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Autor: Malhotra, S.K.

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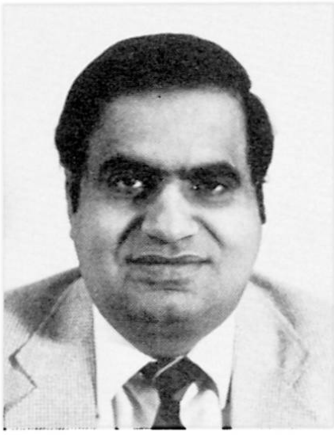
Effects of Eccentricity of Loading and End Restraint on Timber Columns

Effets de charges excentriques et conditions aux extrémités de colonnes en bois

Die Wirkung der Exzentrizität der Belastung und der Endeinspannung auf Holzsäulen

S.K. MALHOTRA
Professor
Techn. Univ. of Nova Scotia
Halifax, NS, Canada



S.K. Malhotra received his Ph.D. from Technical University of Nova Scotia, Halifax, Canada. He has been involved with teaching and research in timber engineering and structural engineering for many years, and has published numerous papers. He holds many appointments on boards and technical committees of professional, research and design code development organizations.

SUMMARY
This paper highlights an analytical and experimental research project on the behaviour of eccentrically-loaded timber columns. The theoretical predictions are compared with test results. The findings of this research are discussed in relation to the current design practice of using linear interaction formula for such timber members, in which compressive and bending stresses are combined linearly. The effect of end restraints on the strength of timber columns is also investigated.

RESUME
Quelques points importants d'un projet de recherche sur le comportement des colonnes en bois de construction avec une charge excentrique sont donnés. Les prédictions théoriques sont comparés avec les résultats expérimentaux. Les résultats de cette étude sont discutés en relation avec la pratique des projets actuels en utilisant des équations d'interactions linéaires pour les pièces de bois dans lesquelles des forces de compression et de cintrage sont combinées linéairement. L'effet des conditions aux extrémités des colonnes est aussi examiné.

ZUSAMMENFASSUNG
Es werden hier einige besonders hervorzuhebende Ergebnisse eines analytischen und experimentellen Versuchsprojekts über das Verhalten von exzentrisch belasteten Holzsäulen präsentiert. Theoretische Ergebnisse werden mit experimentellen Ergebnissen verglichen. Die Ergebnisse dieser Forschungsarbeit werden in bezug auf die gegenwärtige Konstruktionspraxis der Anwendung von Formeln linearer Interaktion für solche Holzsäulen, wo Druck- und Biegespannung linear zusammengesetzt sind, diskutiert. Die Wirkung der Endeinspannung auf die Festigkeit der Holzsäulen wird ebenfalls untersucht.

1. INTRODUCTION

The current practice for the design of timber columns [4] is based on the concept of pure, axially loaded columns. Columns in actual structures such as frames and trusses have some unavoidable bending moments along the member due to various factors such as nominal accentricity of axial load, initial crookedness in the column length, non-homogenity of the material, fabrication tolerances, etc. Due to these unavoidable bending moments, the concept of pure, axially loaded column is an idealization in actual situation.

The analysis of a structural member subjected simultaneously to axial compressive force and bending moment due to eccentricity of loading requires consideration of both deflection problem as a beam and stability problem as a column. Thus, the analysis of such members is rather tedious compared to the column analysis which is a pure buckling problem. The complexity of the problem for members in non-linear range, develops mainly from the highly involved relationship between the generalized stresses and strains in the column cross-section for this range.

The behaviour of columns, under eccentric loading, of structural materials other wood, has been the subject of research for many years in the recent past [6]. While some investigations have been conducted on centrally loaded timber columns, there has been relatively little systematic research on eccentrically loaded timber columns and the literature available on the subject is quite scanty [1, 3, 7, 19].

The broad objectives of the research reported in this paper are to study the behaviour of timber columns subjected to axial load with small eccentricity and to investigate the effect of end restraints on the strength of timber columns. Theoretical investigation and a testing program were carried out to achieve these objectives. In the present research, only those members are considered in which the plane of moment due to eccentricity is identical with the plane of actual buckling. The problem of "lateral buckling" of structural wood members is beyond the scope of this research.

