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EARTH PRESSURES

RÉPARTITION DES EFFORTS DANS LES TERRES

SPANNUNGEN IM ERDKÖRPER

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The following is a short account of some experimental work carried out in the University of St. Andrews at University College Dundee for the study and determination of the laws applying to Earth Pressures.

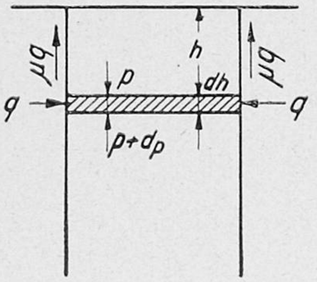
The inital stage was an endeavour to measure the overturning moment of materials such as sand, garden soil and gravel on retaining walls with varying internal batters and thereby to distinguish between the merits of Rankine's Theory of Conjugate Pressures and Coulomb's Wedge Theory. An account of these experiments was given in a paper[1]), read before the Institution of Civil Engineers in London and the results appear to favour the latter more than the former theory.

Subsequently in devising apparatus to illustrate and verify the theory of Bins and Piles the possibility suggested itself of using the apparatus for determining the ratio of conjugate pressures in earthwork when bounded by rigid containing walls and thence to deduce the conditions of stress about an element in a large mass of earth.

General Theory.

The theory applicable to these conditions is well known but has been amplified for the purpose of this investigation.

The conditions of equilibrium of a small horizontal lamina of thickness dh at a depth h beneath the free upper surface of a column of granular material bounded by vertical walls and having a constant horizontal section of area A and weight w per unit volume are:



$$dp \cdot A + q \cdot U \cdot \mu \cdot dh + U \cdot c \cdot dh = w \cdot A \cdot dh$$

where q = horizontal stress per unit area
U = perimeter of cross section
μ = co-efficient of internal friction
c = cohesion per unit area

If, as is generally accepted, the lateral pressure is a lineal function of the vertical pressure due to the weight of the filling, and if μ is constant

[1]) "Overturning Moment on retaining walls" Proc. I. C. E. CCIX part I (1919/20).

throughout the mass and since cohesion may be considered constant at all pressures then, for a column of given perimeter

$$q \cdot U \cdot \mu \text{ may be written } K \cdot p \text{ and } c \cdot U = C$$

$$\therefore \quad dp = \frac{dh}{A}(w \cdot A - K \cdot p - C)$$

$$\text{and} \quad \log_e\left(w - \frac{K}{A} \cdot p - \frac{C}{A}\right) = -\frac{K}{A} \cdot h + D$$

where D is a constant.

Where there is no superload S on the upper surface.

$$p = \frac{A}{K}\left(w - \frac{C}{A}\right)\left(1 - e^{-\frac{K}{A} \cdot h}\right) \tag{1}$$

With a superload S on the surface area.

Surface pressure $= S/A = p_1$

$$p = \frac{A}{K}\left(w - \frac{C}{A}\right)\left(1 - e^{-\frac{K}{A} \cdot h}\right) + p_1 \cdot e^{-\frac{K}{A} \cdot h} \tag{2}$$

where $p =$ pressure per unit area at a depth h due to the weight of the column of material above and to that portion of the superload transmitted through the material.

For any one material where w and C are constant the value of p in these equations depends on the cross sectional area and on the depth of the column beneath the surface, while the constant K varies with the perimeter of the section and may be written kU

where $k = u \cdot \frac{q}{p} =$ co-efficient of internal friction $\times$ ratio of conjugate pressures
$=$ a constant by assumption

If therefore the cross-section of the column be kept constant and h varied, we have from equation (1)

$$\frac{p_1}{1 - e^{-\frac{kU}{A} \cdot h_1}} = \frac{p_2}{1 - e^{-\frac{kU}{A} \cdot h_2}} = \frac{p}{1 - e^{-\frac{k}{m} \cdot h}} = \text{constant}$$

where $m =$ ratio of area/perimeter of column.

By measuring p_1 and p_2 at depths h_1 and h_2 for a known m the value of k can be determined.

The experimental value thus obtained can be compared with the value of k got by the use of the Rankine theory.

Thus for a horizontal surface

$$k = \mu \cdot \frac{q}{p} = \tan \Phi \cdot \frac{1 - \sin \Phi}{1 + \sin \Phi}$$

Differentiating and equating to zero to determine the max value of k we find $\Phi = 30^{\circ}$ and $k = .192$.

