two-hinged suspension bridges.

6. Derivation of Working Formulas for H.

The basic equation for H, Eq. (21), may be simplified, for any particular loading condition, by substituting detailed expressions for the terms that depend upon the loading, transferring some of the terms containing H, and re-solving for H.

For the case of partial loading, with a uniform load p covering the left segment k $(=l-m)$ of the main span (Case III, Art. 1), and with no applied load in the side spans, the simplification of the general equation for H is as follows:

For this loading condition, the principal summation term in the numerator of Eq. (21) takes the following form upon substituting the respective expressions for M' (as defined by Eqs. 1) and integrating for the two segments of the main span and for the two side spans:

$$\Sigma K \int_0^l \left(M' - \frac{p_x}{c^2}\right) dx = pk\left[\frac{k}{12}(3l-2k) - \frac{1}{c^2}\right] + \frac{(M_1+M_2)}{2} \cdot \Sigma'(Kl) \qquad (32)$$

The integration summation term in the denominator of Eq. (21) may be written in the form:

$$-\Sigma K \int_0^l (C_1 e^{cx} + C_2 e^{-cx})\, dx = -\int_0^k (C_1 e^{cx} + C_2 e^{-cx})\, dx - \int_k^l (C_3 e^{cx} + C_4 e^{-cx})\, dx - 2K_1 \int_0^{l_1} (C_1' e^{c_1 x_1} + C_2' e^{-c_1 x_1})\, dx_1$$

Substituting for the main-span constants C_1, C_2, C_3, C_4 the values given by Eqs. (27) and (28), and for the side span constants C'_1 and C'_2 values similar to those given by Eqs. (26) except that p is zero for the unloaded side spans, the foregoing summation reduces, upon integration, to the form:

$$- \Sigma K \int_0 (C_1 e^{cx} + C_2 e^{-cx})\, dx = -\frac{p}{Hc^3} \frac{(e^{ck}-1)(e^{cm}+1)}{(e^{cl}+1)} + \Sigma\left[K \frac{(e^{cl}-1)}{(e^{cl}+1)} \cdot \frac{2}{\varrho c^3}\right] + \\ + \Sigma'\left[K \cdot \frac{2}{c} \frac{(e^{cl}-1)}{(e^{cl}+1)} \cdot \frac{(T_1+T_2)}{2H}\right] \qquad (33)$$

Substituting the expressions given by Eqs. (32) and (33) for the respective summations in Eq. (21), utilizing Eqs. (15), introducing the abbreviation

$$L_c = l - \frac{2}{c} \frac{(e^{cl}-1)}{(e^{cl}+1)} = (\text{approx.})\, l - \frac{2}{c} \qquad (34)$$

and solving for H, we finally obtain:

$$H = \frac{F(p) + \frac{(T_1+T_2)}{2} \cdot \Sigma'(KL_c) \mp \varrho c^2 EI \omega t L_t}{D} \qquad (35)$$

where D, the denominator of the H-formula, is given by:

$$D = \tfrac{8}{3} \Sigma(Kfl) - \Sigma\left(\frac{KL_c}{\varrho c^2}\right) + \varrho c^2 \frac{E}{E_c} \cdot \frac{I}{A_0} \cdot L_s \qquad (36)$$

and $F(p)$, for this loading condition (Case III), is given by:

$$F(p) = pk\left[\frac{k}{12}(3l-2k) - \frac{1}{c^2}\right] + \frac{p}{c^3} \frac{(e^{ck}-1)(e^{cm}+1)}{(e^{cl}+1)} \qquad (37)$$

Eq. (36) for the denominator D is found to remain unchanged for all other loading conditions; it contains no terms involving the load intensity p, the load length k, nor the temperature change t. Eq. (36) is therefore the expression for the denominator of the H-formula for any condition of loading. It should be noted, however, that D is not a constant; although the expression for D remains unchanged, it contains the variable c which depends upon H, and therefore the value of D varies with the loading. The calculations for a given structure are facilitated by computing in advance the values of D for varying values of H, and tabulating or plotting the results for reference in the subsequent computations. (For an illustration, see Fig. 4.) It may also be noted that Eq. (36) for D contains no terms representing continuity, and is therefore applicable to two-hinged as well as to continuous suspension bridges.

In the numerator of the H-formula (Eq. 35), $F(p)$ represents the terms containing p and is the only part of the formula that has different expressions for different loading conditions.

The second term in the numerator of Eq. (35) contains $(T_1 + T_2)$, representing the effect of continuity. By simply omitting this term, Eq. (35) reduces to the H-formula for two-hinged suspension bridges.

The last term in Eq. (35), representing temperature effect, has the minus sign for a rise in temperature above normal, and the plus sign for a drop in temperature below normal. In other words, H is diminished by a rise in temperature and augmented by a drop in temperature.

From the expression for $F(p)$ given by Eq. (37) for Case III, the $F(p)$ expressions for any other loading conditions are easily written. Thus, interchanging k and m yields Case IV.

For full loading of the main span, substitute l for k and zerro for m in Eq. (37). This yields

$$F(p) = \frac{p\,l^3}{12} - \frac{p}{c^3} \cdot L_c \tag{38}$$

for the main span fully loaded (Case II). Write $p = 0$ to obtain Case I.

Subtracting from $F(p)$ for the main span fully loaded (Case II) the $F(p)$ expressions for partial loading from each end (Cases III and IV), we obtain $F(p)$ for the case of partial loading near the middle of the span (Case V). Subtract Case V from Case II to obtain $F(p)$ for Case VI.

By substituting the side-span torms K_1, l_1, c_1 and L_{c1} in the $F(p)$ expressions for main-span loadings, we obtain the corresponding $F(p)$ expressions for side-span loadings (Cases VI to X).

The ten loading cases in Art. 11 are tabulated in complementary pairs. The $F(p)$ expression for each case may be obtained by subtracting the complementary case from $F(p)$ for the full span loaded (Case II or VIII).

By doubling $F(p)$ for Case VIII and adding to $F(p)$ for Case II, we obtain the following expression for the condition of full loading of all three spans:

$$F(p) = \Sigma\left[K\frac{p\,l^3}{12}\right] - \Sigma\left[K L_c \cdot \frac{p}{c^2}\right] \tag{39}$$

The methods used above for deducing all other $F(p)$ expressions, for use in the H-formula, from the expression given by Eq. (37) for one loading condition have consisted of the simple processes of substitution, addition and subtraction. We have made use of the fact that the expression for $F(p)$ (though not its value) is algebraically additive for combinations of loadings.

It is also of interest to note that unsymmetrically loaded spans are reversible (left to right) without affecting the value of $F(p)$ or the expression for H (whereas the integration constants C are altered by such reversal). That is because the directional variable x does not occur in the formulas for $F(p)$. Furthermore, an unsymmetrically loaded main span is reversible (left to right) and unequally loaded side spans are interchangeable without affecting the value of H, since the shifting of any load to a symmetrical position about the center line of the entire structure does not alter the value of $(T_1 + T_2)$; but an unsymmetrically loaded side span cannot be reversed about its own center line without altering the value of $(T_1 + T_2)$ and therefore of H. (This last distinction does not apply to non-continuous suspension bridges.)

The abbreviation L_c, introduced in this Article, will be found convenient in condensing the formulas and applications of the Deflection Theory. The approximate value $L_c = l - \frac{2}{c}$ is almost exact, ranging from about 0.9985 to 0.9996 of the exact value. For the side span terms, the approximation $L_{c1} = l_1 - \frac{2}{c_1}$ is less permissible, ranging from about .80 to .90 of the exact value. If both approximations are used in the combination $\Sigma'(kL_c)$, the resulting summation will range from about .96 to .98 of the exact value for this term; in the combination $\Sigma(kL_c)$, the resulting summation will range from about .93 to .97 of the exact value. The final error in the value of H will, however, be negligible.

7. Evaluation of the Deflection Functions $\Delta\mu$.

The functions T_1, T_2, and $(T_1 + T_2)$, occurring in the working formulas for H, C_1 and C_2, involve in turn (by Eqs. (15) and (23)) the functions $\Delta\mu_1$, $\Delta\mu_2$, and $\Delta\mu_f$, respectively. It is therefore necessary to evaluate these functions, representing the values of $\Delta\mu$ at the two towers and at mid-span, respectively.

The values of μ expressed by Eqs. (6) and (7) are for geometrically symmetrical side spans, as initially assumed, and therefore represent equal values of μ_1 and μ_2 at the two towers. If, however, the two side spans have unequal sags f_1 and f_2, the Theorem of Three Moments yields differing values of μ_1 and μ_2, as follows:

$$\mu_1 = \frac{2f + 2ir\left(f_1 + \dfrac{f_1 - f_2}{1 + 2ir}\right)}{3 + 2ir}, \qquad \mu_2 = \frac{2f + 2ir\left(f_2 + \dfrac{f_2 - f_1}{1 + 2ir}\right)}{3 + 2ir} \tag{40}$$

Differentiating these expressions, we obtain:

$$\Delta\mu_1 = \frac{2}{3 + 2ir}\left[\Delta f + ir\left(\Delta f_1 + \frac{\Delta f_1 - \Delta f_2}{1 + 2ir}\right)\right] \tag{41a}$$

$$\Delta\mu_2 = \frac{2}{3 + 2ir}\left[\Delta f + ir\left(\Delta f_2 + \frac{\Delta f_2 - \Delta f_1}{1 + 2ir}\right)\right] \tag{41b}$$

Taking one-half the sum of these two expressions, we obtain:

$$\Delta\mu_f = \frac{\Delta\mu_1 + \Delta\mu_2}{2} = \Delta(\varepsilon f) = \frac{2\Delta f + ir(\Delta f_1 + \Delta f_2)}{3 + 2ir} \tag{42}$$

Taking the difference of the two expressions in Eqs. (41), we obtain:

$$(\Delta\mu_1 - \Delta\mu_2) = \frac{2ir}{1 + 2ir}(\Delta f_1 - \Delta f_2) \tag{43}$$

Eq. (42) may also be written by differentiating the dependent variables in Eq. (6), using the mean of the two side-span sag-changes, Δf_1 and Δf_2 (since $\Delta\mu_f$ is a symmetrical function, equally affected by a change in either side span).

Eqs. (41), (42) and (43) will yield the desired $\Delta\mu$ functions when the sag-changes Δf, Δf_1 and Δf_2 are known.

To obtain an expression for Δf, for any main-span loading condition, substitute the appropriate expressions for C_1 and C_2 in the basic deflection formula (Eq. 10), also substitute $x = \frac{l}{2}$, $y = f$, $\eta = \Delta f$ and $\Delta\mu = \Delta\mu f$.

$$\Delta f = A + \frac{\Delta\mu_f}{e_c} \tag{44}$$

where

$$e_c = \frac{(e^{cl} + 1)}{(e^{\frac{cl}{2}} - 1)^2} = \text{(approximately) } 1 \tag{45}$$

and A is an expression that does not contain any deflection terms and depends only upon the loading condition. The expression for A is not affected by loading conditions in the side spans. For the generalized case of the main span divided into any number of segments k, each having a different uniform load p_k (which may be zero for any segments) the expression for A is found to be:

$$A = \frac{H}{EIc^2}\left[\frac{1}{2Hc^2}\frac{\Sigma(p_f - p_k)(e^{ck}+e^{-ck}-2)}{\left(e^{\frac{cl}{2}}+e^{-\frac{cl}{2}}\right)} + \frac{1}{e_c}\left(-\frac{p_f}{Hc^2}+\frac{1}{\varrho c^2}+\frac{M_1+M_2}{2H}+\varepsilon f\right)+\frac{M_f}{H}-f\right] \quad (46)$$

where p_f denotes the uniform load (if any) at mid-span, and M_f denotes the simple-span bending moment (M_0) at mid-span. Upon substituting appropriate values for k and p_k for each loading segment, also for p_f, Eq. (46) reduces to the respective formulas for A tabulated for the six main-span loading cases in Art. 11.

To obtain an expression for Δf_1, for any loading condition in the left side span, substitute the appropriate expressions for C_1 and C_2 in the basic deflection formula (Eq. 10) modified for side-span application by using η_1, c_1, x_1, y_1, and ϱ_1, also substitute $x_1 = \frac{l_1}{2}$, $y_1 = f_1$, $\eta_1 = \Delta f_1$, and $\Delta\mu = \frac{1}{2}\Delta\mu_1$. This will yield:

$$\Delta f_1 = B_1 + \frac{\Delta\mu_1}{2e_{c_1}} \quad (47)$$

where (similar to Eq. 45)

$$e_{c_1} = \frac{(e^{c_1 l_1}+1)}{\left(e^{\frac{c_1 l_1}{2}}-1\right)^2} \quad (48)$$

and B_1 is an expression (similar to A) that does not contain any deflection terms and depends only upon the loading condition in the left side span. Similarly, for any loading condition in the right side span, a corresponding expression for Δf_2 is obtained:

$$\Delta f_2 = B_2 + \frac{\Delta\mu_2}{2e_{c_1}} \quad (49)$$

where B_2 is an expression parallel to B_1 but depending only upon the loading condition in the right side span. The expression for B_1 or B_2 is not affected by loading conditions in the other spans. For the generalized case of a side span divided into any number of segments k, each having a different uniform load p_k (which may be zero for any segments) the expression for B_1 or B_2 is found to be:

$$B_{1,2} = \frac{H}{EIc^2}\left[\frac{1}{2Hc_1^2}\frac{\Sigma(p_{f_{1,2}} - p_k)(e^{c_1 k}+e^{-c_1 k}-2)}{\left(e^{\frac{c_1 l_1}{2}}+e^{-\frac{c_1 l_1}{2}}\right)} + \frac{1}{e_{c_1}}\left(-\frac{p_{f_{1,2}}}{Hc_1^2}+\frac{1}{\varrho_1 c_1^2}+\frac{M_{1,2}}{2H}+\frac{\varepsilon f}{2}\right)+\frac{M_{f_{1,2}}}{H}-f_{1,2}\right] \quad (50)$$

where p_{f1} or p_{f2} denotes the uniform load (if any) at the middle of the respective side span, and M_{f1} or M_{f2} denotes the simple-span bending moment (M_0) at the same point. Upon substituting appropriate values for k and p_k for each loading segment, also for $p_{f1,2}$, Eq. (50) reduces to the respective formulas for $B_{1,2}$ tabulated for the four side-span loading cases in Art. 11.

Substituting the expressions for Δf, Δf_1 and Δf_2, from Eqs. (44), (47) and (49), in Eq. (42) for $\Delta\mu_f$, we obtain:

$$\Delta\mu_f = \tfrac{1}{2}(\Delta\mu_1 + \Delta\mu_2) = \frac{2A + ir(B_1+B_2)}{(3+2ir) - \left(\frac{2}{e_c}+\frac{ir}{e_{c_1}}\right)} \quad (51)$$

Substituting the same expressions for Δf_1 and Δf_2 in Eq. (43), we obtain:

$$\tfrac{1}{2}(\Delta \mu_1 - \Delta \mu_2) = \frac{ir(B_1 - B_2)}{(1 + 2ir) - \left(\frac{ir}{e_{c_1}}\right)} \tag{52}$$

Add the values given by Eqs. (51) and (52) to obtain $\Delta\mu_1$, and take their difference to obtain $\Delta\mu_2$. This yields:

$$\Delta \mu_{1,2} = \frac{2A + ir(B_1 + B_2)}{(3 + 2ir) - \left(\frac{2}{e_c} + \frac{ir}{e_{c_1}}\right)} \pm \frac{ir(B_1 - B_2)}{(1 + 2ir) - \left(\frac{ir}{e_{c_1}}\right)} \tag{53}$$

Eq. (53) is an exact formula giving $\Delta\mu_1$ and $\Delta\mu_2$ in terms of A, B_1 and B_2, representing the loadings in the three respective spans. Denoting the denominator in the first term by $(1 + ir)_c$ and that in the second term by $(1 + ir)_{c_1}$ Eq. (53) may be written in the abbreviated form:

$$\Delta \mu_{1,2} = \frac{2A + ir(B_1 + B_2)}{(1 + ir)_c} \pm \frac{ir(B_1 - B_2)}{(1 + ir)_{c_1}} \tag{53a}$$

As indicated in Eq. (45), e_c is approximately equal to unity. (Values of e_c have been found to range between 1.03 and 1.08 for 1200-ft. main spans and beetwen 1.05 and 1.09 for 800-ft. spans. For shorter spans, the error increases.) A similar approximation for e_{c1} is not permissible. (Values of e_{c1} have been found to range between 1.50 and 1.84 for 400-ft. and 500-ft. side spans.)