2. CENTRALLY LOADED TIMBER COLUMNS

As a result of a theoretical investigation and a comprehensive testing program, Malhotra and Mazur [12] and Malhotra [13] developed a rational approach to the analysis and design of centrally loaded solid timber columns. In this approach, the Euler-Engesser tangent modulus buckling load is regarded as the lower limit of the load the ideal column can carry without too large a deflection and is taken as the basis for the design of centrally loaded columns. A stress-strain function containing three parameters is selected to solve the column buckling problem in the inelastic range of stress. A column buckling stress equation, covering a full range of column slenderness ratios, is derived in terms of these three parameters and the column slenderness ratio.

3. ECCENTRICALLY LOADED TIMBER COLUMNS

Some studies have been done to treat the problem of an eccentrically loaded column as a stress problem, in which the failure load of the column is taken as the load that initiates the yield stress (for wood, ultimate compressive stress) in the extreme fiber in compression. The most mentioned works on the subject are secant formula, modified secant formula and Perry-Robertson formula. Derivations and other details of these formulas are given in [1, 5, 11, 17, 18].

In the stability approach to eccentrically loaded columns, a critical column load is determined at which increase of lateral deflections takes place without any increase of load. A brief account of the development of stability theory of eccentrically loaded columns is given by Bleich [2].

