Statistical Study of Resistance of Steel Members

Etude statistique de la résistance d'éléments de construction métallique

Statistische Untersuchung der Tragfähigkeit von Bauteilen aus Stahl

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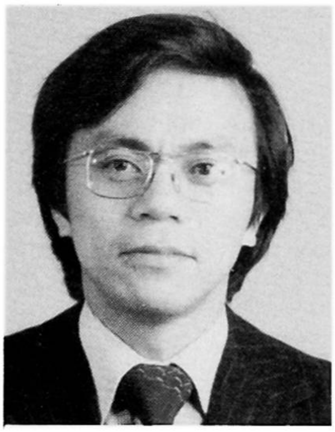

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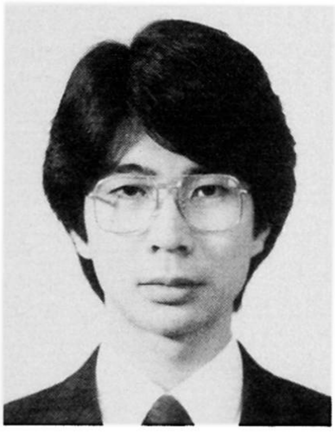

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SUMMARY
This study presents a reliability-based design method for structural steel frames, which enables the designer to control numerically the structural safety. It allows to compute and formulize the statistical resistance values of members, on the basis of widely varying experimental data.

RESUME
L'étude présente une méthode de calcul de charpentes métalliques, fondée sur la notion de fiabilité et permettant au projeteur de contrôler numériquement la sécurité structurale. La méthode permet aussi – sur la base de valeurs expérimentales – de calculer une valeur statistique de la résistance des éléments de la charpente.

ZUSAMMENFASSUNG
Der Beitrag beschreibt ein Bemessungsverfahren für Rahmentragwerke aus Stahl, das sich auf die statistische Erfassung der Materialfestigkeit abstützt. Es erlaubt dem projektierenden Ingenieur die Bruchsicherheit numerisch zu kontrollieren und den Widerstand des Tragwerkes aufgrund experimenteller Daten statistisch zu erfassen.

1. INTRODUCTION

Building structures can never be free from initial imperfections, nonuniformity of material properties, variation of constraint on structural connections, irregularities of loads and uncertainties in analysis of load transfer when the loads are converted to load effects. Hence, the safety of structures cannot be discussed without considering these uncertain factors. When remarkable progress in precision of fabrication of members and in techniques of structural analysis are considered, it is highly desirable that a reliability-based design method predicated on a probalistic approach be put in practical use. However, it still appears that some more time will have to be elapsed before such method can be commonly accepted in design practice. In this connection, the insufficiency of statistical information on these uncertain factors may be pointed out as a major factor which is preventing the early acceptance of such methods.

This research is intended to estimate the limit performance to the maximum extent, to present a design method which can control the scale of structural safety by numerical values, to substantiate the statistical information on the member resistance which is pointed out to be particularly insufficient, and to define clearly in the framework of statistics the real meaning of design strength as set out in various design criteria currently in use.

2. INTRODUCING FAILURE CRITERIA

The design method by the use of standardized values is developed beginning with the derivation of the failure probability of structures. In development of the following formulas, upper case letters are used to indicate random variables and the standard random variable, mean value and standardized deviation of X are expressed by $\tilde{X}$, $\bar{X}$ and σ_x respectively. In Fig. 1, the resistance and the load effect are expressed as independent normal random variables R and S respectively and the failure region R < S is expressed by the standardized coordinate $\tilde{R}$ - $\tilde{S}$. Then, the joint probability density function $f(\tilde{R}, \tilde{S})$ in this coordinate is expressed by the following formula:

$$f(\tilde{R},\tilde{S}) = \phi(K) / \sqrt{2\pi} \quad (1)$$

in which $\phi(\ )$ is the standard normal probability density function, and K is equal to $\sqrt{R^2 + S^2}$. Since the failure standardized value k_f is given by shortest distance from the origin to the boundary, k_f and the failure probability p_f may be expressed as follows:

$$k_f = (\bar{R} - \bar{S}) / \sqrt{\sigma_R{}^2 + \sigma_S{}^2} \quad (2)$$

$$p_f = \iint_{\sigma_R \cdot \tilde{R} - \sigma_S \cdot \tilde{S} + \bar{R} - \bar{S} < 0} f(\tilde{R}, \tilde{S}) d\tilde{R} d\tilde{S} = 1 - \Phi(k_f) \quad (3)$$