Substituting Eq. (51) in Eq. (44), we obtain:

$$\Delta f = \frac{\left(3 + 2ir - \frac{ir}{e_{c_1}}\right)A + \frac{ir}{e_c}(B_1 + B_2)}{(1 + ir)_c} \tag{54}$$

Similarly, from Eqs. (47), (49), and (53), we obtain:

$$\Delta f_{1,2} = \frac{2A + ir(B_1 + B_2)}{2e_{c_1}(1 + ir)_c} \pm \frac{ir(B_1 - B_2)}{2e_{c_1}(1 + ir)_{c_1}} + B_{1,2} \tag{55}$$

The values of the mid-span deflections Δf, Δf_1 and Δf_2 are thus determined by Eqs. (54) and (55) when A, B_1 and B_2 are known.

The functions A, B_1 and B_2, tabulated for the ten loading cases in Art. 11, will not be needed for the computation of stresses in a suspension bridge, since these functions are eliminated by the further simplification in Art. 8. They are useful, however, for calculating the $\Delta\mu$ and Δf functions by Eqs. (51) to (55), when the values of these deflection functions are separately desired.

8. Evaluation of T_1 and T_2.

The formulas developed in the preceding articles are sufficient for the complete analysis of a continuous suspension bridge by the Deflection Theory. The entire operation can be made more direct, however, by eliminating a number of inter-related functions. Working formulas for T_1 and T_2, defined in Eqs. (15), will now be developed so as to eliminate the necessity of calculating M_1, M_2, A, B_1, B_2, $\Delta\mu_1$, $\Delta\mu_2$, $\Delta\mu_f$, and εf in practical application.

The formulas for A and B (Eqs. 46, 50, and Art. 11) may be written in generalized form as follows:

$$EIc^2A = \frac{p}{2c^2}\cdot F_A(e) + \frac{1}{e_c}\left[-\frac{pf}{c^2} + \frac{H}{\varrho c^2} + \frac{M_1+M_2}{2} + H\cdot\varepsilon f\right] + M_f - Hf \quad (61a)$$

$$EIc^2B_{1,2} = \frac{p}{2c_1^2}\cdot F_B(e) + \frac{1}{e_{c_1}}\left[\frac{-pf_{1,2}}{c_1^2} + \frac{H}{\varrho_1 c_1^2} + \frac{M_{1,2}}{2} + \frac{H\cdot\varepsilon f}{2}\right] + M_{f_{1,2}} - Hf_1 \quad (61b)$$

where $F_A(e)$ and $F_B(e)$ are temporary abbreviations for the terms (if any) containing the exponential functions.

From the seven equations (15, 53, and 61), the five deflection functions (A, B_1, B_2, $\Delta\mu_1$ and $\Delta\mu_2$) are to be eliminated so as to leave two formulas (for T_1 and T_2) that do not involve these deflection functions. For clarity of presentation, the contributions of certain groups of terms will be evaluated separately.

The terms containing εf and f in Eqs. (61), when substituted in Eq. (53) for $\Delta\mu_{1,2}$, yield the following contribution to $EIc^2\cdot\Delta\mu_{1,2}$:

$$H\frac{\frac{2}{e_c}(\varepsilon f) - 2f + \frac{ir}{e_{c_1}}(\varepsilon f) - 2irf_1}{(3+2ir) - \left(\frac{2}{e_c}+\frac{ir}{e_{c_1}}\right)} \pm H\frac{ir(0)}{(1+2ir) - \left(\frac{ir}{e_{c_1}}\right)} = -H\cdot\varepsilon f \quad (62)$$

which exactly cancels the term $H\cdot\varepsilon f$ in Eqs. (15) for T_1 and T_2.

The terms containing M_1, M_2, M_f, and $M_{f1,2}$ in Eqs. (61), when substituted in Eq. (55) for $\Delta\mu_{1,2}$ and then combined with the term $M_{1,2}$ in Eqs. (15), yield the following contribution to the terms $M_{1,2}+EIc^2\cdot\Delta\mu_{1,2}$ in the formula for $T_{1,2}$:

Contribution of M and M_f terms to $T_{1,2}$

$$= \frac{\frac{(M_1+M_2)}{2}\left(\frac{2}{e_c}+\frac{ir}{e_{c_1}}\right) + 2M_f + ir(M_{f_1}+M_{f_2})}{(3+2ir) - \left(\frac{2}{e_c}+\frac{ir}{e_{c_1}}\right)} \pm \frac{\left[\frac{(M_1-M_2)}{2e_{c_1}} + (M_{f_1}-M_{f_2})\right]ir}{(1+2ir) - \left(\frac{ir}{e_{c_1}}\right)} + M_{1,2} \quad (63)$$

Inspection of the formulas for $M_{1,2}$ (tabulated in Art. 11) shows that, for any condition of loading, the value of $M_{1,2}$ may be written in the form:

$$M_{1,2} = \frac{F(M)}{(3+2ir)} \pm \frac{F'(M)}{(1+2ir)}$$

so that

$$\frac{M_1+M_2}{2} = \frac{F(M)}{(3+2ir)}, \quad \text{and} \quad \frac{M_1-M_2}{2} = \frac{F'(M)}{(1+2ir)}$$

Substituting these three expressions in Eq. (63), a simplifying cancellation results, and Eq. (63) reduces to:

Contribution of M and M_f terms to $T_{1,2}$

$$= \frac{F(M) + 2M_f + ir(M_{f_1}+M_{f_2})}{(1+ir)_c} \pm \frac{F'(M) + ir(M_{f_1}-M_{f_2})}{(1+ir)_{c_1}}$$

This expression may be segregated into separate contributions from the load in the three respective spans, as follows:

Main Span contribution to $\quad T_{1,2} = \frac{N}{(1+ir)_c} \pm \frac{N'}{(1+(ir)_{c1}} \quad$ (64a)

Left Side Span contribution to $T_{1,2} = \frac{irN_1}{(1+ir)_c} \pm \frac{irN_1}{(1+ir)_{c_1}}$ (64b)

Right Side Span contribution to $T_{1,2} = \frac{irN_2}{(1+ir)_c} \mp \frac{irN_2}{(1+ir)_{c_1}}$ (64c)

where
$$N = F(M) + 2M_f \quad (65a)$$
$$N' = F'(M) \quad (65b)$$
$$N_{1,2} = F_{1,2}(M) + M_{f_{1,2}} \quad (65c)$$

In writing Eqs. (64b) and (64c), use was made of the fact that, for load in either side span, $F_{1,2}(M) = F_{1,2}'(M)$.

Upon substituting the respective values of $M_{1,2}$, M_f and $M_{f1,2}$, the N-functions defined by Eqs. (65) reduce to the following expressions for the ten loading conditions of Art. 11:

Case I: $N \pm N' = 0$

Case II: $N \pm N' = 0$

Case III:
$$(k > m):\ N \pm N' = \frac{pm^2}{8l^2}(k^2 - m^2 \mp k^2)$$
$$(k < m):\ N \pm N' = \frac{pk^2}{8l^2}(k^2 - m^2 \mp m^2)$$

Case IV:
$$(k > m):\ N \pm N' = \frac{pm^2}{8l^2}(m^2 - k^2 \pm k^2)$$
$$(k < m):\ N \pm N' = \frac{pk^2}{8l^2}(m^2 - k^2 \pm m^2)$$

Case V:
$$N \pm N' = -\frac{p}{8l^2}[k^4 + z^4 - k^2(l-k)^2 - z^2(l-z)^2 \pm k^2(l-k)^2 \mp z^2(l-z)^2]$$

Case VI:
$$N \pm N' = \frac{p}{8l^2}[k^4 + z^4 - k^2(l-k)^2 - z^2(l-z)^2 \pm k^2(l-k)^2 \mp z^2(l-z)^2]$$

Case VII: $N_{1,2} = 0$

Case VIII: $N_{1,2} = 0$

Case IX:
$$(k > m):\ N_{1,2} = \frac{pm^4}{8l_1^2}$$
$$(k < m):\ N_{1,2} = \frac{pk^2}{8l_1^2}(2m^2 - k^2)$$

Case X:
$$(k > m):\ N_{1,2} = -\frac{pm^4}{8l_1^2}$$
$$(k < m):\ N_{1,2} = -\frac{pk^2}{8l_1^2}(2m^2 - k^2)$$

Where double signs occur in the foregoing expressions, use the upper sign in the value for T_1 and the lower sign in the value for T_2.

It is interesting to note that, for any span fully loaded, the contribution of M_f exactly cancels the contribution of the M_1 and M_2 terms in the foregoing N-functions. Hence these functions reduce to zero for any span fully loaded or unloaded (Cases I, II, VII, VIII). These N-functions are algebraically additive for combinations of loading conditions, and are algebraically complementary for complementary loading conditions.

To take account of the difference of denominators in Eqs. (64), the terms following the double signs in the foregoing expressions denote N' and should be written over the denominator $(1+ir)_{c1}$; and the foregoing expressions for $N_{1,2}$ need to be multiplied by the factor

$$\frac{ir}{(1+ir)_c} \pm \frac{ir}{(1+ir)_{c_1}},$$

with this double sign inverted for the right side-span contribution, as shown by Eqs. (64 b) and (64 c).

The contributions of the terms containing f and the moment terms in Eqs. (61) to the values of $T_{1,2}$ have now been evaluated. The contributions of the remaining terms of Eqs. (61) are easily written and are as follows:

Main Span contribution to $T_{1,2}$

$$= \frac{\frac{p}{c^2} F_A(e) + \frac{2}{e_c}\left(-\frac{pf}{c^2} + \frac{H}{\varrho\, c^2}\right)}{(1+ir)_c} \tag{66a}$$

Left Side Span contribution to $T_{1,2}$

$$= \frac{ir\left[\frac{p}{2c_1^2} F_{B_1}(e) + \frac{1}{e_{c_1}}\left(-\frac{pf_1}{c_1^2} + \frac{H}{\varrho_1 c_1^2}\right)\right]}{(1+ir)_c} \pm \frac{ir\left[\frac{p}{2c_1^2} F_{B_1}(e) - \frac{pf_1}{e_{c_1} c_1^2}\right]}{(1+ir)_{c_1}} \tag{66b}$$

Right Side Span contribution to $T_{1,2}$

$$= \frac{ir\left[\frac{p}{2c_1^2} F_{B_2}(e) + \frac{1}{e_{c_1}}\left(-\frac{pf_2}{c_1^2} + \frac{H}{\varrho_1 c_1^2}\right)\right]}{(1+ir)_c} \mp \frac{ir\left[\frac{p}{2c_1^2} F_{B_2}(e) - \frac{pf_2}{e_{c_1} c_1^2}\right]}{(1+ir)_{c_1}} \tag{66c}$$

Combining the foregoing expressions for the contributions of the f-terms (Eq. 62), the moment terms (Eqs. 64), and the remaining terms (Eqs. 66); substituting the values of N, N', and $N_{1,2}$, (tabulated above); and restoring the original expressions for $F_A(e)$, $F_{B1,2}(e)$ taken from the respective formulas for A and B for the various conditions of loading, — there are obtained the working formulas for the individual span contributions to T_1 and T_2. These working formulas are tabulated for the ten loading cases in Art. 11. In that tabulation, groups of terms containing p, occurring in the T-formulas, are denoted by $G(p)$ and are separately recorded. These load functions $G(p)$ are separately useful in the computation of H, as will be developed in the next Article.

The T-Formulas tabulated in Art. 11 give the individual span contributions. For any loading condition on the structure, the respective contributions from all three spans must be added together to give the total value of T_1 or T_2.

The expressions for $T_{1,2}$ hereinabove derived give exact values for the tower moments T_1 and T_2. Closely approximate values of these expressions for $T_{1,2}$ are obtained by calling $e_c = 1$ to make $(1+ir)_c = (1+ir)_{c1}$ and these approximate expressions for $T_{1,2}$, as well as the exact expressions, are recorded for the ten loading cases in Art. 11. This approximation eliminates the small contribution of the far side-span loading to $T_{1,2}$, so that it may then be considered that the contributions to T_1 or T_2 come only from the two contiguous spans in either case. In the tabulation in Art. 11, the approximate expressions for the contributions to $T_{1,2}$

from side-span loadings are written on this basis. (For an 800-ft. main span, this approximation will introduce errors in the values of T_1 or T_2, ranging from 5 percent for large tower moments to 20 percent in small values of the tower moments. For shorter spans, the errors will be greater.)

The physical significance of T_1 and T_2, as defined by Eqs. (15) should be recalled. Since M_1 is the bending moment, at the left tower, in the unsuspended continuous truss; $H \cdot \varepsilon f$ is the bending moment, at the tower, due to the suspender forces, with deflection neglected; and $EIc^2 \cdot \Delta\mu_1 = (H_w + H) \cdot \Delta\mu_1$ is the deflection correction in the bending moment at the tower; therefore, T_1 is the total resultant bending moment in the stiffening truss at the left tower. Similarly, T_2 is the total resultant bending moment in the stiffening truss at the right tower. Obviously these functions vanish in the two-hinged suspension bridge.

With the values of T_1 and T_2 evaluated, the following direct expressions for the $\Delta\mu$ functions, written from Eqs. (15), may now be used, superseding the formulas of Art. 7:

$$EIc^2 \cdot \Delta\mu_{1,2} = T_{1,2} - M_{1,2} - H \cdot \varepsilon f \tag{67}$$

$$EIc^2 \cdot \Delta\mu_f = \frac{T_1 + T_2}{2} - \frac{M_1 + M_2}{2} - H \cdot \varepsilon f \tag{68}$$

9. Reduction of Working Formula for *H*.

The working formula for H established in Art. 6 (Eq. 35) contains, in the numerator, the continuity term:

$$\frac{(T_1 + T_2)}{2} \cdot \Sigma'(KL_c)$$

Physically interpreted, $\frac{1}{2}(T_1 + T_2)$ is the mid-span height of the "closing line" that represents the total continuity correction for bending moments.

The values of T_1 and T_2 contributed by individual span loading cases, as deduced in the preceding Article, are tabulated in Art. 11. The expressions for T_1 and T_2 do not contain H except in the terms $\frac{H}{\varrho c^2}$ and $\frac{H}{\varrho_1 c_1^2}$; all of the other terms are load-terms, containing p. Hence, for any combination of span-loadings, the expression for $\frac{1}{2}(T_1 + T_2)$ will contain a group of load terms, which may be represented by the symbol $G(p)$; and, in addition, a function of $\frac{H}{\varrho c^2}$ and $\frac{H}{\varrho_1 c_1^2}$. This non-load function takes the form:

$$\frac{2H\left(\frac{1}{e_c} \cdot \frac{1}{\varrho c^2} + \frac{1}{e_{c_1}} \cdot \frac{ir}{\varrho_1 c_1^2}\right)}{(1 + ir)_c} = \frac{16H}{c^2 l} \cdot \frac{\Sigma'(n/e_c)}{(1 + ir)_c}$$

Accordingly, for any condition of loading, we may write:

$$\left(\frac{T_1 + T_2}{2}\right) = G(p) + \frac{16H}{c^2 l} \cdot \frac{\Sigma'(n/e_c)}{(1 + ir)_c} \tag{69}$$

Substituting this value for $\frac{1}{2}(T_1 + T_2)$ in Eq. (35), and transferring the non-load term to the denominator (so as to eliminate H), we obtain the following reduced working formula for H:

$$H = \frac{F(p) + G(p) \cdot \Sigma'(KL_c) \mp \varrho c^2 EI \omega t L_t}{D'} \tag{70}$$

where

$$D' = D - \frac{16}{c^2 l} \cdot \frac{\Sigma'(n/e_c)}{(1+ir)_c} \cdot \Sigma'(KL_c)$$

$$= \tfrac{8}{3}\Sigma(Kfl) - \Sigma\left(\frac{KL_c}{\varrho c^2}\right) + \varrho c^2 \frac{E}{E_c} \cdot \frac{I}{A_0} \cdot L_s - \frac{16}{c^2 l} \cdot \frac{\Sigma'(n/e_c)}{(1+ir)_c} \cdot \Sigma'(KL_c) \tag{71}$$

The values of $G(p)$ are easily written from the complete expressions for T_1 and T_2, deduced as outlined in the preceding Article. For the main-span contribution, $G(p)$ is the same as $T_{1,2}$, with the $\frac{H}{\varrho c^2}$ term and the $\pm$ load terms omitted. For each side-span contribution, $G(p)$ is that portion of the expression for $T_{1,2}$ (with the $\frac{H}{\varrho_1 c_1^2}$ term omitted) whose denominator is $(1+ir)c$. These values of $G(p)$ are recorded for the ten loading cases in Art. 11. They are algebraically additive for any combinations of loading. The values from all three spans must be combined to give the total value of $G(p)$ to be used in the numerator of Eq. (70).

For practical application, Eq. (70) is preferable to Eq. (35) as a working formula for H, in that terms containing H have been eliminated, and the computation of $G(p)$ is somewhat more direct than the computation of $\frac{1}{2}(T_1 + T_2)$.