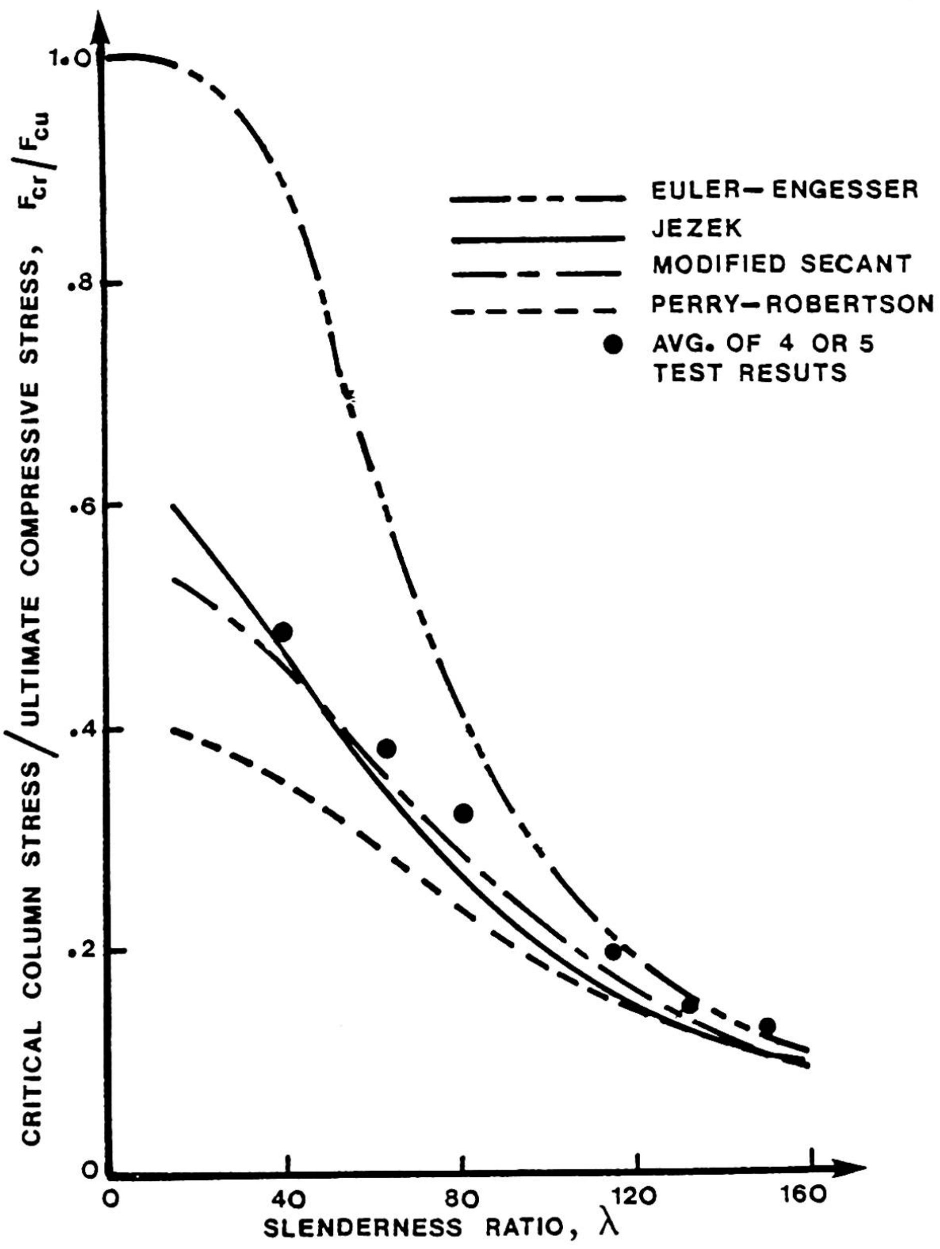


Fig. 1 Prediction and Experimental Column Stress Values for 64 by 89 mm Columns, Subjected to 13 mm Eccentricity

The serious difficulty encountered by various researchers in solving the problem of instability of eccentrically loaded columns is due to the non-linear stress-strain relationship for the column material. Assuming a simplified stress-strain relationship, linear in tension and ideally elastoplastic in compression, Jezek [8, 9] offered a relatively uncomplicated analytical solution. In the present research, Jezek's approach is adopted to predict the critical load of eccentrically loaded timber columns. Details of the theoretical development for steel columns can be obtained from [2, 8, 9] and for timber columns from [1].

An extensive testing program was undertaken to verify the theory. In all, some 560 columns of construction No. 1 grade eastern spruce lumber available in the Maritime Provinces of Canada were tested. Columns of two cross-sectional dimensions, 38 by 89 mm and 64 by 89 mm, were tested for four values of