Further, from equation (2) we have that if a column of material be superloaded with a load S over its surface area A, the increase in pressure per unit area at a depth h is given by $\frac{S}{A} \cdot e^{-\frac{k \cdot h}{m}}$. Therefore for a given depth and section the increase of pressure varies directly as the increase of surface load.

Apparatus.

The apparatus designed to determine the value of k provided a means of supporting a column of granular material of known weight in such a way as to make it possible to measure the load transmitted vertically downwards through the column while the load supported by shear stresses, that is by the friction of the containing walls was obtained by difference. It consisted essentially of a steelyard over which was supported, independently, bottom-

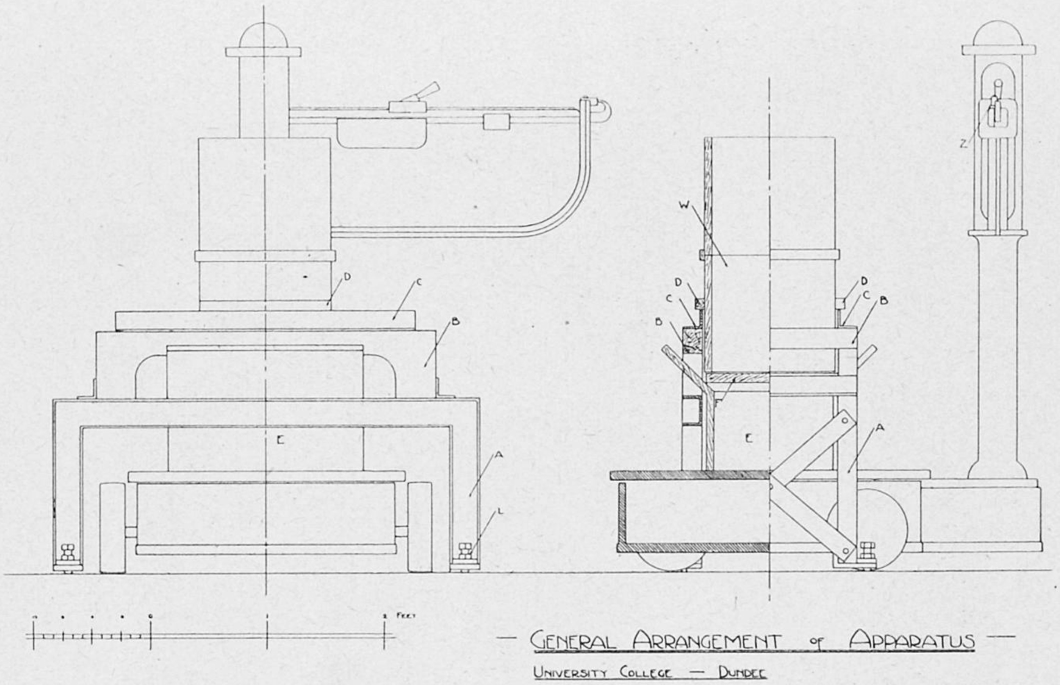


Fig. 1.

less containers of various heights and varing sections. The main features of the apparatus are shown in the accompanying photographs. (Figures 1 and 2.)

Results.

The first series of experiments was carried out on two materials, sea sand and garden soil with uncompacted specific weights of 93 and 66 lbs. per cub. ft. respectively when perfectly dry, and 85/101 lbs. and 78/81 lbs. with water contents up to 20%. The areas used were .5 sq. ft. circular section and 1 and 4 sq. ft. (square section) with heights varying from 1 to 3 feet and with superficial loads ranging by 100 lbs. differences to 600 lbs. per sq. ft. The results of the tests carried out on these two distinct types of material point to the conclusion that not only is k a constant for any one material as assumed but also that a variation in the type of material does not greatly affect its value. Thus:

Material	Density	k mean value	range of k
Sea sand	93 lbs.	.26	.25 — .28
„ „ 5% Water Content	84 „	.23	.23
„ „ 15% „ »	101 „	.23	.23
Garden soil	66 „	.25	.23 — .28

The lower values in each case were obtained with superficial loads.