In the new working formula for H (Eq. 70), the first term $F(p)$ in the numerator is the load term with continuity disregarded; it is identical with the load-term in the H-formula for the two-hinged suspension bridge. The second term in the numerator, containing $G(p)$, is the load term due to continuity. The remaining term in the numerator, containing t, represents the effect of temperature change.

In the denominator D' (Eq. 71), the first three terms (as in Eq. 36) are identical with the denominator D of the H-formula for two-hinged suspension bridges. The fourth term in the denominator is new, and represents the contribution of the geometrical constants of the structure to the correction for continuity.

In all H-formulas (Eqs. 21, 35, 70 and 71), the terms due to continuity are identified by their containing the modified summation Σ'. For application to two-hinged suspension bridges, all terms containing Σ' vanish.

10. Loading Conditions for Maximum Moments and Shears.

In the continuous suspension bridge, loading conditions for maximum stress are not as uniform and definite as in the non-continuous structure, nor are they as easily predictable from a study of the basic equations.

A study of the loading conditions which produced maximum moment and shear for continuous suspension bridges that have been designed yields the guiding indications for loading placement tabulated in Fig. 2 for maximum moments and in Fig. 3 for maximum shears.

For maximum positive moment at any point, it is apparent from the basic formula Eq. (2) or Eq. (3) that the loading must be such as to make M' as large as possible while keeping H small. Hence a length of span embracing the given point must generally be covered with live load, and the other spans must generally be unloaded (since their loading would contribute additional H). For points near the towers, however, these rules are changed, since adjacent loading contributes negative M' and since $(y-\mu)$ is negative near the towers.

For maximum negative moment at any point, the reverse conditions must be satisfied. The loading diagrams for maximum $+M$ and $-M$ are complementary, and are shown in pairs in Fig. 2.

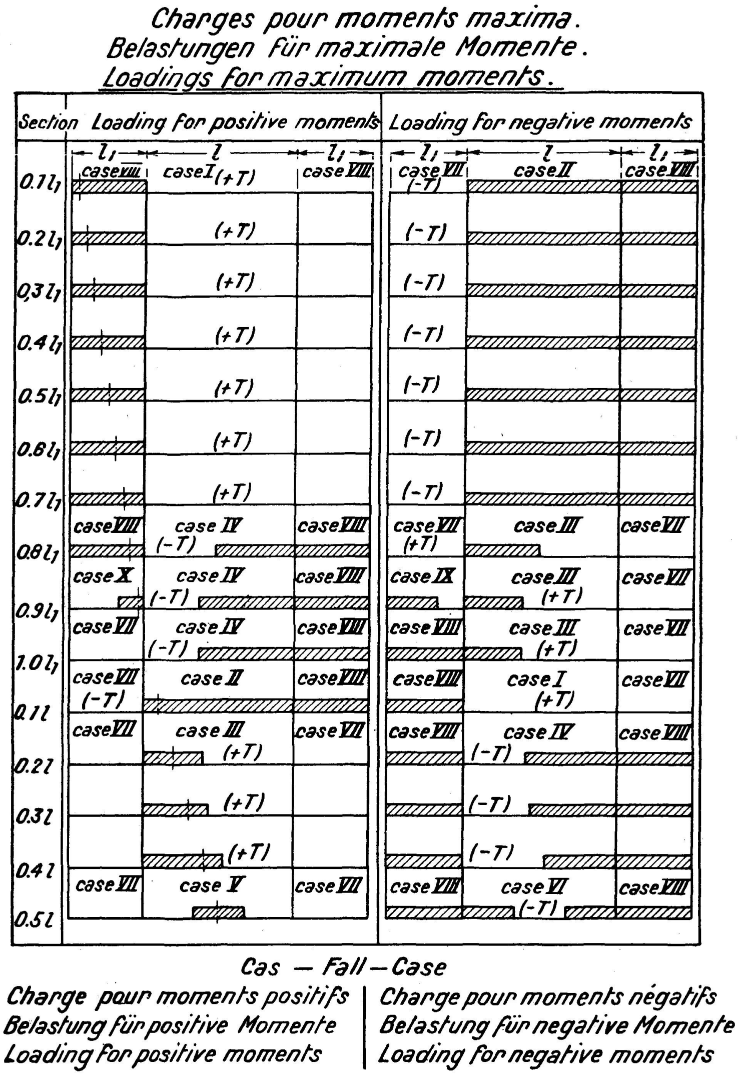


Fig. 2.

For maximum positive (or negative) shear at any point, a length of span extending to the right (or left) from the given point must be covered with live load; the partial loading always stops at the section where the maximum shear is sought; in addition some loading generally has to be placed in one or both of the other spans. The complementary loading diagrams for maximum $+V$ and $-V$ at the various points of the structure are shown in pairs in Fig. 3. (A cyclical sequence of loaded and unloaded segments may be observed in these diagrams.)

The load-lengths for maximum positive and negative moments and for shears generally have to be found by trial in the Deflection Theory. The load-lengths determined by the Elastic Theory may be used as a guide for the trial values to be substituted in the more exact theory. Generally, three trial values of the load-length suffice to determine the maximum value of the function sought.

In Figs. 2 and 3, roman numerals are marked over the loading diagrams in the individual spans. These refer to the numbered designations of the respective basic loading cases tabulated in Art. 11. Thus, for instance, the

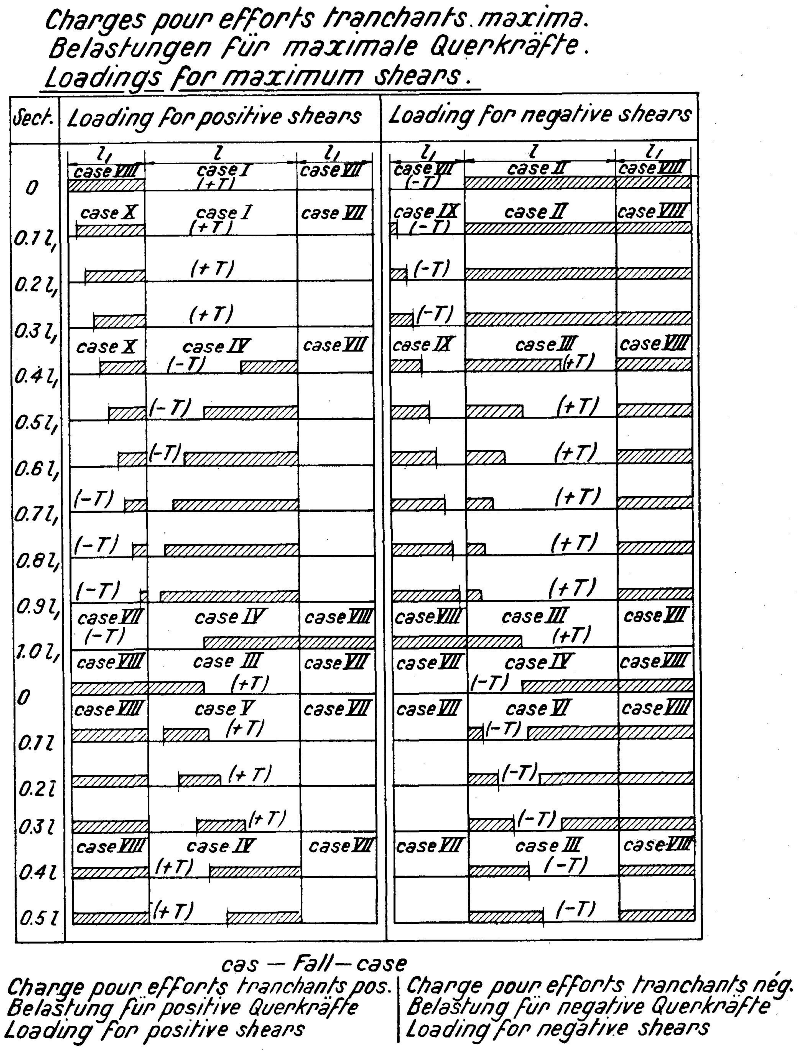


Fig. 3.

load condition shown in Fig. 2 for maximum positive moment at the 0.1 point of the main span is resolved into the elementary loading Cases VII, II, and VIII, the formulas for which are listed in the next Article.

The symbols ($+T$) and ($-T$) marked on the loading diagrams indicate whether a rise or fall in temperature should be assumed in conjunction with the indicated loading to yield the greatest resultant value of the stress sought.

The loading diagrams given in Fig. 2 for maximum moments at any points will also generally represent, with little or no modification, the loading placements for maximum deflections at the same points. An exception to this statement occurs for the mid-point of the main span whose maximum deflection is generally produced by a fully loaded main span.

In the case of structures of unusual geometrical proportions, the loading diagrams given in Figs. 2 and 3 may be subject to some modification. A preliminary analysis by the Elastic Theory will generally indicate the proper loading placements, as well as the trial values for the load-lengths. The ten component loading cases given in Art. 11 are fundamental, and are expected to cover all conditions arising in practical application.

11. Working Formulas for Primary Loading Cases.

By considering each span separately, the various loading conditions that are useful in design (as illustrated in Art. 10) may be resolved into ten primary loading cases: six for main-span conditions, and four for side-span conditions. These ten cases, in combination, cover all loading conditions of practical importance; and they are presented below in complementary pairs.

For each primary loading case, the pertinent working formulas are given in the accompanying charts for the following functions:

$F(p)$: a load term in the numerator of the H-formula, Eq. (35) or Eq. (70). The derivation of the formulas for $F(p)$ is explained and illustrated in Art. 6. (See Eqs. 37, 38, 39). The values of $F(p)$ contributed by the three spans are to be combined for substitution in either H-formula, Eq. (35) or (70). (The expressions for $F(p)$ are independent of continuity, and are the same as for two-hinged suspension bridges.)

$G(p)$: another load term in the numerator of the H-formula, Eq. (70); also required in the evaluation of $T_{1,2}$. The values of $G(p)$ contributed by the three spans are to be combined for substitution in the H-formula, Eq. (70). (The function $G(p)$ is introduced and explained in Art. 9; it is the load term due to continuity.)

C_1, C_2: the constants of integration, for substitution in the basic formulas (10), (12) and (13), for η, M and V at any point in the given loading segment. (The derivation of these formulas, and others, for C_1 and C_2 has been given in Art. 5.)

T_1, T_2: total bending moments at the towers, defined by Eqs. (15), and explained and evaluated in Art. 8. Both exact and approximate formulas for $T_{1,2}$ are listed. If the exact formulas are used, the contributions of all three spans are to be combined to give each total value of $T_{1,2}$: If the approximate formulas are used, the contributions of only the two contiguous spans are to be combined to give each total value of T: main span and left side span to give T_1, main span and right side span to give T_2. The resulting total values of T_1 and T_2, either exact or approximate, are for substitution in the formulas for C_1 and C_2, also for substitution in Eq. (35) for H. (T_1 and T_2 vanish in the non-continuous structure.)

M_1, M_2: continuous-beam bending moments at the towers, contributed by the indicated applied loading, with no suspender forces acting. (See Fig. 1 and Eqs. 1.) The formulas given here for M_1 and M_2 are derived from the Theorem of Three Moments. The contributions from the three spans are to be combined to obtain the total values of M_1 and M_2 (These formulas for M_1 and M_2 are given here for reference in the development of the theory, but are not required in the practical application unless it is desired to use the A and B functions.)

$A, B_{1,2}$: constants defined in Eqs. (44), (47) and (49), for substitution in the formulas for the deflection functions $\Delta\mu_f$, $\Delta\mu_1$, $\Delta\mu_2$, Δf, Δf_1, and Δf_2 (Eqs. 51 to 55). The value of A for the main-span loading case is to be used in conjunction with the values of B_1 and B_2 for the respective side-span loading cases when substituting in the formulas for the deflection functions. (The constants A and $B_{1,2}$ are introduced in Art. 7, and generalized formulas for them are given in Eqs. 46 and 50. The formulas for A and $B_{1,2}$ are given here only for reference in the development of the theory or for the direct computation of the deflection functions; they are not required in the practical stress analysis, since they have been eliminated in the evaluation of $T_{1,2}$.)

12. Effect of Temperature Variation with No Load on Spans.

This condition of the structure is covered by Cases I and VII, Art. 11. Since there is no load on the spans.

$$p = 0, \quad M_1 = M_2 = 0, \quad F(p) = 0, \quad G(p) = 0, \quad T_1 = T_2$$

Accordingly, Eq. (70) reduces to:

$$H_t = \mp \frac{\varrho c^2 EI \omega t L_t}{D'} \tag{72}$$

and Eq. (69) yields:

$$T_1 = T_2 = \frac{2Ht\left(\frac{1}{e_c} \cdot \frac{1}{\varrho c^2} + \frac{1}{e_{c_1}} \cdot \frac{ir}{\varrho_1 c_1^2}\right)}{(1 + ir)_c} = \frac{16Ht}{c^2 l} \cdot \frac{\Sigma'(n/e_c)}{(1 + ir)_c} \tag{73}$$

Eqs. (25) for the integration constants in the main span reduce to:

$$C_1 = -\frac{1}{(e^{cl} + 1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H_t}\right)$$

$$C_2 = C_1 e^{cl}$$

With these values, the general formula for moments (Eq. 12) reduces to:

$$M_t = -H_t\left(C_1 e^{cx} + C_2 e^{-cx} + \frac{1}{\varrho c^2}\right) \tag{74}$$

This give the moment curve (for temperature effect) in the main span. At mid-span $\left(x = \frac{l}{2}\right)$ it yields:

$$\text{Max. } M_t = -H_t\left(2C_1 e^{\frac{cl}{2}} + \frac{1}{\varrho c^2}\right) \tag{75}$$

Since H_t is negative for a rise in temperature, M_t is positive at mid-span (Eq. 75) and negative at the towers (Eq. 73). By Eq. (74), the temperature moment changes sign ($M_t = 0$) at an intermediate point x given by the equation:

$$\frac{(e^{cl} + 1)}{[e^{cx} + e^{c(l-x)}]} = \frac{T_{1,2}}{H_t} \cdot \varrho c^2 + 1 = \frac{2}{(1 + ir)_c}\left(\frac{1}{e_c} + \frac{1}{e_{c_1}} \cdot \frac{v}{r}\right) + 1 \tag{76}$$

For maximum $+M$ at any point between this section x and the symmetrical section $(l-x)$, highest temperature should be used. (Approximately $x = 0.13\,l$).

For any point in the side spans, Eqs. (26) yield:

$$C_1 = -\frac{1}{(e^{c_1 l_1}+1)} \cdot \frac{1}{\varrho_1 c_1^2} - \frac{1}{(e^{c_1 l_1} - e^{-c_1 l_1})} \cdot \frac{T_{1.2}}{H_t}$$

$$C_2 = -C_1 - \frac{1}{\varrho_1 c_1^2}$$

and Eq. (12) gives:

$$M_t = -H_t\left(C_1 e^{c_1 x_1} + C_2 e^{-c_1 x_1} + \frac{1}{\varrho_1 c_1^2}\right) \tag{77}$$

Eq. (77) gives the moment graph (for temperature effect) in the side span. At the free end ($x_1 = 0$), it yields $M_t = 0$. At the tower end ($x = l_1$), it yields $M_t = T_1 = T_2$. At the mid-point of the side-span $\left(x_1 = \frac{l_1}{2}\right)$, Eq. (77) yields:

$$M_t = -H_t\left(C_1 e^{\frac{c_1 l_1}{2}} + C_2 e^{-\frac{c_1 l_1}{2}} + \frac{1}{\varrho_1 c_1^2}\right)$$

This value is positive for a rise in temperature. In the side span, as in the main span, M_t is positive at mid-span and negative at the tower. By Eq. (77), the temperature moment changes sign ($M_t = 0$) at an intermediate point x_1 given by:

$$\frac{e^{c_1 l_1}+1}{e^{c_1 x_1}+1} = \frac{T_{1,2}}{H_t} \cdot \frac{\varrho_1 c_1^2}{(1 - e^{-c_1 l_1})} + 1 \tag{78}$$

(Approximately $x_1 = 0.72\ l_1$).

Eq. (76) determines two division points in the main span, and Eq. (78) determines a division point in each side span. For sections near the towers, between the adjacent division points, use lowest temperature for maximum $+M$; for all other sections of the main and side spans, use highest temperature for maximum $+M$. The exact locations of these division points will be modified somewhat when temperature effect is considered in conjunction with the live load, and the use of $+t$ or $-t$ for maximum $+M$ in the critical range near a division point will have to be determined by trial.

The signs of t to be used for maximum $+M$ and $-M$, respectively, at different sections of the spans, are indicated in Fig. 2.