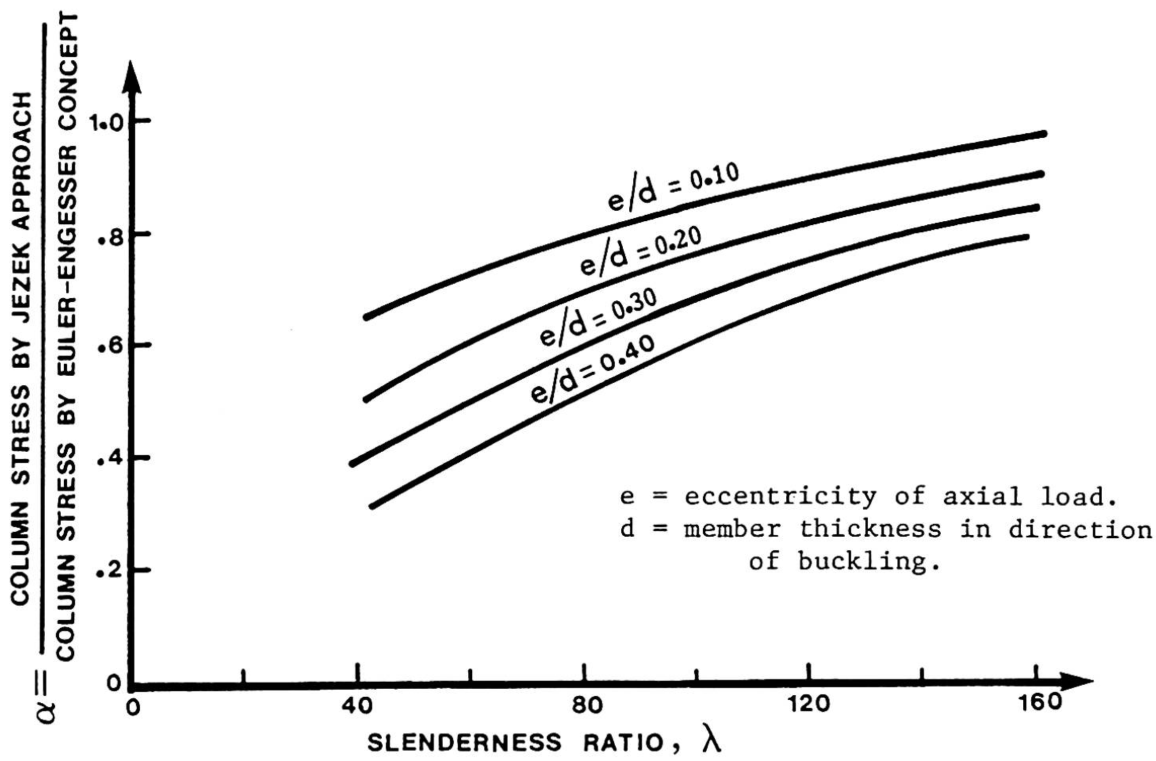


Fig. 2 Effect of Loading Eccentricity on Column Strength

eccentricities of loads. All columns for the verification of the theory were tested with pinned-end conditions. In addition, a number of series of 38 x 89 mm columns were tested with other end restraints. The details of the test results are given in [1, 12, 13, 14].

Good agreement is observed between the experimental results and theoretical predictions by the Jezek formula and by modified secant equation. The maximum difference between the predictions and test results is about 16 percent and the average difference is around 6 percent. The difference is noted to decrease with the increase in eccentricity. As an illustration, Fig. 1 is presented here. This figure represents columns of size 64 by 89 mm in cross-section, subjected to load with 13 mm eccentricity.

The effect of eccentricity on strength of columns can be seen in Fig. 2. This graph is plotted for 64 by 89 mm columns, by using the predictions by Euler-Engesser (for centrally loaded columns) and Jezek formulas. It can be observed that an eccentricity has smaller effect on the strength of long columns than on that of short and intermediate columns. Similar behaviour is noted for 38 by 89 mm columns.

The test results were also compared in relation to the following linear interaction formula:

$$\frac{P}{P_{cr}} + \frac{M}{M_u} = 1$$

in which: P = axial thrust; P_{cr} = critical column strength (buckling strength due to concentric axial load only); M = bending moment due to eccentricity of load, including amplification effect; M_u = ultimate bending moment capacity of cross-section for pure bending case.