It has already been shown that the maximum value of k should be obtained when $\Phi = 30^0$ and therefore when $\mu = .6$ the ratio of conjugate pressures

$$= \frac{k}{\mu} = \frac{.255}{.6} = .425$$

This result is in striking agreement with the conclusions come to by Dr. Charles TERZAGHI in his Report[2]) on pressure in granular masses in which he states:

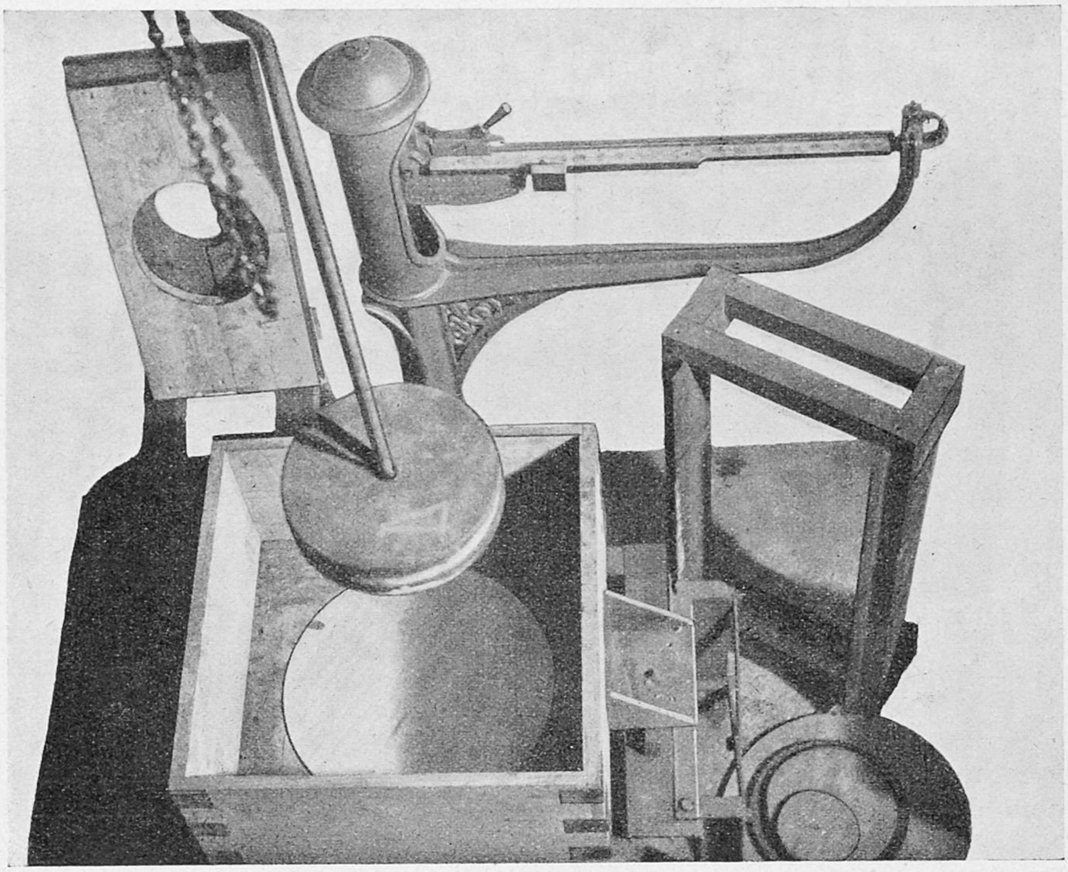


Fig. 2.

"The ratio between the horizontal and vertical pressures is found to be independent of the absolute value of the pressure and equal to .42", and:

"The earth pressure against a perfectly rigid wall seems to be fairly independent of the density of the backfilling."

Pressures at various depths of a column of earth.

General equation (2)

$$p = \frac{m}{k}\left(w - \frac{C}{A}\right)\left(1 - e^{-k \cdot \frac{h}{m}}\right) + p_1 e^{-k \cdot \frac{h}{m}}$$

shows that the pressure p for any material and any surface loading diminishes

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[2]) „Old Earth Pressure Theories and New Test Results" (Engineering News Record, Sept. 30 1920).

as h increases or as m the area/perimeter ratio diminishes and very rapidly approaches a limiting value (when m is not too great) of

$$\frac{m}{k} \cdot \left(w - \frac{C}{A}\right)$$

which in non-cohesive materials equals

$$\frac{m}{k} \cdot w$$

In the case of the sand experiments with a square section of 1 foot side the value of the

$$\text{limit} = \frac{1}{4} \cdot \frac{93}{.26} = 90 \text{ lbs. approximately}$$