A convenient rule, in practical application, is always to use highest temperature (in conjunction with the appropriate live load placement) to obtain the absolute maximum M (disregarding sign) at any point of any span, and lowest temperature (with the complementary live load placement) to obtain the maximum of opposite sign.

Following a procedure similar to that represented by Eqs. (74) to (78), temperature criteria for maximum $+V$ and $-V$ may be derived from the general formula for shears (Eq. 13). The temperature criteria for shears are indicated in Fig. 3.

The deflection constants A and $B_{1,2}$ (Cases I and VII) reduce to

$$A = \frac{H_t}{EIc^2}\left[\frac{1}{e_c}\left(\frac{1}{\varrho c^2} + \varepsilon f\right) - f\right]$$

$$B_1 = B_2 = \frac{H_t}{EIc^2}\left[\frac{1}{e_{c_1}}\left(\frac{1}{\varrho_1 c_1^2} + \frac{\varepsilon f}{2}\right) - f_1\right]$$

The temperature deflection at the mid-point of the main span is then obtained by substituting these constants in Eq. (54) for Δf, which, upon reduction, yields:

$$\text{Max. } \eta_t = (\Delta f)_t = \frac{H_t}{EIc^2}\left[\frac{1}{e_c}\left(\frac{1}{\varrho c^2} + \frac{T_{1,2}}{H_t}\right) - f\right] \tag{79}$$

The temperature deflection at the mid-point of the side span is given by substituting A and $B_{1,2}$ in Eq. (55) for $\Delta f_{1,2}$, which, upon reduction, yields:

$$(\Delta f_{1,2})_t = \frac{H_t}{EIc^2}\left[\frac{1}{e_{c_1}}\left(\frac{1}{\varrho_1 c_1^2} + \frac{T_{1,2}}{2H_t}\right) - f_1\right] \tag{80}$$

13. Simplified Formulas for Spans Fully Loaded or Unloaded.

Let p, p_1 and p_2 be the intensities of loading covering the main span, the left side span and the right side span, respectively. Then, by simply writing zero for one or more of these loading intensities, the fully unloaded condition of the corresponding span or spans is represented. The simple-beam moments at mid-span will be:

$$M_f = \frac{p\,l^2}{8}, \qquad M_{f_1} = \frac{p_1 l_1^2}{8}, \qquad M_{f_2} = \frac{p_2 l_2^2}{8}$$

Introduce the following abbreviations for the three respective spans:

$$R = M_f - H\cdot f, \qquad R_1 = M_{f_1} - H\cdot f_1, \qquad R_2 = M_{f_2} - H\cdot f_2 \tag{81}$$

so that R, R_1 and R_2 denote the bending moments at mid-spans, without continuity and deflection corrections; also:

$$S = \left(\frac{p}{c^2} - \frac{H}{\varrho c^2}\right)\cdot\frac{1}{e_c}, \quad S_1 = \left(\frac{p_1}{c_1^2} - \frac{H}{\varrho_1 c_1^2}\right)\cdot\frac{1}{e_{c_1}}, \quad S_2 = \left(\frac{p_2}{c_1^2} - \frac{H}{\varrho_1 c_1^2}\right)\cdot\frac{1}{e_{c_1}} \tag{82}$$

These respective functions are inter-related:

$$S = \frac{8}{c^2 l^2}\cdot\frac{1}{e_c}R, \qquad S_1 = \frac{8}{c_1^2 l_1^2}\cdot\frac{1}{e_{c_1}}\cdot R_1, \qquad S_2 = \frac{8}{c_1^2 l_2^2}\cdot\frac{1}{e_{c_1}}\cdot R_2$$

The continuous-beam moments, M_1 and M_2, for this condition of loading, are given by the relations:

$$\frac{M_1 + M_2}{2} = -\frac{2M_f + ir(M_{f_1} + M_{f_2})}{3 + 2ir} \tag{83a}$$

$$\frac{M_1 - M_2}{2} = -\frac{ir}{1+2ir}(M_{f_1} - M_{f_2}) = -\frac{ir}{1+2ir}(R_1 - R_2) \tag{83b}$$

$$\frac{M_1 + M_2}{2} + H\cdot\varepsilon f = -\frac{2R + ir(R_1 + R_2)}{3 + 2ir} \tag{83c}$$

The formulas for the total values of T_1 and T_2 (Cases II and VIII, Art. 11) become:

$$T_{1,2} = -\frac{2S + ir(S_1 + S_2)}{(1+ir)_c} \mp \frac{ir(p_1 - p_2)}{(1+ir)_{c_1}\cdot e_{c_1}\cdot c_1^2} \tag{84a}$$

$$\frac{T_1 + T_2}{2} = -\frac{2S + ir(S_1 + S_2)}{(1+ir)_c} \tag{84b}$$

For the symmetrical case, with both side spans equally loaded or unloaded and the main span either fully loaded or unloaded, $p_1 = p_2$ and Eq. (84 a) reduces to:

$$T_1 = T_2 = -\frac{2S + ir(S_1 + S_2)}{(1 + ir)_c} \qquad (84')$$

With the foregoing abbreviations, the formulas for A and $B_{1,2}$ for the same Cases reduce to:

$$EIc^2 \cdot A = R - S + \left(\frac{M_1 + M_2}{2} + H\varepsilon f\right) \cdot \frac{1}{e_c}$$

$$EIc^2 B_{1,2} = R_{1,2} - S_{1,2} + \left(\frac{M_{1,2}}{2} + H \cdot \frac{\varepsilon f}{2}\right) \cdot \frac{1}{e_{c_1}}$$

Substituting these values of A and $B_{1,2}$ with the values of M_1 and M_2 from Eqs. (83) in the equations (44 and 51) for Δf, also in the equations (47, 49, and 53) for $\Delta f_{1,2}$, and reducing, we obtain the following simplified formulas for mid-span deflections:

$$EIc^2 \cdot \Delta f = R - S + \frac{1}{e_c} \frac{(T_1 + T_2)}{2} \qquad (85a)$$

$$EIc^2 \cdot \Delta f_{1,2} = R_{1,2} - S_{1,2} + \frac{1}{e_{c_1}} \cdot \frac{T_{1,2}}{2} \qquad (85b)$$

Eqs. (67) and (68), with reference to Eqs. (83), yield:

$$EIc^2 \cdot \Delta \mu_1 = T_1 + \frac{2R + ir(R_1 + R_2)}{3 + 2ir} + \frac{ir(R_1 - R_2)}{1 + 2ir} \qquad (86a)$$

$$EIc^2 \cdot \Delta \mu_f = \frac{T_1 + T_2}{2} + \frac{2R + ir(R_1 + R_2)}{3 + 2ir} \qquad (86b)$$

With the aid of Eq. (85 a), Eq. (3 a) reduces to the following expression for total resultant moment M at the mid-point $\left(x = \frac{l}{2}\right)$ of the main span:

$$M_{\frac{l}{2}} = S + \left(1 - \frac{1}{e_c}\right) \frac{(T_1 + T_2)}{H} = \text{(approx.) } S \qquad (87a)$$

(The approximate value is only about 3 or 4 percent too small.) Hence S represents, very closely, t h e t o t a l b e n d i n g m o m e n t a t t h e m i d d l e o f t h e m a i n s p a n. Similarly, Eqs. (3 b) and (85 b) yield the following expression for the resultant moment M at the mid-point $\left(x_1 = \frac{l_1}{2}\right)$ of the side span:

$$M_{\frac{l_1}{2}} = S_1 + \left(1 - \frac{1}{e_{c_1}}\right) \frac{T_1}{2} = \text{(approx.) } S_1 \qquad (87b)$$

indicating that S_1 represents, approximately, the total bending moment at the middle of the side span. (This approximate value may be about 10 percent too large.)

Accordingly, the functions R, S, and T, running through the formulas of this Article, represent moments at mid-spans and at towers, as above defined. (See Fig. 4.)

Eqs. (87) are also obtained by substituting Eqs. (86) in the fundamental Equation (3).

For any condition of loading, the general equation for moments (Eq. 12) may be written in the form:

$$M = e_c \cdot S - H(C_1 e^{cx} + C_2 e^{-cx}) \qquad (88$$

For the main span fully loaded or unloaded (Case I or II), the constants of integration become:

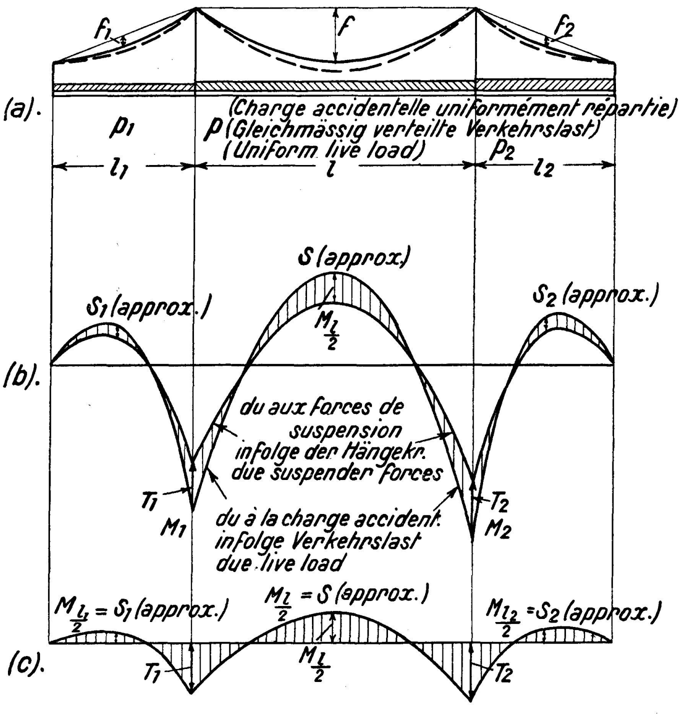
$$C_1 = \frac{1}{(e^{cl}+1)}\,\frac{(e_c \cdot S - T_1)}{H} + \frac{1}{(e^{cl}-e^{-cl})}\,\frac{(T_1 - T_2)}{H} = (\text{approx.})\,\frac{1}{(e^{cl}+1)}\,\frac{(e_c \cdot S - T_2)}{H}$$

$$C_2 = -C_1 + \frac{e_c \cdot S - T_1}{H}$$

and those for the side span fully loaded or unloaded (Case VII or VIII) become:

$$C_1 = \frac{1}{(e^{c_1 l_1}+1)}\,\frac{(e_{c_1} \cdot S_1)}{H} - \frac{1}{(e^{c_1 l_1}-e^{-c_1 l_1})} \cdot \frac{T_1}{H} = (\text{approx.})\,\frac{1}{(e^{c_1 l_1}+1)}\,\frac{(e_{c_1} \cdot S_1 - T_1)}{H}$$

$$C_2 = -C_1 + \frac{e_{c_1} \cdot S_1}{H}$$



Moments des travées chargées totalement, montrant les variations de S et T.
Momente in den vollbelasteten Feldern, den Verlauf von S und T darstellend.
Moments in spans fully loadet showing significance of S and T.

Fig. 4.

(The approximate expression for C_1 in the main span is exact for symmetrical loading and correct within a fraction of 1 percent for unsymmetrical loading. The approximate expression for C_1 in the side span yields a value that may be about 3 percent too small.)

Substitution of the foregoing integration constants in the general formula for moments, Eq. (88), applied to the mid-span points $x = \frac{l}{2}$ and $x_1 = \frac{l_1}{2}$

yields expressions identical with Eqs. (87). (This is another check on the mutual consistency of all the formulas involved.)

With the integration constants given above, the general formula for shears (Eq. 13) yields the following values:

At $x = \frac{l}{2}$,

$$V_{\frac{l}{2}} = -\frac{c}{\left(e^{\frac{cl}{2}} - e^{-\frac{cl}{2}}\right)}(T_1 - T_2) \tag{89a}$$

At $x_1 = \frac{l_1}{2}$,

$$V_{\frac{l_1}{2}} = \frac{c_1}{\left(e^{\frac{c_1 l_1}{2}} - e^{-\frac{c_1 l_1}{2}}\right)} \cdot T_1 \tag{89b}$$

At the ends of the main span ($x = 0$ or $x = l$),

$$V_{0,l} = \pm c \cdot \frac{(e^{cl} - 1)}{(e^{cl} + 1)}\left(e_c \cdot S - \frac{T_1 + T_2}{2}\right) - c \frac{(e^{cl} + 1)}{(e^{cl} - 1)} \cdot \frac{(T_1 - T_2)}{2} \tag{89c}$$

which yields, approximately,

$$V_{0,l} = \pm c(e_c \cdot S - T_{1,2})$$

(This approximate value is correct within a fraction of 1 percent).
At the ends of the side span ($x_1 = 0$ or $x_1 = l_1$),

$$V_{0,l_1} = \pm c_1 \frac{(e^{c_1 l_1} - 1)}{(e^{c_1 l_1} + 1)}\left(e_{c_1} \cdot S_{1,2} - \frac{T_{1,2}}{2}\right) + c_1 \frac{(e^{c_1 l_1} + 1)}{(e^{c_1 l_1} - 1)} \cdot \frac{T_{1,2}}{2} \tag{89d}$$

which yields, approximately,

$$V_0 = c_1 \frac{(e^{c_1 l_1} - 1)}{(e^{c_1 l_1} + 1)}(e_{c_1} \cdot S_{1,2}), \qquad \text{at the free end,}$$

and

$$V_{l_1} = -c_1 \left[\frac{(e^{c_1 l_1} - 1)}{(c^{c_1 l_1} + 1)} \cdot e_{c_1} \cdot S_{1,2} - T_{1,2}\right],$$

at the tower end of the side span. (The approximate value of V_0 is exact for symmetrical loading and may be 4 percent too large for unequally loaded side spans. The approximate value of V_{l_1} may be 1 percent too large.)

The total truss reaction at the tower is obtained by combining Eqs. (89 c) and (89 d), which yields, approximately,

$$\text{Truss Reaction} = \left[c \cdot e_c \cdot S + c_1 \cdot e_{c_1} \frac{(e^{c_1 l_1} - 1)}{(e^{c_1 l_1} + 1)} \cdot S_{1,2}\right] - (c + c_1)\, T_{1,2} \tag{90}$$

(The approximate value given by Eq. 90 may be 2 percent too large in the case of unequally loaded side spans, but it is practically exact in the case of symmetrical loading.)

Though the maximum value of the truss reaction at the tower is not given by Eq. (90) which is written for fully loaded or unloaded spans, that equation indicates that for a maximum truss reaction a combination of the greatest values of S (obtained when H is small) and of the greatest negative value of T is required. The loading condition which will generally answer these requirements and produce the maximum truss reaction is the same as that which produces maximum negative tower moment. This loading condition is shown in Fig. 2; it consists of the adjacent side span fully loaded and the main span partially loaded near the tower at highest temperature.

Since the truss reaction at the tower is obtained by combining Eqs. (89 c) and (89 d), it follows that the maximum value of the shear at the tower, in the main span and in the side span, is also obtained under the same loading condition which produces maximum negative tower moment.

For substitution in the H-formula (Eq. 35), the value of $F(p)$, for the spans fully loaded or unloaded, is given by Eq. (39); and the value of $\frac{T_1 + T_2}{2}$ is given by Eq. (84 b).

It may be preferred to use the reduced working formula for H (Eq. 70). The total value of $G(p)$ to be substituted in Eq. (70) is given by Cases II and VIII, Art. 11, as:

$$G(p) = -\frac{1}{(1+ir)_c}\left[\frac{2}{e_c}\cdot\frac{p}{c^2} + \frac{ir}{e_{c_1}}\frac{(p_1+p_2)}{c_1^2}\right]$$

The contribution due to temperature variation may be included without any change in the formulas of this Article. All that is necessary is to include the temperature term in evaluating H (Eq. 35 or Eq. 70), which enters the formulas of this article in Eqs. (81) and (82).

14. Application to Multiple-Span Suspension Bridges.

The Generalized Deflection Theory developed in Articles 2 to 6, inclusive, is directly applicable to multiple-span suspension bridges, either continuous or hinged at the towers.

Deflections, moments, and shears in multiple-span suspension bridges are accurately expressed by the general formulas in Art. 3 (Fundamental Equations). All of the general equations in that Article (Eqs. 1—4, 7—15, 3 a and 3 b) remain valid without modification. It is only the special expression for the coefficient of continuity ε (given by Eqs. 5 and 6 for the special case of symmetrical three-span bridges) that needs to be re-written for cases of more than three spans. The values of ε for any case may be written with the aid of the Theorem of Three Moments.