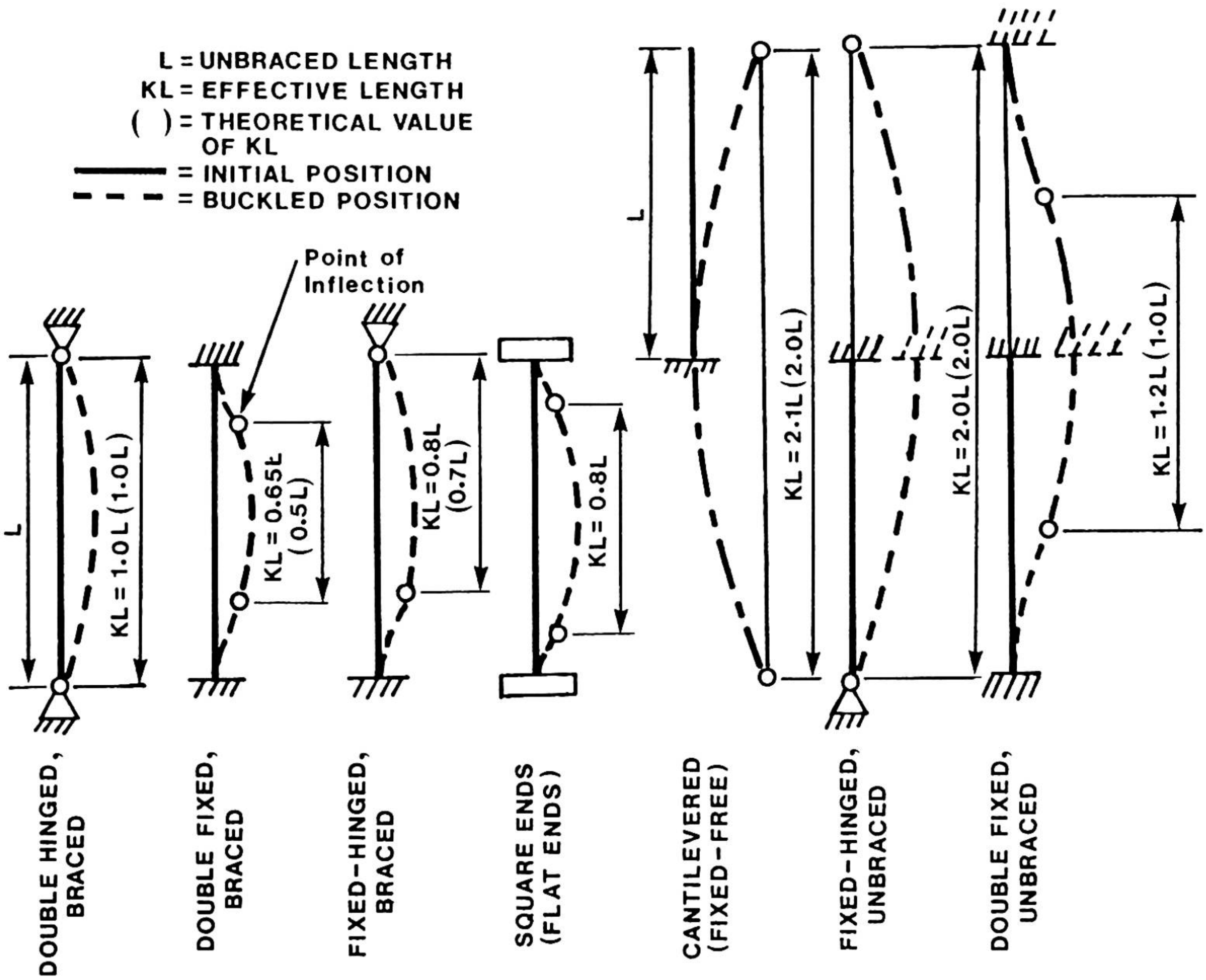


Fig. 3 Effective Length Values for Centrally Loaded Columns with Various End Conditions

As the testing in the present study was done for small eccentricities of loads only, a very limited range of interaction curve was covered. For this limited information, it was noted that the linear interaction equation underestimates somewhat the strength of intermediate and long compression members. A better agreement was obtained if M/M_u in the equation were replaced by $(M/M_u)^2$.

4. EFFECT OF END RESTRAINT

The end restraints of a column significantly influence the load-carrying capacity of the column. Generally, a column held in position and restrained against rotation at both its ends (fixed ends) is much stronger than a pinned-end column of same length and cross-section. The effect of column end restraints is recognised in the design procedure by the introduction of effective length concept. The slenderness ratio of a column is computed on the basis of its effective length rather than on unbraced length. The effective length is taken as the distance between the points of inflection on the buckled column. Commonly accepted design values and theoretical values of effective length, are given in Fig. 3 (discussion on square-end conditions is given below).

It should be noted that the effective length of a column with square ends (or flat ends) is usually taken in design to be the same as the unbraced length. Series of tests on timber columns with square ends [14] have shown that the