where $\Phi(\ )$ is the standard normal distribution function.
In applying to the design the failure probability thus derived, it is general practice to establish the allowable failure probability p_d firstly and then proceed with the design within the allowable range of p_d. Thus,

$$p_f = 1 - \Phi(k_f) < p_d \quad (4)$$

However, p_f which is necessary for the design presents itself as a problem generally at the skirt of the probability distribution, and it shows a delicate reaction even to minute changes in k_f. To provide a practical solution to such problem, the above formula is transformed as follows:

$$\Phi^{-1}(1 - p_d) < k_f \quad (5)$$

In the above formula, the term $(1 - p_d)$ is a probability indicating the reliability which is a complementary event of failure, and thus $\Phi^{-1}(1 - p_d)$ indicates the standardized value in the standard normal distribution. If this is defined as a design standardized value, then k_d and k_f can be checked on the same order. In this case, k_d is determined on the basis of engineering judgement by adjusting it to the requirements of the existing design criteria and by considering the significance of the structure. Any probability distribution other than the normal distribution may be considered by two methods. In one method, a corrective coefficient is obtained for each distribution pattern and k_f is multiplied by such coefficient. In this case, even the value of p_f can be obtained at the time of designing. For computation of the corrective coefficient, the failure probability is obtained first by the Monte Carlo simulation, and then the analytical value is converted to the standardized value in the standard normal probability distribution. As an alternative, the standardized value converted in the same manner as described above may be compared with k_f, and k_d itself may be corrected for each distribution. In this report, the first manner described above is followed, but k_f is presented without being modified to reflect the distribution. Consequently, from this k_d, the design of members having a reliability of $\Phi(k_d)$% is carried out to satisfy the following formula:

$$k_f = (\bar{R} - \bar{S}) \,/\, \sqrt{\sigma_R^2 + \sigma_S^2} > k_d \tag{6}$$

Where the unity of various loads is involved, each load effect S_j is regarded as acting independently, and the load effect is obtained by the following formula:

$$\bar{S} = \sum_{j=1}^{n} \bar{S}_j \,, \qquad \sigma_S = \sqrt{\sum_{j=1}^{n} \sigma_{S_j}^2} \tag{7}$$

In the design based on reliability theory, such limit state as failure of the structure is assumed. This makes it necessary to assess the resistance also in terms of the ultimate strength. Particularly, in case of those members like beams or beams-columns with low slenderness ratio whose resistance is not markedly reduced even when they are deformed by loading exceeding their maximum strength, their resistance must be established with due consideration of their deformation capacity. In this study, the energy that can be absorbed by the member is defined by the strength, using an ideal elasticity model. If the resistance which defines the deformation capacity is expressed as R' and the deformations are taken as θ_1 and θ_2 as shown in Fig. 2, the resistance R^* used in the energy estimation may be given by the following formula:

$$R^* = \sqrt{2\theta_2 \,/\, \theta_1} \cdot R' \tag{8}$$

The load effect S^* can then be obtained from the acting energy W as follows:

$$S^* = \sqrt{2 \cdot W \cdot R' \,/\, \theta_1} \tag{9}$$

By converting R and S into R^* and S^*, the design by means of the energy estimation may be carried out following the same procedure as the design which uses Eq. (6).

3. DESIGN OF STEEL MEMBERS BASED ON RELIABILITY THEORY

Columns and beams are designed by using Formula (6) as described in the preceding section. The statistical values of the respective resistances R_c and R_b which appear in the next section are computed based on the experimental data.

In case of beams-columns, their resistances are governed by two stress conditions; therefore, the method as described above cannot be used for these members. In this report, the design method is presented by expressing the resistance R_{bc} of beam-columns by the following interaction formulas and then by deriving the design standardized value. The resistance R_{bc} and the load effect S may be taken as follows:

$$R_{bc} = \frac{P_r}{p_u} + \frac{M_r}{m_u} = \bar{R}_{bc} \pm K \cdot \sigma_{R_{bc}} \tag{10}$$

$$\left(\frac{M_s}{m_u}, \frac{P_s}{p_u}\right) = \left(\frac{\bar{M}_s}{m_u} + K \cdot \sigma_{\frac{M_s}{m_u}} \cdot \cos\omega, \quad \frac{\bar{P}_s}{p_u} + K \cdot \sigma_{\frac{P_s}{p_u}} \cdot \sin\omega\right) \tag{11}$$