For the case of a symmetrical four-span continuous suspension bridge (two equal main spans and two equal end-spans), the expressions for ε will be:

(At the two side towers:) $$\varepsilon_1 = \frac{2(1+2irv)}{3+4ir} \qquad (91\,a)$$

(At the middle tower:) $$\varepsilon_2 = \frac{2(1+2ir-irv)}{3+4ir} \qquad (91\,b)$$

and the corresponding expressions for εf will be:

$$\mu_1 = \varepsilon_1 f = \frac{2(f+2irf_1)}{3+4ir} \qquad (92\,a)$$

$$\mu_2 = \varepsilon_2 f = \frac{2(f+2irf-irf_1)}{3+4ir} \qquad (92\,b)$$

For the case of a symmetrical five-span continuous suspension bridge (spans, l_2, l_1, l, l_1, l_2), the expressions for εf will be:

(At the two outside towers:)

$$\mu_1 = \varepsilon_1 f = \frac{2\,[f_1(3+i_1 r_1) + f_2(2i_2 r_2 + 3i_3 r_3) - f]}{3(2+i_1 r_1) + 2(2i_2 r_2 + 3i_3 r_3)} \qquad (93a)$$

(At the two inside towers:)

$$\mu_2 = \varepsilon_2 f = \frac{2\left[f_1 (i_1 r_1 + 2 i_2 r_2) + 2f(1 + i_3 r_3) - f_2 (i_2 r_2)\right]}{3(2 + i_1 r_1) + 2(2 i_2 r_2 + 3 i_3 r_3)} \tag{93b}$$

where

$$i_1 r_1 = \frac{I}{I_1} \cdot \frac{l_1}{l}$$

$$i_2 r_2 = \frac{I}{I_2} \cdot \frac{l_2}{l}$$

$$i_3 r_3 = \frac{I_1}{I_2} \cdot \frac{l_2}{l_1} = \frac{i_2 r_2}{i_1 r_1}$$

For other numbers and proportions of spans, the appropriate expressions for ε and μ may be similarly written.

If the multiple-span suspension bridge is non-continuous at the towers, the values of ε and μ are, of course, zero; and the terms containing these functions and their derivatives vanish from all formulas.

In Art. 4 (Derivation of the Basic Equation for H), all of the formulas (Eqs. 16—24) remain valid without change, except for a slight modification in writing the two continuity terms in Eq. (21) for H. The continuity term in the numerator, $E I c^2 \cdot \Delta \mu_f \cdot \Sigma' (K l)$, takes the more generalized form: $E I c^2 \cdot \Sigma (K l \cdot \Delta \mu_f)$, where $\Delta \mu_f$ is the mean of the values of $\Delta \mu$ at the two ends of any span; and the continuity term in the denominator, $\varepsilon f \cdot \Sigma' (K l)$, takes the more generalized form: $\Sigma (K l \cdot \mu_f)$, where μ_f is the mean of the values of $\mu = \varepsilon f$ at the two ends of any span. (At the free ends of the continuous structure, μ and $\Delta \mu$ are zero.) With only the two continuity terms thus modified for greater generality, Eq. (21) takes the following generalized form:

$$H = \frac{\Sigma K \int_0^l \left(M' - \frac{p_x}{c^2}\right) dx - \varrho c^2 E I \omega t L_t + E I c^2 \cdot \Sigma (K l \cdot \Delta \mu_f)}{\Sigma K \left[- \int_0^l (C_1 e^{cx} + C_2 e^{-cx})\, dx + \tfrac{2}{3} f l - \frac{l}{\varrho c^2} \right] + \varrho c^2 \frac{E}{E_c} \cdot \frac{I}{A_0} \cdot L_s - \Sigma (K l \cdot \mu_f)} \tag{94}$$

Eq. (94) is the generalized form of the basic H-equation, applicable to multiple spans as well as to the common three-span type. If the spans are non-continuous, the terms containing μ_f and $\Delta \mu_f$ vanish.

The expressions for L_s and L_t given by Eqs. (18) and (19) are unchanged for the generalized case of any number of spans.

All of Art. 5 (Evaluation of the Integration Constants) remains valid for the generalized case, without any modification. Eqs. (25) to (31), inclusive, and all of the formulas for C_1 and C_2 tabulated in Art. 11, are applicable, without any change, to suspension bridges having any number and proportions of spans. (Formulas established for the "side spans" of the three-span bridge remain valid for the "end spans" of the multiple-span structure.)

In Art. 6 (Derivation of Working Formulas for H), covering Eqs. (32) to (39), only the continuity terms (identified by Σ') occurring in Eqs. (32), (33), and (35) need to be rewritten for complete generality. In Eq. (32), the continuity term $\frac{(M_1 + M_2)}{2} \cdot \Sigma'(K l)$ takes the generalized form $\Sigma\left(K l \cdot \frac{M_1 + M_2}{2}\right)$, where M_1 and M_2 are the continuous beam bending moments at the two ends of any span. In Eq. (33), the continuity term

$$\Sigma'\left[K \cdot \frac{2}{c} \cdot \frac{(e^{cl}-1)}{(e^{cl}+1)} \cdot \frac{(T_1+T_2)}{2H}\right]$$

takes the generalized form

$$\Sigma\left[K \cdot \frac{2}{c} \cdot \frac{(e^{cl}-1)}{(e^{cl}+1)} \cdot \frac{(T_1+T_2)}{2H}\right]$$

where T_1 and T_2 are the total resultant bending moments at the two ends of any span. (For any number of spans $M_{1,2}$ and $T_{1,2}$ are zero at the free ends of the structure.) In the H-formula (Eq. 35), the continuity term $\frac{(T_1+T_2)}{2} \cdot \Sigma'(KL_c)$ correspondingly takes the generalized form $\Sigma\left(KL_c \cdot \frac{T_1+T_2}{2}\right)$ and the working formula for H becomes:

$$H + \frac{F(p) + \Sigma\left(KL_c \cdot \frac{T_1+T_2}{2}\right) \mp \varrho c^2 EI \omega t L_t}{D} \qquad (95)$$

Eq. (36) for D remains unchanged; and all of the expressions for $F(p)$ (including Eqs. 37 to 39 and all of the formulas for $F(p)$ tabulated in Art. 11) remain valid for the generalized case, without modification.

Articles 7 to 13 are not directly applicable to the general case of multiple-span suspension bridges, since the formulas there developed are based on the special values of ε and μ given by Eqs. (5) and (6) for the symmetrical three-span suspension bridge. For symmetrical continuous bridges of four or five spans, working formulas paralleling those of Articles 7 to 13 may be developed from the respective expressions for ε and μ given in Eqs. (91) and (92), or (93). A generalized expression for ε or μ would be required for developing the corresponding working formulas for the general case of any number of spans. However, the formulas in Articles 2 to 6, inclusive, with the slight generalizing modifications noted in this Article, suffice for the complete analysis of multiple-span suspension bridges. The formulas in the subsequent Articles are a convenience for continuous spans, but not a necessity. Without them, the interdependent functions are determined by successive substitution. These interdependent functions vanish when the spans are non-continuous, and Articles 7 to 13 then lose their significance.

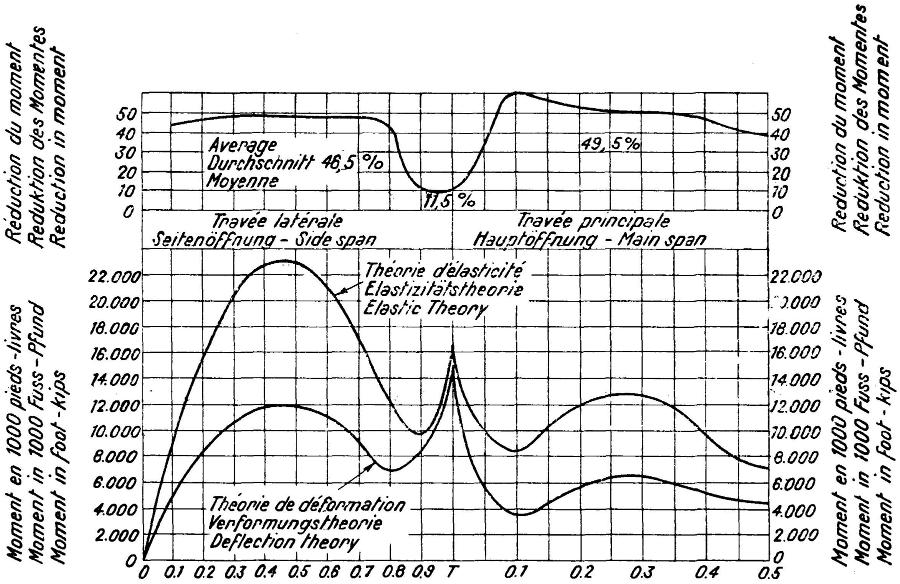
For non-continuous multiple-span suspension bridges, the formulas of Articles 2 to 6 are completely sufficient, without any modification.

15. Practical Application to Continuous Spans.

In order to test the practical applicability of the Generalized Deflection Theory and, at the same time, to establish data for comparisons of types of structure and theories of analysis, the theory and formulas developed in this paper have been applied to the analysis of a three-span continuous suspension bridge.

Design I. The structure selected for the numerical application of the theory has an 800-ft. main span and two 400-ft. side spans, and had been previously designed as a two-hinged suspension bridge. The trusses have a constant depth of 12 ft. throughout and are spaced 45 ft. center to center. For the first comparative design, herein referred to as Design I, the same moments of inertia ($I = 1960$ in.^{2}ft.^{2}, $I_1 = 2420$ in.^{2}ft.^{2}) were assumed as in the two-hinged design, in order to ascertain comparative rigidity under conditions of equal economy.

C o m p a r i s o n   w i t h   E l a s t i c   T h e o r y. The stresses in the continuous spans were first computed by the Elastic Theory. This preliminary analysis incidentally yielded the approximate loading conditions to be used as a guide for assuming trial load-lengths in the more exact analysis. The stresses were then computed more accurately by the Generalized Deflection Theory, using the formulas and procedure developed in this paper. The maximum bending moments yielded for Design I by the two respective theories are plotted in Fig. 5. The percentage reductions obtained by the application of the Deflection Theory are also plotted. These reductions in ma-



Comparaison des moments maximum des poutres de raidissement, calculés d'après la théorie d'élasticité et la théorie de déformation, et pourcentage de la réduction des moments:

Vergleich der maximalen Momente in den Versteifungsfachwerkträgern, berechnet nach der Elastizitäts- und nach der Verformungstheorie, nebst Prozentsatz der Verminderung der Momente.

Comparison of maximum moments in stiffening trusses by elastic and deflection theories with percentages of reduction in moment.

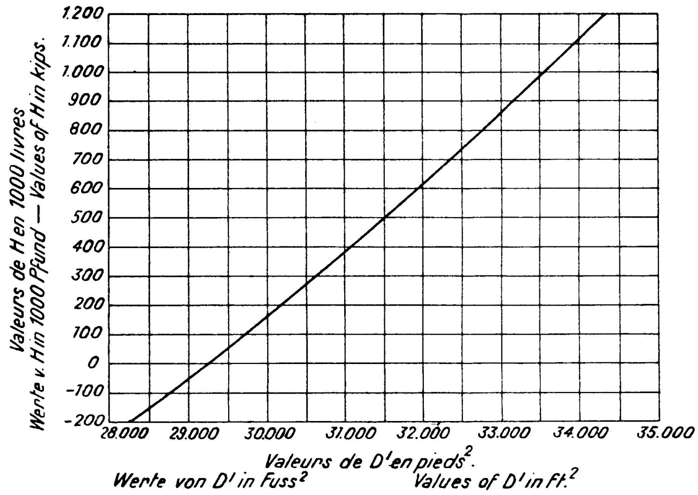
Fig. 5.

ximum bending moments range from 10 percent at the tower to 60 percent in the center span. Except for a comparatively short stretch close to the tower (including about $^1/_8$ of the side span and $^1/_{20}$ of the main span) where the average reduction effected by the Deflection Theory is only about 12 percent, the reductions are approximately 50 percent in the main span and 45 percent in the side span. A comparison of the total areas under the bending moment graphs for the two respective theories, as plotted in Fig. 5, shows that the reduction or saving yielded by application of the Deflection Theory to a continuous stiffening truss is 45.5 percent as an average for the entire length of the structure.

The percentages of reduction from the Elastic Theory in the case of continuous spans are closely comparable to the reduction percentages previously established for two-hinged suspension bridges. The direct application of any approximate factor of reduction, however, is modified in the continuous structure by the variation of reduction ratio near the towers.

Comparison of Deflections. The maximum deflections in the two-hinged design, computed by the Deflection Theory (using Eqs. 85 with $T_1 = T_2 = 0$) were 5.16 ft. in the main span and 3.30 ft. in the side spans. In the continuous structure of Design I (having the same assumed moments of inertia to represent equal economy), the maximum deflections were also computed by Eqs. (85 a) and (85 b), and were found to be 5.24 ft. in the main span and only 2.92 ft. in the side spans. The comparison of maximum deflections shows a reduction of 11.5 percent in the side spans of the continuous design and an increase of 1.5 percent in the center span, or an average reduction of 5 percent for the entire structure. Hence, for designs of equal economy, the continuous structure of 800-ft. main span is, on the whole, about 5 percent more rigid than the two-hinged type.

Design II. In order to ascertain comparative economy for equal rigidity, the assumed moments of inertia for the continuous structure were



Courbe pour la détermination de D'
Kurve zur Bestimmung von D'
Graph for determination of D'

Fig. 6.

modified for a second analysis. Following the indications of the foregoing comparison of deflections yielded by Design I, the new assumptions for Design II were $I = I_1 = 1960$ in.$_2$ ft.$_2$ (as compared with the values of $I =$ 1960 and $I_1 = 2420$ in the two-hinged design). Typical computations for Design II, illustrating the application of the Generalized Deflection Theory to a three-span continuous suspension bridge, are herewith presented.

Generel Data-Calculation of Constants. The following are the dimensional constants:

Main Span: $l = 64$ panels $= 800$ ft., $f = 84$ ft., $n = 0.105$
Side Spans: $l_1 = 32$ panels $= 400$ ft., $f_1 = 21$ ft., $n_1 = 0.0525$
$\sec \alpha_1 = 1.03484$ $\quad \varrho = \varrho_1 = 952.381$ $\quad K_1 = 1.00$
$A = 87.8$ in.^{2} $\quad r = 0.5$ $\quad \nu = 0.25$

The following are the loading constants (all values per cable):
Dead Load: $w = 3850$ lb./ft. Live Load: $p = 1300$ lb./ft.

$$H_w = \frac{w\, l^2}{8 f} = 3667 \text{ kips.}$$

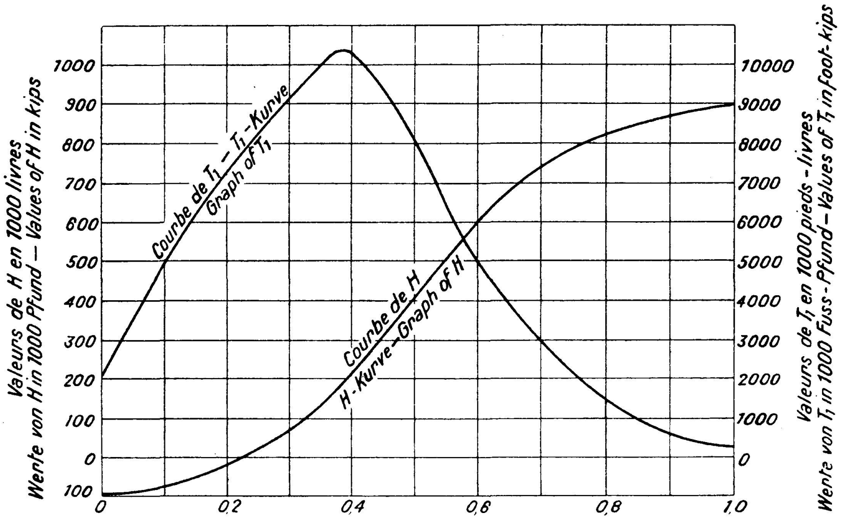
Temperature: $t = \pm 60^{\circ}$ F., $E = 29{,}000{,}000$, $\omega = .0000065$,
$E \omega t = 11{,}310$ lb. per sq. in.

By Formulas (18): $L_s = 2075$, $L_t = 1998$, ($E_c = 25{,}000{,}000$).

The truss constants are as follows:

Main Span: $I = 1960$ in.^{2} ft.^{2}; $EI = 56{,}840{,}000$ ft.^{2} kips
Side Span: $I_1 = 1960$ in.^{2} ft.^{2}; $EI_1 = 56{,}840{,}000$ ft.^{2} kips
$i = 1.000$

Calculation of Values of D'. Preparatory to obtaining values of H for various conditions of loading, the values of D', the denominator of the



Partie chargée de la travée
Belasteter Teil der Oeffnung — Portion of span loaded.