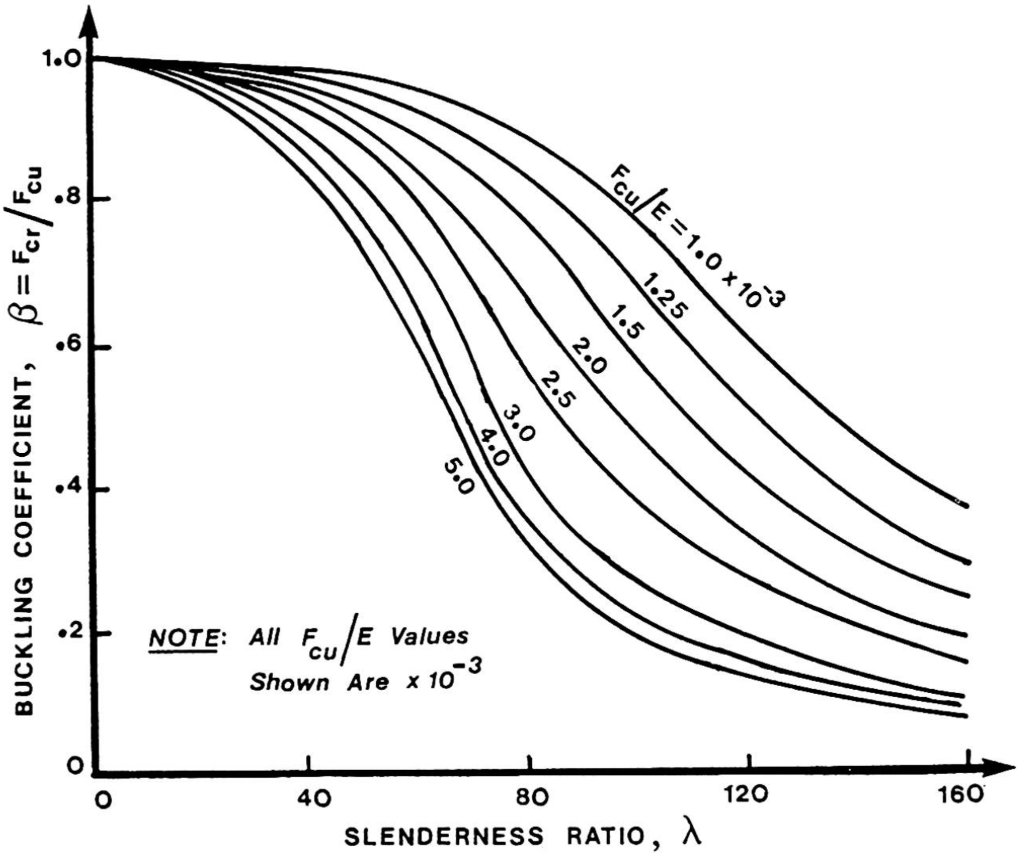


Fig. 4 Buckling Coefficient versus Slenderness Ratio Curves

strength of square-end columns is much higher than that of corresponding pinned-end columns. In the case of some intermediate and large slenderness values, this strength ratio was observed to be more than two to one. Similar results have been reported in [15, 16]. Based on these findings, an effective length (KL) value of 0.8L is recommended for square-end conditions as shown in Fig. 3.

Some exploratory tests on timber columns with other end restraint conditions have shown that the usually accepted values in design are quite reasonable.

5. DESIGN PROCEDURE

By using the Euler-Engesser as the basis, Malhotra [13] proposed "Buckling Coefficient Method of Column Design" for centrally loaded timber columns. In Fig. 4, the buckling coefficient, β, is plotted against λ for a range of F_{cu}/E values. The buckling coefficient is defined as the quantity which, when multiplied by the compressive strength of the column material, gives the stress at which the column will buckle. The β versus λ curves of type given in Fig. 4 can be generated quite easily for a wider range of F_{cu}/E values to provide a comprehensive aid for design.

The above design approach can be applied to columns subjected to axial load with small eccentricity, when used in conjunction with graphs like Fig. 2. To determine the strength of an eccentrically loaded column, one needs to know E, F_{cu}, e/d and λ for the column. For these given values, the appropriate values of α and β can be obtained from design graphs like shown in Figs. 2 and 4. Then, the critical stress of eccentrically loaded column, $F_{cr} = \alpha \times \beta \times F_{cu}$. To compute the allowable column stress, F_{cu} and E should be replaced, in all calculations, by the allowable compressive stress parallel to grain and the design value for modulus of elasticity of the column material.

6. CONCLUSIONS

(1) Theoretical predictions by Jezek approach and by modified secant formula are in good agreement with the experimental results. The influence of eccentricity on column strength is quite significant for short and intermediate columns, whereas there is relatively small effect in the case of long columns.

(2) The linear interaction equation commonly used in the design of members under combined axial and bending loads underestimates the strength of eccentrically loaded intermediate and slender compression members.

(3) Tests indicate that the strength of columns with square-end (or flat-end) condition is significantly higher than that of corresponding columns with pinned-end condition. For some intermediate and large slenderness ratios, this strength ratio is more than two to one.

(4) Based on the theoretical and experimental investigations, a semi-rational approach to the design of columns subjected to loads with small eccentrities is developed.

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