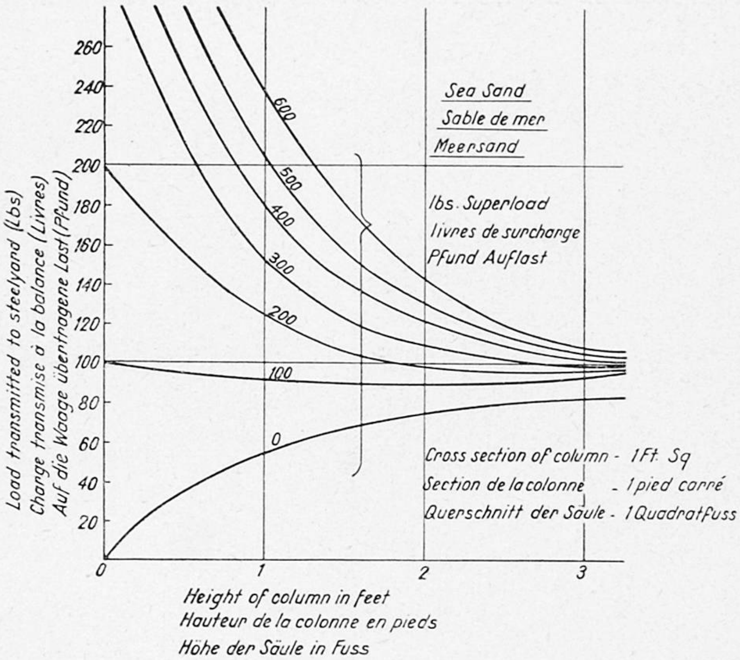

Fig. 3.

The accompanying diagram fig. 3 shows the experimental values of the pressures p obtained with sand and similar results were obtained with the garden soil. These show that the limiting value for such materials is approximately reached comparatively near the surface and that the average pressures on the area vertically beneath the bearing area and at various depths correspond to the distribution as shown by the pressure bulb.

Foundation Pressures.

These results suggest that the foregoing theory might be extended to columns of earth bounded not by rigid walls but by containing walls of the same material as for example the column of earth forming the foundation of footings of walls or piers.

To test this, surface loads were applied to the square columns by means of circular discs of less diameter than the side of the column. The pressure transmitted through the intervening earth was measured at the bottom by loosely fitting discs of the same area and vertically beneath acting on a steelyard. (Figure 4.) The discs were of various diameters but in no case was its diameter greater than .8 of the dimension of the solid container. It was found that the pressure transmitted could be expressed by the foregoing formula but that the value of k had been reduced to .22.

More work would require to be done in order to establish this application to foundation pressures but if verified by further experiment it would indicate that the limiting zone of vertical pressures beneath footings would be generated by a hyperbolic curve and not a straight line [3]).

Distribution of Pressure over Cross-section of Column.

Although the previously described experiments give the average value of the vertical pressures on horizontal sections of the column they give no