Courbes des valeurs H et T_1 pour une charge répartie uniformément, avançant sur la travée principale et pour la plus haute température.
H und T_1 – Kurven für vorwärts schreitende gleichmässig verteilte Last in der Haupt-Oeffnung bei höchster Temperatur.
Graphs of H and T_1 for advancing uniform load in main span at highest temperature.

Fig. 7.

formula for H, were calculated by Eqs. (71) for different values of H from —100 kips to +1200 kips. These computations are made by a systematic tabular method which embraces the step-by-step numerical operations. The principal constants and the values of D' for the different values of H are tabulated below:

H (kips)	c	c_1	e^{cl}	$e^{c_1 l_1}$	D'
- 100	0.007921	0.007921	566	23.77	28742
0	0.008032	0.008032	617	24.85	29223
100	0.008141	0.008141	673	25.95	29683
200	0.008248	0.008248	734	27.09	30150
300	0.008354	0.008354	799	28.26	30602
400	0.008458	0.008458	869	29.47	31048
600	0.008664	0.008664	1024	32.00	31908
800	0.008865	0.008865	1202	34.67	32742
1000	0.009061	0.009061	1407	37.50	33544
1200	0.009253	0.009253	1640	40.50	34321

For convenience of reference and interpolation, the values of D' are plotted against H in Fig. 6.

Values of H for Advancing Uniform Load. The values of H and T_1 for certain lengths of a continuous advancing uniform live load ($p = 1300$ lb. per ft.) in the main span, at highest temperature and with no load in the side spans, are necessary in the computation of the maximum positive moments at several points in the main span. By use of the formulas of Cases III and VII the values of H and T_1 are calculated for the above-described loading for successive load-lengths varying by 0.1 of the span. A trial value of H is usually first assumed for which D' and the other quantities have already been found in the preparation of the D' curve, and H is then computed, using these quantities. From the value thus obtained, a second trial H is selected from which a corrected value of H is calculated, employing a value of D' taken from the graph of Fig. 6. The value of H thus obtained in the second trial is generally in close agreement with that assumed. The operations and results are indicated in condensed form in the following tabulation, and the values of H and T_1 thus obtained are plotted in Fig. 7.

Load k/l	Trial H (kips)	D'	Calculated H	T_1 (ft.-kips)
0	-100	28742	-92.1	- 2015
0.2	- 30	29078	-30.1	- 7315
0.4	220	30241	217	-10267
0.6	594	31908	595	- 4997
0.8	824	32822	836	- 1229
1.0	891	33108	891	- 247.5

Maximum Positive Moments in Main Span

Maximum moments are positive for Points $\frac{x}{l} = 0.2$ to 0.5 and negative for Points $\frac{x}{l} = 0$ and 0.1.

Maximum positive moments for Points $\frac{x}{l} = 0.2$, 0.3, and 0.4 are calculated by Eq. (12) for the load condition of Case III (as indicated in Fig. 2). For each section x, M is calculated for three or four different trial load-lengths k, until the maximum value of M for the section is determined. The values of H and T_1 for each load-length are taken from the H and T_1 graphs, Fig. 7. The tabulation of governing quantities, in abbreviated form, is as follows:

Section: x/l	0.2	0.3	0.4
Load: k/l (assumed)	0.41	0.41	0.53
H (from Fig. 7)	238.6	238.6	467.0
c (by Eq. 9a)	0.00829	0.00829	0.00853
T_1 (from Fig. 7)	-10,046	-10,046	- 7180
$T_1 - T_2$ (by Formula, Case III)	-13,310	-13,310	-14270
$C_1 H$ (by Formula, Casa III)	617.26	617.26	225.4
$C_2 H$ (by Formula, Case III)	24959	24959	18095
e^{cx}	3.768	3.768	15.35
Max. M (by Eq 12)	+ 6590	+ 7610	+ 6501

Maximum Positive Moment at Mid-Span.

Maximum positive moment at the center of the main span, as indicated in Fig. 2, is produced when the central portion of the span is symmetrically loaded with a uniform load for a length m, at highest temperature and with no load on the side spans. In order to find the value of m for which the moment at this point is a maximum, it is necessary to assume different trial values of m and calculate the corresponding values of M by use of the formulas of Case V. Before thus computing each M, however, it is also necessary to obtain, by trial, the value of H corresponding to the assumed value of m. The final quantities in condensed form, are:

m/l (assumed)	0.330
$k = z$	0.335
H (assumed trial value)	488.6 kips
c (by Eq. 9a)	0.00855
e^{cl}	934.6
e^{ck}	9.9
e^{cm}	9.6
D' (by Fig. 6)	31425
H (by Eq. 70)	487.6 kips
T_1 (by formulas, Cases V and VII)	5130 ft.-kips
$C_1 H$ (by formula, Case V)	82.13
$C_2 H$ (by formula, Case V)	76688
$e^{cx} = e^{\frac{cl}{2}}$	30.57
Max. M (by Eq. 12)	5762 ft.-kips

Maximum negative Moment in Main Span Near Tower.

The maximum negative moment at the point $\frac{x}{l} = 0.1$ occurs when the adjacent side span is fully loaded at highest temperature and the other two spans unloaded. For this condition of loading, H is determined by trial, using the formula of Case VIII. The value of M is then calculated by the formulas of Cases I, VII, and VIII, and Eq. (12). A summary of the principal resulting values is as follows:

H (final value by Eq. 70)	−60.5 kips
c (by Eq. 9a)	0.007965
e^{cl}	585.31
e^{cx}	1.89
T_1 (by formulas Cases I and VIII)	−8356
$T_1 - T_2$ (by formulas Cases VII and VIII)	−7030
$C_1 H$ (by formula Case I)	3.94
$C_2 H$ (by formula Casa I)	9353
Max. M (by Eq. 12)	−3957 ft.-kips

Maximum Negative Moment at Tower.

To obtain the maximum negative moment at the tower, it is necessary to load fully the adjacent side span and a portion of the main span next to the tower at highest temperature as shown in Fig. 2. The main span load-length for which the tower moment is a maximum must be determined, by the formulas of Cases III and VIII, as previously outlined for the computation of maximum moment at mid-span. This computation is briefly summarized in the following tabulation:

H (trial value)	200 kips
c	0.008248
e^{cl}	733.84
$e^{c_1 l_1}$	27.09
k	0.365
e^{ck}	11.12
D' (from Fig. 6)	30150
H (by Eq. 70)	197.8 kips
T_1 (by formulas, Cases III and VIII)	−16,387 ft.-kips

Maximum Moments in Side Span.

Maximum moments are positive for sections $\frac{x_1}{l_1} = 0.1$ to 0.7 and negative for sections $\frac{x_1}{l_1} = 0.8$ to 1.0.

The loading condition producing maximum positive moments at sections $\frac{x_1}{l_1} = 0.1$ to 0.7 is shown in Fig. 2 to be the same as that for negative moment at point $\frac{x}{l} = 0.1$ in the main span, the side span fully loaded at highest temperature with no load on the other two spans. The computation of H and T_1 for this loading with that of the moment at mid-span ($x_1 = 0.5\, l_1$) are indicated below; the constants of integration are calculated by the formulas of Case VIII:

H (trial value)	−60.5 kips
$c_1 = c$	0.007965
$e^{c_1 l_1}$	24.193
D' (from Fig. 6)	28932
H (by Eq. 70)	−60.5 kips
T_1 (by formulas, Cases I and VIII)	−8356
$C_1 H$ (by formula, Case VIII)	1199.1
$C_2 H$ (by formula, Case VIII)	20293
M (by Eq. 12)	+11468.5 ft.-kips

Maximum Negative Moment in Side Span Near Tower.

The maximum negative moment at point $\frac{x_1}{l_1} = 0.9$ occurs under a partial loading of the side span and main span, at highest temperature, as shown in Fig. 2. In obtaining the maximum value of this moment, it must be computed by use of the formulas of Cases III, VII, and IX for several trial load-lengths in both spans. The computation of the maximum value of this moment with the corresponding values of k/l and $\frac{m_1}{l_1}$ is given below:

k/l (assumed)	0.39	0.365
m_1/l_1 (assumed)	0.65	0.65
H (trial value)	240 kips	200 kips
$c = c_1$	0.00829	0.00825
e^{cl}	759.0	733.8
$e^{c_1 l_1}$	27.55	27.09
e^{ck}	13.3	11.12
$e^{c_1 m_1}$	8.61	8.52
D' (from Fig. 6)	30332	30150

H	(by Eq. 70)	240.7 kips	198.5 kips
T_1	(by formulas, Cases III and IX)	−13448	−13593
$C_1 H$	(by formula, Case IX)	440.0	479.0
$C_2 H$	(by formula, Case IX)	−67720	−66940
$e^{c_1 x_1}$		19.8	19.5
Max. M	(by Eq. 12)	−8970 ft.-kips	−8980 ft.-kips

The loading condition for maximum negative moment at point $\frac{x_1}{l_1} = 0.8$ in the side span is shown in Fig. 2 to be the same as for positive moments in the main span. This moment is computed in the same manner as for maximum positive moments at sections $\frac{x}{l} = 0.2$, 0.3, and 0.4 in the main span, except for the use in this computation of the integration constants of Case VII.

Minimum Moments. — The loading conditions under which minimum moments are produced in the trusses are shown in Fig. 2. The values of these moments are obtained in the same general manner as outlined for the calculations of maximum moments, and usually occur at lowest temperature in combination with the above live loadings.

Maximum Shears. — The maximum positive and negative shears are calculated for the loading conditions indicated in Fig. 3. The method of procedure is the same as that for the calculation of maximum moments, involving the assumption of successive trial values of H and trial load-lengths to obtain the values for which the shears are a maximum or minimum. The shear is computed from the values of H, C_1, and C_2 by Eq. 13.

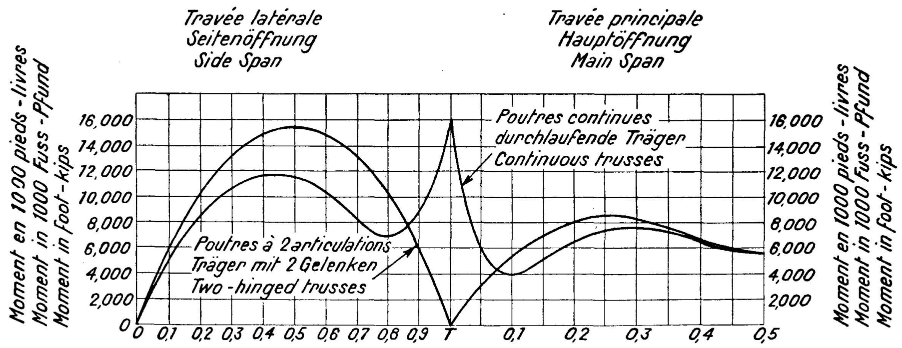
Maximum Deflection in Main Span. — Maximum deflection in the main span is produced by fully loading the main span at highest temperature. H is obtained by trial from the formulas of Case II, the result having been plotted in the H-curve for advancing load in the main span (Fig. 7). The calculation of the deflection at the center of the span is indicated below in condensed form:

H	(trial value)	891 kips
c		0.008955
e^{cl}		1291.9
D'	(from Fig. 6)	33108
H	(by Eq. 70)	891.3 kips
$T_1 = T_2$	(by formulas, Cases II and VII	−247.5
$C_1 H$	(by formula, Case II)	3.704
$C_2 H$	(by formula, Case II)	4785
η	(by Eq. 85a)	5.398 ft.

Maximum Deflection in Side Span. — Maximum downward side span deflection occurs when the one side span is fully loaded at highest temperature. The computation of the necessary constants for this loading condition have already been indicated in the calculation of positive moments in the side span. The downward deflection at the center of the side span is computed by Eq. (85 b) and is found to be 3.223 ft.

It will be noted that the side span deflection is 2.5 percent less than in the two-hinged design, while the main span deflection is 4.5 percent greater. The average rigidity over the entire length differs from that of the two-hinged design by approximately 1 percent.

Comparison of Maximum Moments for Continuous and Two-Hinged Designs. — The maximum bending moments produced by live load and temperature in the continuous spans of Design II, computed as hereinabove outlined, are plotted in Fig. 8. On the same chart are plotted, for comparison, the maximum bending moments produced by live load and temperature in the corresponding two-hinged design. The saving in chord material by the adoption of the continuous type, as indicated by the percentage of difference between the moment areas under these two respective graphs, is 8 percent, and this figure is substantiated by the actual design of the truss members.



Comparaison des moments maxima pour la charge accidentelle et la température pour une poutre en treillis de raidissement continue et ceux pour une poutre avec des articulations sur les appuis médians.

Vergleich der max. Momente infolge Verkehrslast und Temperatur auf das durchlaufende Versteifungsfachwerk und derjenigen auf das Versteifungsfachwerk mit Gelenken über den Mittelstützen.

Comparison of maximum moments for live load and temperature in continuous and two-hinged stiffening trusses.

Fig. 8.

The two-hinged design, represented by the maximum moments plotted in Fig. 8, is a "balanced" design, i. e. the chord sections have been adjusted until the final values of I agree with those assumed. The hingeless design, on the other hand, is not quite "balanced", the calculated bending moments requiring values of I somewhat higher than those assumed. The necessary revision of the continuous design to make it balanced would reduce the indicated saving over the two-hinged design from 8 percent to approximately 5 percent.

Advantages of the Continuous Type. — The foregoing comparative design studies indicate that, for a suspension bridge of 800-ft. main span, the continuous type is approximately 5 percent more rigid than a two-hinged design of the same economy, and about 5 percent more economical than a two-hinged design of the same rigidity. These percentage differences in favor of the continuous type will be greater in shorter spans or with deeper trusses.

As the length of span or its flexibility is increased, the effect of continuity at the towers is lost out on the spans at points proportionately nearer and nearer the towers. It is for this reason that the advantage of the continuous type is greater with shorter spans and deeper stiffening trusses. As the ratio of dead load to live load is increased, the necessity for a stiffening truss is minimized or obviated; accordingly the advantage of the continuous truss will also diminish with increase of dead load. In general, the advantage of the continuous type over the two-hinged design will be governed by the stiffness factor

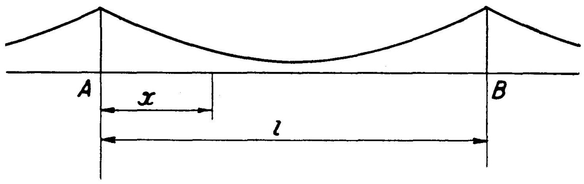
$$S = \frac{1}{l}\sqrt{\frac{EI}{H_w}} = \frac{1}{l^2}\sqrt{\frac{8fEI}{w}}$$

which also governs the percentage correction between the results of the Elastic and Deflection Theories.

The continuous type has an advantage in respect to behavior under lateral forces. The lateral rigidity is greater than in the two-hinged design, and there is a better distribution and absorption of stresses from lateral loading. In fact, for spans of 800 ft. or less, for which the ratio of width to span is greater than 1:20, the chord sections in a continuous design are not affected by wind stresses, since the better distribution of these stresses brings them under the 25 percent increase in allowable stress permitted under this loading by the usual design specifications.

As a general conclusion, we may state that the continuous type of suspension bridge offers advantages over the two-hinged type for spans under 1000 ft. designed for highway loading, and for longer spans when designed for railroad loading.

Case I.



No Load in Main Span

Contributions to Numerator of H:

$$F(p) = 0 \qquad G(p) = 0$$

In Segment AB

$$C_1 = -\frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl} - e^{-cl})}\frac{(T_1 - T_2)}{H}$$

$$C_2 = -C_1 - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right)$$

For Calculating $\Delta\mu$

$$A = \frac{H}{EIc^2}\left[\frac{1}{e_c}\left(\frac{1}{\varrho c^2} + \frac{M_1 + M_2}{2H} + \varepsilon f\right) - f\right]$$

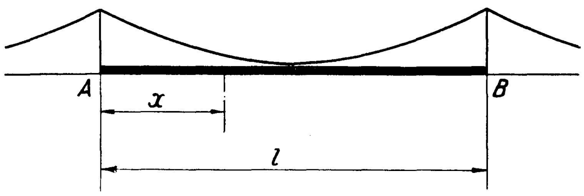
Contributions to M_1 and M_2

$$M_1 = M_2 = 0$$

Contributions to T_1 and T_2

$$T_1 = T_2 = \frac{2}{(1+ir)_c e_c}\left(\frac{H}{\varrho c^2}\right)$$

Case II.