in which P_r and M_r are the existing axial force and the existing moment respectively, p_u and m_u are the nomimal strengths when compressive force only or bending moment only has acted on the members, and ω is the parameter which indicates the state of the load effect. If these two formulas are standardized around the mean value of the load effect, the failure region is converted as shown in Fig. 3. The standardized value at the time of failure can then be obtained from the contact conditions of these two formulas, and consequently the design is carried out by the standardized value given by the following formula:

$$k_f = \left(\bar{R}_{bc} - \frac{\bar{P}_s}{p_u} - \frac{\bar{M}_s}{m_u}\right) \Bigg/ \left(\sqrt{\sigma_{\frac{P_s}{p_u}}^{\;2} + \sigma_{\frac{M_s}{m_u}}^{\;2}} + \sigma_{R_{bc}}\right) > k_d \tag{12}$$

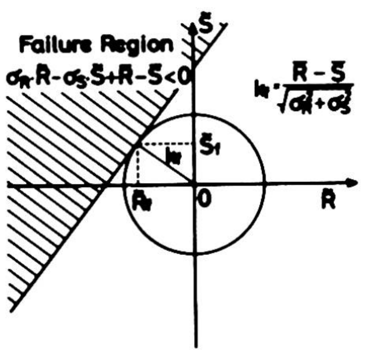

Fig.1 Failure criterion at single stress

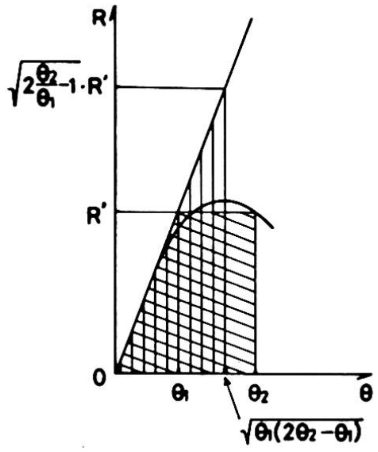

Fig.2 Equivalent resistance

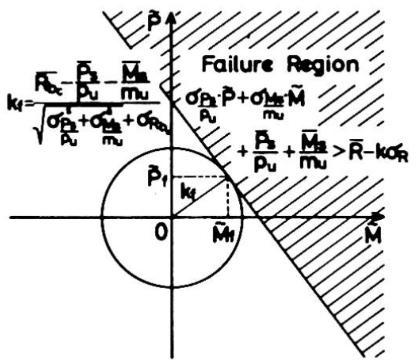

Fig.3 Failure criterion at beam-column

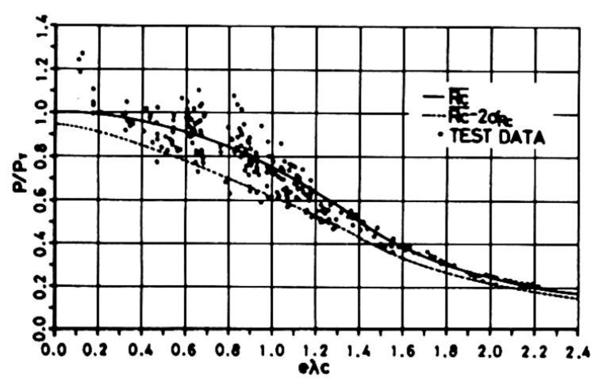

Fig.4 Test data of columns

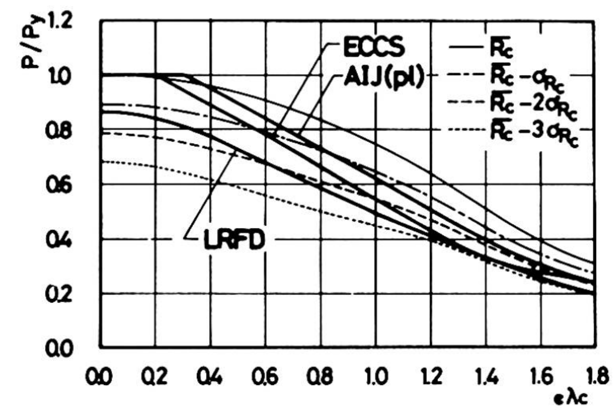

Fig.5 Comparison of mean resistance curves with design column curves

4. MEAN RESISTANCE AND STANDARD DEVIATION OF STEEL MEMBERS

4.1 Resistance of Columns

The statistical values of resistance of columns are obtained by way of the statistical arrangement from the regression analysis results of 270 test data of rolled H-section [4]~[7]. The mean value $\bar{R}_c$ and standard deviation σ_{Rc} of those values may be formulated as follows. These formulas are devised as means of approximation for practical use, using the equivalent slenderness ratio ${}_e\lambda_c$ (= $1/\pi \cdot \sqrt{Y/e_\ell} \cdot \lambda_y$ in which Y = yield axial force, e_ℓ = Young's modulus and λ_y = slenderness atio for minor axis).