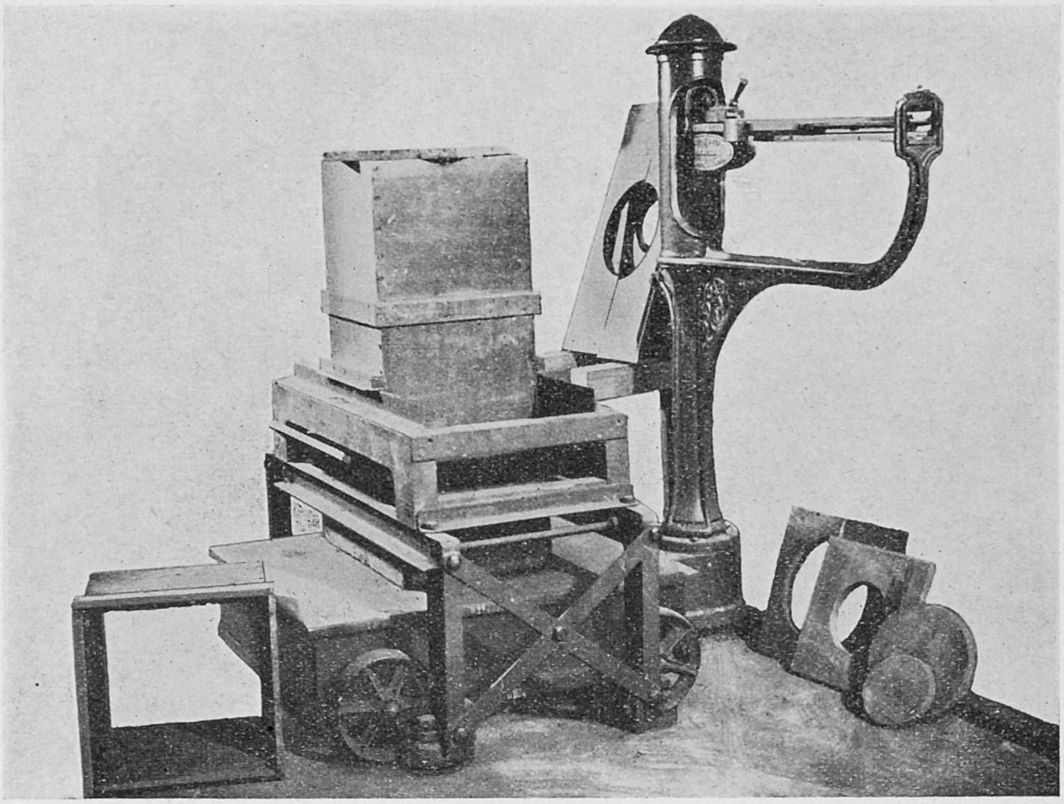

Fig. 4.

indication of the distribution of pressure over the cross section and another series of experiments were carried out in an endeavour to do this.

The arrangement was similar to the preceding one but the surface loads were applied over the whole column area while the pressure transmitted through the column was measured through holes in the bottom plate by discs

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[3]) "Practical Method for the selection of Foundations" W. S. Housel, University of Michigan, Engineering Research Bulletin Oct. 1929.

of increasing areas ranging from .2 to 2 sq. ft. placed centrally. The pressures per sq. ft. registered over these areas have been plotted in the case of a square column of 4 sq. ft. of garden soil and is shown in figure 5.

The distribution is interesting and appears to indicate that "arching" takes place at a radius of about 5 inches from the centre, at which distance the greatest average pressure was measured. At greater radii the influence of the rigid walls is increasingly felt.

In a sand column of the same area a similar diagram was obtained. With columns of smaller area the "arch" tended more to spring from the containing walls.

General Conclusions.

1. That the conjugate pressures in a mass of granular material when subjected to surface loading diminish as the depth beneath the surface in-

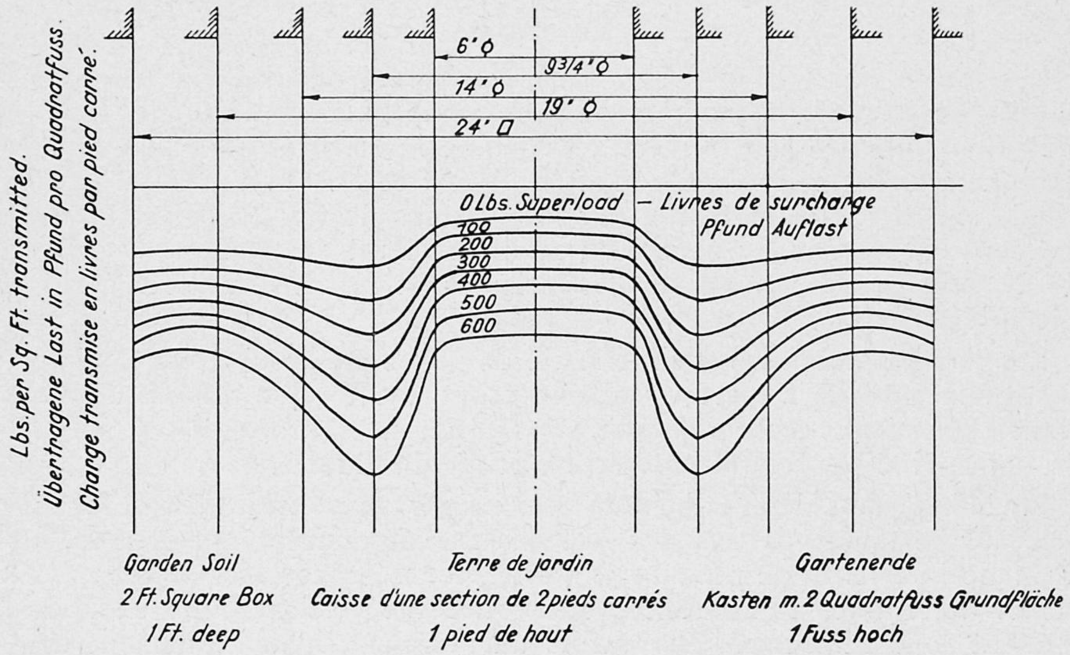

Fig. 5.

creases and as the area/perimeter ratio of the loading surface diminishes. For ordinary area/perimeter ratios these pressures rapidly approach a limiting value as the depth increases.