Main Span Fully Loaded

Contributions to Numerator of H

$$F(p) = \frac{pl^3}{12} - \frac{p}{c^2}L_c \qquad G(p) = \frac{2}{(1+ir)_c e_c}\left(-\frac{p}{c^2}\right)$$

In Segment AB

$$C_1 = \frac{1}{(e^{cl}+1)} \cdot \frac{p}{Hc^2} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl}-e^{-cl})}\,\frac{(T_1 - T_2)}{H}$$

$$C_2 = -C_1 + \frac{p}{Hc^2} - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right)$$

For Calculating $\Delta\mu$

$$A = \frac{H}{EIc^2}\left[\frac{1}{e_c}\left(-\frac{p}{Hc^2} + \frac{1}{\varrho c^2} + \frac{M_1 + M_2}{2H} + \varepsilon f\right) + \frac{p\,l^2}{8H} - f\right]$$

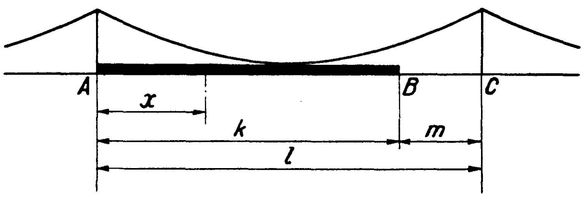
Contributions to M_1 and M_2

$$M_1 = M_2 = -\frac{p\,l^2}{4}\,\frac{1}{(3 + 2ir)}$$

Contributions to T_1 and T_2

$$T_1 = T_2 = +\frac{2}{(1+ir)_c e_c}\left(-\frac{p}{c^2} + \frac{H}{\varrho c^2}\right)$$

Case III.



Main Span Loaded from Left End

Contributions to Numerator of H

$$F(p) = pk\left[\frac{k}{12}(3l - 2k) - \frac{1}{c^2}\right] + \frac{p}{c^3}\,\frac{(e^{ck}-1)(e^{cm}+1)}{(e^{cl}+1)}$$

$$(k > m)\qquad G(p) = \frac{2}{(1+ir)_c}\left[\frac{p}{2c^2}\,\frac{(e^{cm}+e^{-cm}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} - \frac{p}{c^2 e_c} + \frac{p\,m^2}{8l}(k-m)\right]$$

$$(k < m)\qquad G(p) = \frac{2}{(1+ir)_c}\left[-\frac{p}{2c^2}\,\frac{(e^{ck}+e^{-ck}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} - \frac{p\,k^2}{8l}(m-k)\right]$$

In Segment AB

$$C_1 = \frac{p}{2Hc^2}\,\frac{(e^{cm}+e^{-cm}-2e^{-cl})}{(e^{cl}-e^{-cl})} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl}-e^{-cl})}\,\frac{(T_1 - T_2)}{H}$$

$$C_2 = -C_1 + \frac{p}{Hc^2} - \left(\frac{1}{\varrho c^2} + \frac{T_1}{H}\right)$$

For Calculating $\Delta\mu$

$$(k > m)\quad A = \frac{H}{EIc^2}\left[\frac{p}{2Hc^2}\,\frac{(e^{cm}+e^{-cm}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} + \frac{1}{e_c}\left(-\frac{p}{Hc^2} + \frac{1}{\varrho c^2} + \frac{M_1+M_2}{2H} + \varepsilon f\right) + \frac{M_f}{H} - f\right]$$

$$(k < m)\quad A = \frac{H}{EIc^2}\left[-\frac{p}{2Hc^2}\,\frac{(e^{ck}+e^{-ck}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} + \frac{1}{e_c}\left(\frac{1}{\varrho c^2} + \frac{M_1+M_2}{2H} + \varepsilon f\right) + \frac{M_f}{H} - f\right]$$

Contributions to M_1 and M_2

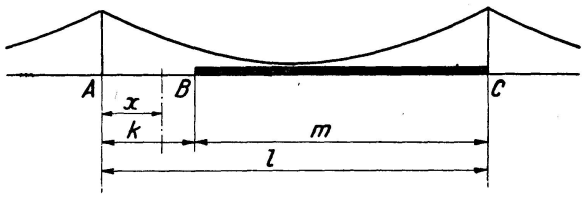
$$M_{1,2} = -\frac{p\,k^2(l+2m)}{4l(3+2ir)} \mp \frac{p}{4l^2}\,\frac{k^2 m^2}{(1+2ir)}$$

Contributions to T_1 and T_2

$$\text{approx.}\quad T_{1,2} = G(p) + \frac{2}{(1+ir)_c}\left[\frac{H}{e_c\,\varrho\,c^2} \mp \frac{p\,k^2\,m^2}{8\,l^2}\right]$$

$$\text{exact}\quad T_{1,2} = G(p) + \frac{2H}{(1+ir)_c e_c\,\varrho\,c^2} \mp \frac{p\,k^2\,m^2}{4\,l^2(1+ir)_{c_1}}$$

Case IV.



Main Span Loaded from Right End

Contributions to Numerator of H

$$F(p) = p\,m\left[\frac{m}{12}(3\,l - 2\,m) - \frac{1}{c^2}\right] + \frac{p}{c^3}\,\frac{(e^{cm}-1)(e^{ck}+1)}{(e^{cl}+1)}$$

$$(k > m)\quad G(p) = \frac{2}{(1+ir)_c}\left[-\frac{p}{2c^2}\,\frac{(e^{cm}+e^{-cm}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} - \frac{p\,m^2}{8\,l}(k-m)\right]$$

$$(k < m)\quad G(p) = \frac{2}{(1+ir)_c}\left[\frac{p}{2\,c^2}\,\frac{(e^{ck}+e^{-ck}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} - \frac{p}{c^2\,e_c} + \frac{p\,k^2}{8\,l}(m-k)\right]$$

In Segment AB

$$C_1 = -\frac{p}{2\,H\,c^2}\,\frac{(e^{cm}+e^{-cm}-2)}{(e^{cl}-e^{-cl})} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho\,c^2} + \frac{T_1}{H}\right) + \frac{1}{(e^{cl}-e^{-cl})}\,\frac{(T_1-T_2)}{H}$$

$$C_2 = -C_1 - \left(\frac{1}{\varrho\,c^2} + \frac{T_1}{H}\right)$$

For Calculating $\Delta\,\mu$

$$(k > m)\quad A = \frac{H}{EIc^2}\left[-\frac{p}{2Hc^2}\,\frac{(e^{cm}+e^{-cm}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} + \frac{1}{e_c}\left(\frac{1}{\varrho\,c^2} + \frac{M_1+M_2}{2H} + \varepsilon f\right) + \frac{M_f}{H} - f\right]$$

$$(\ < m\quad A = \frac{H}{EIc^2}\left[\frac{p}{2Hc^2}\,\frac{(e^{ck}+e^{-ck}-2)}{(e^{cl}+1)\,e^{-\frac{cl}{2}}} + \frac{1}{e_c}\left(-\frac{p}{Hc^2} + \frac{1}{\varrho\,c^2} + \frac{M_1+M_2}{2H} + \varepsilon f\right) + \frac{M_f}{H} - f\right]$$

Contributions to M_1 and M_2

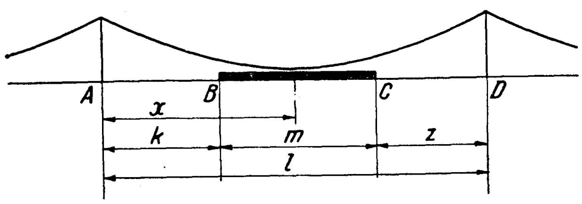
$$M_{1,2} = -\frac{p\,m^2}{4\,l}\,\frac{(l+2\,k)}{(3+2\,ir)} \pm \frac{p\,k^2\,m^2}{4\,l^2(1+2\,ir)}$$

Contributions to T_1 and T_2

$$\text{approx.}\quad T_{1,2} = G(p) + \frac{2}{(1+ir)_c}\left[\frac{H}{e_c\varrho\,c^2} \pm \frac{p\,k^2\,m^2}{8\,l^2}\right]$$

$$\text{exact}\quad T_{1,2} = G(p) + \frac{2H}{(1+ir)_c e_c\,\varrho\,c^2} \pm \frac{p\,k^2\,m^2}{4\,l^2(1+ir)_c}$$

Case V.



Main Span Loaded in Middle Segment

Contributions to Numerator of H

$$F(p) = pm\left[\frac{k(l-k)+z(l-z)}{4} + \frac{m^2}{12} - \frac{1}{c^2}\right] + \frac{p}{c^3}\frac{(e^{cm}-1)(e^{ck}+e^{cz})}{(e^{cl}+1)}$$

$$G(p) = \frac{2}{(1+ir)_c}\left[\frac{p}{2c^2}\frac{(e^{ck}+e^{-ck}+e^{cz}+e^{-cz}-4)}{(e^{cl}+1)e^{-\frac{cl}{2}}} - \frac{p}{c^2 e_c} - \frac{p}{8l^2}\left\{k^4+z^4-k^2(l-k)^2-z^2(l-z)^2\right\}\right]$$

In Segment BC

$$C_1 = \frac{p}{2Hc^2}\frac{[e^{cz}+e^{-cz}-e^{-cl}(e^{ck}+e^{-ck})]}{(e^{cl}-e^{-cl})} - \frac{1}{(e^{cl}+1)}\left(\frac{1}{\varrho c^2}+\frac{T_1}{H}\right) + \frac{1}{(e^{cl}-e^{-cl})}\frac{(T_1-T_2)}{H}$$

$$C_2 = -C_1 + \frac{p}{2Hc^2}(e^{ck}+e^{-ck}) - \left(\frac{1}{\varrho c^2}+\frac{T_1}{H}\right)$$

For Calculating $\Delta\mu$

$$\left(k \text{ and } z < \frac{l}{2}\right)\ A = \frac{H}{EIc^2}\left[\frac{p}{2Hc^2}\frac{(e^{ck}+e^{-ck}+e^{cz}+e^{-cz}-4)}{(e^{cl}+1)e^{-\frac{cl}{2}}} + \frac{1}{e_c}\left(-\frac{p}{Hc^2}+\frac{1}{\varrho c^2}+\frac{M_1+M_2}{2H}+\varepsilon f\right)+\frac{M_f}{H}-f\right]$$

Contributions to M_1 and M_2

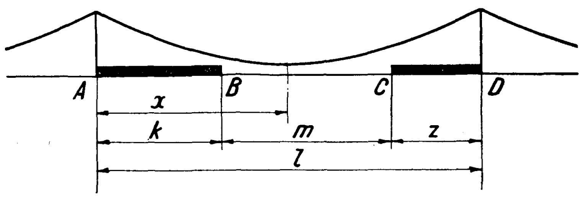
$$M_{1,2} = -\frac{p}{4l}\frac{[m^2(3l-2m)+6kmz]}{(3+2ir)} \mp \frac{p}{4l^2}\frac{[z^2(l-z)^2-k^2(l-k)^2]}{(1+2ir)}$$

Contributions to T_1 and T_2

$$\left(k \text{ and } z < \frac{l}{2}\right)\ T_{1,2} = G(p) + \frac{2}{(1+ir)_c}\left[\frac{H}{\varrho c^2 e_c} \pm \frac{pk^2}{8l^2}(l-k)^2 \mp \frac{pz^2}{8l^2}(l-z)^2\right] \text{ approx}$$

$$\text{exact } T_{1,2} = G(p) + \frac{2H}{\varrho c^2 e_c(1+ir)_c} \pm \frac{p}{4l^2(1+ir)_{c_1}}\left[k^2(l-k)^2 - z^2(l-z)^2\right]$$

Case VI.



Main Span Loaded in End Segments

Contributions to Numerator of H

$$F(p) = \frac{pk^2}{12}(3l-2k) + \frac{pz^2}{12}(3l-2z) - \frac{p}{c^2}(L_c-m) - \frac{p}{c^3}\frac{(e^{cm}-1)(e^{ck}+e^{cz})}{(e^{cl}+1)}$$

$$G(p) = \frac{2}{(1+ir)_c}\left[-\frac{p}{2c^2}\frac{(e^{ck}+e^{-ck}+e^{cz}+e^{-cz}-4)}{(e^{cl}+1)e^{-\frac{cl}{2}}} + \frac{p}{8l^2}\left\{k^4+z^4-k^2(l-k)^2-z^2(l-z)^2\right\}\right]$$

In Segment BC

$$C_1 = \frac{p}{2Hc^2}\frac{[e^{-cl}(e^{ck}+e^{-ck})-(e^{cz}+e^{-cz})]}{(e^{cl}-e^{-cl})} - \frac{1}{(e^{cl}+1)}\left(-\frac{p}{Hc^2}+\frac{1}{\varrho c^2}+\frac{T_1}{H}\right) + \frac{1}{(e^{cl}-e^{-cl})}\frac{(T_1-T_2)}{H}$$

$$C_2 = -C_1 - \frac{p}{2Hc^2}(e^{ck}+e^{-ck}-2) - \left(\frac{1}{\varrho c^2}+\frac{T_1}{H}\right)$$

For Calculating $\Delta\mu$

$$\left(k \text{ and } z < \frac{l}{2}\right)\ A = \frac{H}{EIc^2}\left[-\frac{p}{2Hc^2}\frac{(e^{ck}+e^{-ck}+e^{cz}+e^{-cz}-4)}{(e^{cl}+1)e^{-\frac{cl}{2}}} + \frac{1}{e_c}\left(\frac{1}{\varrho c^2}+\frac{M_1+M_2}{2H}+\varepsilon f\right)+\frac{M_f}{H}-f\right]$$

Contributions to M_1 and M_2

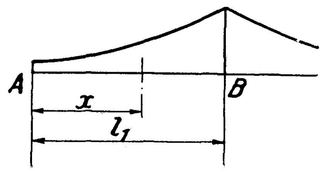
$$M_{1,2} = -\frac{p}{4l}\frac{(l+2m)(l-m)^2-6kmz}{(3+2ir)} \pm \frac{p}{4l^2}\frac{[z^2(l-z)^2-k^2(l-k)^2]}{(1+2ir)}$$

Contributions to T_1 and T_2

approx. $$T_{1,2} = G(p) + \frac{2}{(1+ir)_c}\left[\frac{H}{\varrho c^2 e_c} \mp \frac{pk^2}{8\,l^2}(l-k)^2 \pm \frac{pz^2}{8\,l^2}(l-z)^2\right]$$

exact $$T_{1,2} = G(p) + \frac{2H}{\varrho c^2 e_c (1+ir)_c} \mp \frac{p}{4\,l^2(1+ir)_{c_1}}\left[k^2(l-k)^2 - z^2(l-z)^2\right]$$

Case VII.



Side Span Unloaded

Contributions to Numerator of H

$$F(p) = 0 \qquad\qquad G(p) = 0$$

In Segment AB

$$C_1 = -\frac{1}{(e^{c_1 l_1}+1)}\frac{1}{\varrho_1 c_1^2} - \frac{1}{(e^{c_1 l_1} - e^{-c_1 l_1})}\frac{T_{1,2}}{H}$$

$$C_2 = -C_1 - \frac{1}{\varrho_1 c_1^2}$$

For Calculating $\Delta\mu$

$$B_{1,2} = \frac{H}{EIc^2}\left[\frac{1}{e_{c_1}}\left(\frac{1}{\varrho_1 c_1^2} + \frac{M_{1,2}}{2H} + \frac{\varepsilon f}{2}\right) - f_1\right]$$

Contributions to M_1 and M_2

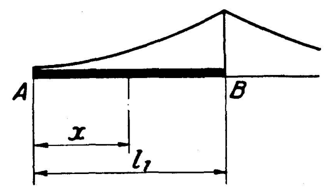
$$M_1 = M_2 = 0$$

Contributions to T_1 (or T_2 for this Loading in Right Side Span)

approx. $$T_1 = \frac{2ir}{(1+ir)_c e_{c_1}}\left(\frac{H}{\varrho_1 c_1^2}\right)$$

exact $$T_{1,2} = \frac{ir}{(1+ir)_c e_{c_1}}\left(\frac{H}{\varrho_1 c_1^2}\right)$$

Case VIII.



Side Span Fully Loaded

Contributions to Numerator of H

$$F(p) = K_1\frac{p\,l_1^3}{12} - K_1\frac{p}{c_1^2}L_{c_1}$$

$$G(p) = \frac{ir}{(1+ir)_c e_{c_1}}\left(-\frac{p}{c_1^2}\right)$$

In Segment AB

$$C_1 = \frac{1}{(e^{c_1 l_1}+1)}\left(\frac{p}{Hc_1^2} - \frac{1}{\varrho_1 c_1^2}\right) - \frac{1}{(e^{c_1 l_1} - e^{-c_1 l_1})}\frac{T_{1,2}}{H}$$

$$C_2 = -C_1 + \frac{p}{Hc_1^2} - \frac{1}{\varrho_1 c_1^2}$$

For Calculating $\Delta\mu$

$$B_{1,2}=\frac{H}{EIc^2}\left[\frac{1}{e_{c_1}}\left(-\frac{p}{Hc_1^2}+\frac{1}{\varrho_1 c_1^2}+\frac{M_{1,2}}{2H}+\frac{\varepsilon f}{2}\right)+\frac{p\,l_1^2}{8H}-f_1\right]$$

Contributions to M_1 and M_2

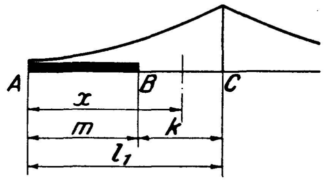
$$M_{1,2}=-\frac{p\,l_1^2}{8}\,ir\left[\frac{1}{(3+2ir)}\pm\frac{1}{(1+2ir)}\right]$$

Contributions to T_1 (or T_2 for this Loading in Right Side Span)

approx. $$T_1=\frac{2ir}{(1+ir)_c e_{c_1}}\left[-\frac{p}{c_1^2}+\frac{H}{\varrho_1 c_1^2}\right]$$

exact $$T_{1,2}=G(p)\pm\frac{(1+ir)_c}{(1+ir)_{c_1}}G(p)+\frac{ir}{(1+ir)_c e_{c_1}}\left(\frac{H}{\varrho_1 c_1^2}\right)$$

Case IX.