$$0.0 \leqq {}_e\lambda_c < 1.4 \qquad \bar{R}_c = 1-0.25\,{}_e\lambda_c{}^2 \qquad (13\text{-a})$$

$$1.4 \leqq {}_e\lambda_c \qquad \bar{R}_c = 1/{}_e\lambda_c{}^2 \qquad (13\text{-b})$$

$$0.0 \leqq {}_e\lambda_c < 0.7 \qquad \sigma_{Rc} = 0.07\exp\{-3.46({}_e\lambda_c-0.7)^2\} \qquad (14\text{-a})$$

$$0.7 \leqq {}_e\lambda_c \qquad \sigma_{Rc} = 0.06\exp\{-2({}_e\lambda_c-0.07)^2\}+0.01 \qquad (14\text{-b})$$

In Fig. 4, the characteristic values by these formulas are compared with the experimental data. When the correlative variation of yield stress Y and area of cross section A is taken into account, the resistance $R_{c,d}$ and $\sigma_{Rc,d}$ may be expressed as follows, by taking their nominal values as y and a respectively.

$$\bar{R}_{c,d} = \frac{\bar{A}}{a} \cdot \frac{\bar{Y}}{y} \cdot \bar{R}_c \qquad (15)$$

$$\sigma_{R_{c,d}} = \frac{\bar{A}}{a} \cdot \frac{\bar{Y}}{y} \cdot \bar{R}_c \cdot \sqrt{\left(\frac{\sigma_A}{\bar{A}}\right)^2 + \left(\frac{\sigma_Y}{\bar{Y}}\right)^2 + \left(\frac{\sigma_{Rc}}{\bar{R}_c}\right)^2} \qquad (16)$$

Fig. 5 presents in a comparative way the characteristic strength curves by the two formulae above and the solutions by the present design formula. The design strength in plastic design spec. of AIJ is similar to $\bar{R}_c$ curve in a region of ${}_e\lambda_c \leqq 0.4$ and $\bar{R}_c - 1.8\cdot\sigma_{Rc}$ curve in the elastic region. In elasto-plastic region, the strength is represented by the straight line connecting these two levels. LRFD curve locates between $\bar{R}_c - 2\cdot\sigma_{Rc}$ and $\bar{R}_c - 3\cdot\sigma_{Rc}$ curves. ECCS curve has the intermediate property between AIJ and LRFD curves.

4.2 Resistance of Beams

The statistical values of resistance of beams are obtained, in the same way as for columns, from 223 test data of rolled and welded H-section under uniform moment [8]~[11]. Their $\bar{R}_b$ and σ_{Rb} may be formulated as follows, using ${}_e\lambda_b$ = $\sqrt{M_p / M_{e\ell}}$ (in which $M_{e\ell}$: elastic lateral buckling moment).

$$0.0 \leqq {}_e\lambda_b < 0.4 \qquad \bar{R}_b = 1.0 \qquad (17\text{-a})$$

$$0.4 \leqq {}_e\lambda_b < 1.4 \qquad \bar{R}_b = -0.49\,{}_e\lambda_b + 1.196 \qquad (17\text{-b})$$

$$1.4 \leqq {}_e\lambda_b \qquad \bar{R}_b = 1/{}_e\lambda_b{}^2 \qquad (17\text{-c})$$

$$0.0 \leqq {}_e\lambda_b < 0.9 \qquad \sigma_{Rb} = 0.07\exp\{-({}_e\lambda_b-0.9)^2\} \qquad (18\text{-a})$$

$$0.9 \leqq {}_e\lambda_b \qquad \sigma_{Rb} = 0.056\exp\{-3({}_e\lambda_b-0.9)^2\}+0.014 \qquad (18\text{-b})$$

Fig. 6 indicates on a comparative basis the characteristic values by these formulas and the experimental data. When the randomness of yield stress Y and plastic section modulas Z are taken into account, the resistance $R_{b,d}$ may be expressed as follows, by taking their nominal values as y and z respectively.