2. That the ratio of the conjugate pressures under these conditions is constant at varying depths and is fairly independent of the density of the material.

The experimental work in connection with this investigation was very ably carried out by W. F. CASSIE, B. Sc., Ph. D., in the Engineering Laboratory, University College, Dundee, Scotland.

Summary.

The apparatus designed for this experimental work consisted essentially of a steelyard, over which bottomless containers of various heights and various sections were independently supported (figs. 1 and 2). This made it possible to measure the load transmitted vertically downwards through the column of earth, whilst the load supported by the friction of the containing walls was obtained by difference.

The results for sea-sand are given in fig. 3. It is interesting to note that the ratio of the conjugate pressures is constant at various depths and is also fairly independent of the density of the material.

Then the load was applied, by means of discs, to a part of the square column only, and measured at the bottom by discs of the same area (fig. 4). Thus it was possible to determine the vertical friction exercised not by the rigid walls but by the earth itself. More research work is required in order to be able to apply this method to foundation pressures.

Finally the distribution of pressure over the cross-section was determined by measuring the force transmitted through the column to discs of various areas; the results are shown in fig. 5.

Résumé.

Les essais qui sont exposés dans ce rapport ont été exécutés avec un appareil construit de la manière suivante: une balance supporte, d'un côté, un récipient indépendant, ouvert à sa partie inférieure, dont la hauteur est variable, ainsi que la section de base (figures 1 et 2). On a mesuré les efforts transmis, en direction verticale, par les prismes de terre ainsi constitués, le frottement exercé par les parois du récipient étant déterminé par différence entre les efforts s'exerçant sur les prismes et les efforts transmis à leur partie inférieure.

Les résultats obtenus avec le sable de mer sont groupés sur la figure 3. Il est intéressant de remarquer que le rapport entre les compressions horizontale et verticale reste constant pour différentes profondeurs et qu'il ne varie qu'à peine pour différentes compacités du matériau.

En interposant des plaques de dimensions appropriées, on a localisé la charge sur certaines parties des prismes rectangulaires, munis d'ailleurs à leur partie inférieure de plaques de dimensions égales (voir figure 4). On a pu ainsi déterminer l'importance du frottement vertical qui s'exerçait non seulement sous l'influence des parois du récipient, mais également dans la masse. Des travaux plus étendus sont encore nécessaires, si l'on veut pouvoir étendre l'emploi de ce procédé à l'étude de la théorie des fondations.

L'auteur a ensuite déterminé la répartition de la compression sur la section, en mesurant l'effort transmis par le prisme sur des plaques de dimensions différentes; les résultats obtenus sont indiqués sur la figure 5.

Zusammenfassung.

Der für die vorliegenden Versuche konstruierte Apparat bestand in der Hauptsache aus einer Wage, über der, unabhängig von ihr, unten offene Behälter mit veränderlicher Höhe und Grundfläche angebracht wurden (Fig. 1 und 2). Es konnte so die in senkrechter Richtung durch die Erdsäule übertragene Kraft gemessen werden, während die von den Gefäßwänden ausgeübte

Reibung sich aus der Differenz zwischen aufgebrachter und übertragener Kraft ergab.

Die Ergebnisse für Meersand sind in Fig. 3 zusammengestellt. Interessant ist, daß das Verhältnis zwischen horizontalem und vertikalem Druck für verschiedene Tiefen konstant bleibt und sich auch für wechselnde Dichte des Materiales kaum ändert.

Dann wurde die Last mittels Scheiben nur auf einen Teil der quadratischen Erdsäule aufgebracht, und unten die Übertragung auf Scheiben mit gleicher Fläche gemessen (Fig. 4). So war es möglich, die vertikale Reibung zu bestimmen, die nicht durch die Gefäßwände, sondern durch das Erdreich selbst ausgeübt wurde. Es würden noch weitergehende Arbeiten erforderlich sein, um dieses Verfahren auf die Theorie der Gründungen anwenden zu können.

Endlich ist die Verteilung des Druckes über den Querschnitt ermittelt worden durch Messung der Kraft, die durch die Erdsäule auf Scheiben von verschiedener Grundfläche übertragen wurde; die Ergebnisse sind in Fig. 5 dargestellt.

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