Side Span Loaded from Free End

Contributions to Numerator of H

$$F(p)=K_1 p m\left[\frac{m}{12}(3l_1-2m)-\frac{1}{c_1^2}\right]+K_1\frac{p}{c_1^3}\frac{(e^{c_1 m}-1)(e^{c_1 k}+1)}{(e^{c_1 l_1}+1)}$$

$$(k>m)\quad G(p)=\frac{ir}{(1+ir)_c}\left[-\frac{p}{2c_1^2}\frac{(e^{c_1 m}+e^{-c_1 m}-2)}{(e^{c_1 l_1}+1)e^{-\frac{c_1 l_1}{2}}}+\frac{p\,m^4}{8\,l_1^2}\right]$$

$$(k<m)\quad G(p)=\frac{ir}{(1+ir)_c}\left[\frac{p}{2c_1^2}\frac{(e^{c_1 k}+e^{-c_1 k}-2)}{(e^{c_1 l_1}+1)e^{-\frac{c_1 l_1}{2}}}-\frac{p}{c_1^2 e_{c_1}}+\frac{p\,k^2}{8\,l_1^2}(2m^2-k^2)\right]$$

In Segment BC

$$C_1=\frac{p\,e^{-c_1 l_1}}{2Hc_1^2}\frac{(e^{c_1 m}+e^{-c_1 m}-2)}{(e^{c_1 l_1}-e^{-c_1 l_1})}-\frac{1}{(e^{c_1 l_1}+1)}\frac{1}{\varrho_1 c_1^2}-\frac{1}{(e^{c_1 l_1}-e^{-c_1 l_1})}\frac{T_{1,2}}{H}$$

$$C_2=-C_1-\frac{p}{2Hc_1^2}(e^{c_1 m}+e^{-c_1 m}-2)-\frac{1}{\varrho_1 c_1^2}$$

For Calculating $\Delta\mu$

$$(k>m)\ B_{1,2}=\frac{H}{EIc^2}\left[-\frac{p}{2Hc_1^2}\frac{(e^{c_1 m}+e^{-c_1 m}-2)}{(e^{c_1 l_1}+1)e^{-\frac{c_1 l_1}{2}}}+\frac{1}{e_{c_1}}\left(\frac{1}{\varrho_1 c_1^2}+\frac{M_{1,2}}{2H}+\frac{\varepsilon f}{2}\right)+\frac{M_{f_{1,2}}}{H}-f_1\right]$$

$$(k<m)\ B_{1,2}=\frac{H}{EIc^2}\left[\frac{p}{2Hc_1^2}\frac{(e^{c_1 k}+e^{-c_1 k}-2)}{(e^{c_1 l_1}+1)e^{-\frac{c_1 l_1}{2}}}+\frac{1}{e_{c_1}}\left(-\frac{p}{Hc_1^2}+\frac{1}{\varrho_1 c_1^2}+\frac{M_{1,2}}{2H}+\frac{\varepsilon f}{2}\right)+\frac{M_{f_{1,2}}}{H}-f_1\right]$$

Contributions to M_1 and M_2

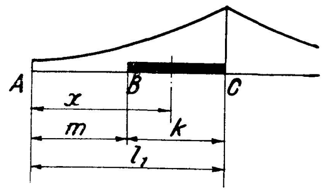
$$M_{1,2}=-\frac{p}{8\,l_1^2}\,ir\left[\frac{1}{(3+2ir)}\pm\frac{1}{(1+2ir)}\right]m^2(2l_1^2-m^2)$$

Contributions to T_1 (or T_2 for this Loading in Right Side Span)

approx. $$T_1=2G(p)+\frac{2ir}{(1+ir)_c e_{c_1}}\left(\frac{H}{\varrho_1 c_1^2}\right)$$

exact $$T_{1,2}=G(p)\pm\frac{(1+ir)_c}{(1+ir)_{c_1}}G(p)+\frac{ir}{(1+ir)_c e_{c_1}}\left(\frac{H}{\varrho_1 c_1^2}\right)$$

Case X.



Side Span Loaded from Tower End

Contributions to Numerator of H

$$F(p) = K_1 p k \left[\frac{k}{12}(3 l_1 - 2k) - \frac{1}{c_1^2}\right] + K_1 \frac{p}{c_1^3} \frac{(e^{c_1 k} - 1)(e^{c_1 m} + 1)}{(e^{c_1 l_1} + 1)}$$

$$(k > m) \quad G(p) = \frac{i r}{(1 + i r)_c}\left[\frac{p}{2 c_1^2} \frac{(e^{c_1 m} + e^{-c_1 m} - 2)}{(e^{c_1 l_1} + 1) e^{-\frac{c_1 l_1}{2}}} - \frac{p}{c_1^2 e_{c_1}} - \frac{p m^4}{8 l_1^2}\right]$$

$$(k < m) \quad G(p) = \frac{i r}{(1 + i r)_c}\left[- \frac{p}{2 c_1^2} \frac{(e^{c_1 k} + e^{-c_1 k} - 2)}{(e^{c_1 l_1} + 1) e^{-\frac{c_1 l_1}{2}}} + \frac{p k^2}{8 l_1^2}(k^2 - 2 m^2)\right]$$

In Segment BC

$$C_1 = - \frac{p}{2 H c_1^2} \frac{(e^{c_1 m} + e^{-c_1 m}) e^{-c_1 l_1} - 2}{(e^{c_1 l_1} - e^{-c_1 l_1})} - \frac{1}{(1 + e^{c_1 l_1})} \frac{1}{\varrho_1 c_1^2} - \frac{1}{(e^{c_1 l_1} - e^{-c_1 l_1})} \frac{T_{1,2}}{H}$$

$$C_2 = - C_1 + \frac{p}{2 H c_1^2}(e^{c_1 m} + e^{-c_1 m}) - \frac{1}{\varrho_1 c_1^2}$$

For Calculating $\Delta \mu$

$$(k > m) \quad B_{1,2} = \frac{H}{E I c^2}\left[\frac{p}{2 H c_1^2} \frac{(e^{c_1 m} + e^{-c_1 m} - 2)}{(e^{c_1 l_1} + 1) e^{-\frac{c_1 l_1}{2}}} + \frac{1}{e_{c_1}}\left(- \frac{p}{H c_1^2} + \frac{1}{\varrho_1 c_1^2} + \frac{M_{1,2}}{2H} + \frac{\varepsilon f}{2}\right) + \frac{M_{f_{1,2}}}{H} - f_1\right]$$

$$(k < m) \quad B_{1,2} = \frac{H}{E I c^2}\left[- \frac{p}{2 H c_1^2} \frac{(e^{c_1 k} + e^{-c_1 k} - 2)}{(e^{c_1 l_1} + 1) e^{-\frac{c_1 l_1}{2}}} + \frac{1}{e_{c_1}}\left(\frac{1}{\varrho_1 c_1^2} + \frac{M_{1,2}}{2H} + \frac{\varepsilon f}{2}\right) + \frac{M_{f_{1,2}}}{H} - f_1\right]$$

Contributions to M_1 and M_2

$$M_{1,2} = - \frac{p}{8 l_1^2} i r \left[\frac{1}{(3 + 2 i r)} \pm \frac{1}{(1 + 2 i r)}\right] k^2 (l_1 + m)^2$$

Contributions to T_1

$$\text{approx.} \quad T_1 = 2 G(p) + \frac{2 i r}{(1 + i r)_c e_{c_1}} \left(\frac{H}{\varrho_1 c_1^2}\right)$$

$$\text{exact} \quad T_{1,2} = G(p) \pm \frac{(1 + i r)_c}{(1 + i r)_{c_1}} G(p) + \frac{i r}{(1 + i r)_c e_{c_1}} \left(\frac{H}{\varrho_1 c_1^2}\right)$$

Summary.

This paper presents an extension of the Deflection Theory to cover suspension bridges with continuous stiffening trusses.

The more general adoption of the continuous type of suspension bridge, offering advantages of economy and rigidity, has been retarded by the lack of an accurate theory for its analysis. The Deflection Theory for simple-span suspension bridges has been available to the profession for over 40 years; but the corresponding theory for the suspension bridge with continuous stiffening truss has thus far been lacking.

In order to supply this deficiency, the writer has undertaken to develop the Deflection Theory for continuous suspension bridges, with working formulas for practical application.

The resulting analysis, presented in this paper, is a generalized Deflection Theory for suspension bridges, applicable to both continuous and non-continuous types. By simply dropping the recognizable terms due to continuity, the formulas are reduced to those for the two-hinged suspension bridge. Moreover, the general formulas are also found to be applicable to multiple-span suspension bridges, with or without continuity.

In the development of the analysis herein presented, maximum simplicity of formulas and ease of practical application have been governing considerations. Incidentally, new simplifications are here developed and introduced in the working formulas hitherto published for the two-hinged type.

Interdependent functions necessarily introduced in the earlier portions of the theoretical analysis are resolved and eliminated in deducing the final working formulas.

Simplified approximate formulas are also given as alternatives, with their departure from exact values indicated, to facilitate preliminary or approximate evalution. These approximate formulas also facilitate the interpretation of the expressions or relations represented.

To show the practical workability of the Generalized Theory here developed, this paper includes a numerical example of the application of the formulas to the analysis of the stresses and deflections in a continuous suspension bridge of 800-ft. main span. For the continuous stiffening truss, the Deflection Theory is found to yield an average reduction of 45 percent in the bending moments as previously found by the common Elastic Theory. In comparison with the two-hinged type, the continuous design is found to be approximately 5 percent more rigid for the same economy, or 5 percent more economical for the same rigidity. For shorter spans, these percentages of superior efficiency would be increased.

Résumé.

Le présent mémoire a pour but d'exposer une extension de la théorie des déformations que subissent les ponts suspendus comportant des éléments raidisseurs continus en treillis.

Les ponts suspendus du type continu présentent des avantages des points de vue de l'économie et de la rigidité. Toutefois, l'absence d'une théorie suffisamment précise concernant l'étude de cette disposition n'a pas été sans en entraver la généralisation. La théorie des ponts suspendus comportant une seule ouverture est connue des milieux techniques depuis plus de 40 ans; par contre, cette théorie ne s'étend pas encore aux ponts comportant des éléments de renforcement continus en treillis.

Afin de combler cette lacune, l'auteur a entrepris de mettre la question au point du point de vue théorique et de fournir des formules susceptibles d'être employées dans les calculs pratiques.

Les résultats ainsi obtenus, et qui font l'objet du présent mémoire, constituent d'ailleurs une théorie généralisée des ponts suspendus, théorie que l'on peut appliquer aussi bien aux ponts du type continu que du type non continu. Il suffit de laisser de côté, dans les formules, les expressions qui traduisent la continuité; les formules simplifiées ainsi obtenues sont ensuite valables dans le cas des ponts suspendus comportant des articulations aux appuis (cas des ponts suspendus à deux articulations sur trois appuis). En outre, les formules générales elles-mêmes peuvent s'appliquer aux ponts

suspendus à travées multiples, avec ou sans articulations aux appuis. L'auteur s'est d'ailleurs préoccupé tout particulièrement, au cours des études qui sont exposées dans ce mémoire, des possibilités pratiques d'application, en ce qui concerne la simplicité et la facilité. Il a en outre apporté quelques simplifications nouvelles aux formules qui ont été publiées jusqu'à maintenant pour le type à deux articulations. Les fonctions réciproques introduites par nécessité au début de l'étude théorique ont été résolues, puis éliminées dans l'établissement des formules définitives.

Afin de rendre plus facile un calcul préliminaire ou approximatif, des formules très simplifiées et approchées ont été également mises au point; toutes indications sont d'ailleurs données en ce qui concerne les écarts auxquels il faut s'attendre par rapport aux valeurs exactes. Ces formules facilitent également la compréhension des expressions et notations employées.

Afin de montrer les possibilités de mise en oeuvre de la théorie généralisée, dans la pratique, le mémoire contient en outre un exemple numérique d'emploi de ces formules pour l'étude des contraintes et des déformations dans un pont suspendu continu ayant une ouverture principale d'environ 250 m. Pour les éléments de renforcement continus en treillis, la théorie des déformations donne pour les moments fléchissants une réduction moyenne de 45 % par rapport à la théorie courante de l'élasticité. Par comparaison avec le type à deux articulations, on a constaté qu'à égalité de prix, le type continu est de 5 % plus rigide et qu'à égalité de rigidité, il est de 5 % plus économique. Ce pourcentage est d'ailleurs encore plus élevé pour les faibles portées.

Zusammenfassung.

Die vorliegende Abhandlung stellt eine Erweiterung der Verformungstheorie betreffend Hängebrücken mit durchlaufenden Versteifungsfachwerken dar.

Die allgemeinere Anwendung des durchlaufenden Typus der Hängebrücken, der bezüglich Wirtschaftlichkeit und Steifigkeit Vorteile bietet, wurde durch den Mangel an einer genauen Theorie für dessen Untersuchung hintangehalten. Die Verformungstheorie für Hängebrücken über eine Öffnung ist im Ingenieurfach schon seit über 40 Jahren bekannt; die entsprechende Theorie für die Hängebrücken mit durchlaufendem Versteifungsfachwerk hingegen war noch mangelhaft.

Um diesem Mangel abzuhelfen, hat es der Verfasser unternommen, die Verformungstheorie für durchlaufende Hängebrücken sowie Formeln für ihre praktische Anwendung, zu entwickeln.

Die resultierende Untersuchung, die in dieser Abhandlung vorgelegt wird, ist eine verallgemeinerte Verformungstheorie für Hängebrücken, anwendbar sowohl für Brücken vom durchlaufenden als auch vom nicht durchlaufenden Typus. Wenn man in den Formeln die erkennbaren Glieder der Kontinuität einfach fallen läßt, so vereinfachen sich die Formeln und gelten für den Fall der Hängebrücken mit Gelenken über den Stützen (für sog. Zweigelenkhängebrücken über drei Öffnungen). Außerdem eignen sich die allgemeinen Formeln zur Anwendung auf Hängebrücken mit vielen Öffnungen mit oder ohne Gelenken über den Stützen. In der Entwicklung der Untersuchung, die hier gezeigt wird, wurde hauptsächlich auf äußerste Einfachheit und Leichtigkeit in der praktischen Anwendung getrachtet. Daneben wurden neue Vereinfachungen entwickelt und in die bis jetzt veröffentlichten

Formeln für den Zweigelenktyp eingesetzt. Die am Anfang der theoretischen Untersuchungen notwendigerweise eingeführten gegenseitigen abhängigen Funktionen werden gelöst und beim Ableiten der Endformeln eliminiert.

Um eine vorläufige bezw. approximative Kalkulation zu erleichtern, werden auch vereinfachte Näherungsformeln als alternative angegeben; es wird gezeigt, inwieweit sie von genauen Werten variieren können. Sie erleichtern ebenfalls die Erklärung der dargestellten Ausdrücke und Beziehungen.

Um die praktische Ausführbarkeit der verallgemeinerten Theorie, die hier entwickelt wurde, zu zeigen, enthält diese Abhandlung ein numerisches Beispiel der Anwendung der Formeln für die Untersuchung der Spannungen und Verformungen in einer durchlaufenden Hängebrücke mit einer Hauptöffnung von 800 Fuß. Für die durchlaufenden Versteifungsfachwerke ergibt die Verformungstheorie eine durchschnittliche Reduktion der Biegungsmomente von 45 % gegenüber der gewöhnlichen Elastizitätstheorie. Im Vergleich mit dem Zweigelenktypus fand man, daß das durchlaufende System ungefähr 5 % starrer ist bei gleicher Wirtschaftlichkeit, oder um 5 % wirtschaftlicher bei gleicher Steifigkeit. Bei kleineren Spannweiten würden diese Prozentsätze noch höher sein.