$$R_{b,d} = \frac{\bar{Z}}{z} \cdot \frac{\bar{Y}}{y} \cdot \bar{R}_b \qquad (19)$$

$$\sigma_{R_{b,d}} = \frac{\bar{Z}}{z} \cdot \frac{\bar{Y}}{y} \cdot \bar{R}_b \cdot \sqrt{\left(\frac{\sigma_Z}{\bar{Z}}\right)^2 + \left(\frac{\sigma_Y}{\bar{Y}}\right)^2 + \left(\frac{\sigma_{Rb}}{\bar{R}_b}\right)^2} \qquad (20)$$

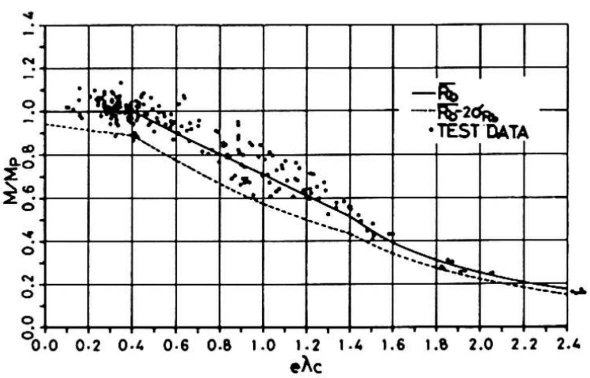

Fig.6 Test data of beams

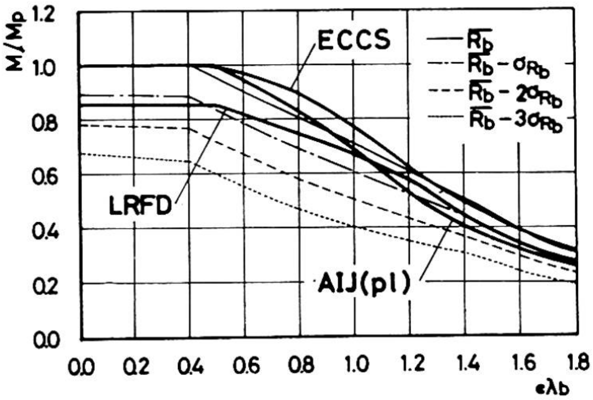

Fig.7 Comparison of mean resistance curves with design beam curves

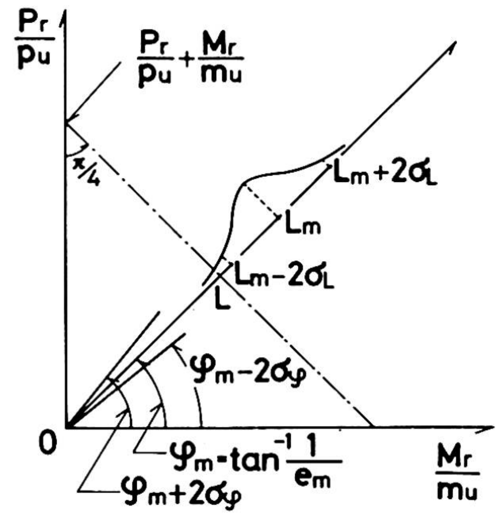

Fig.8 Evaluate method of resistance of beam-column

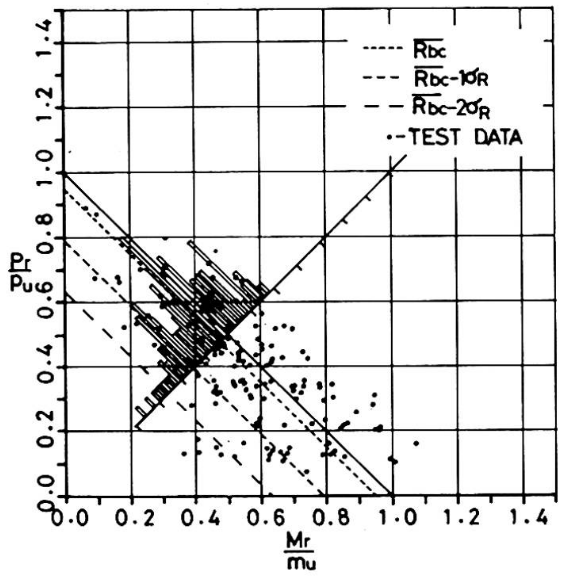

Fig.9 Test data based on Eq.(21)

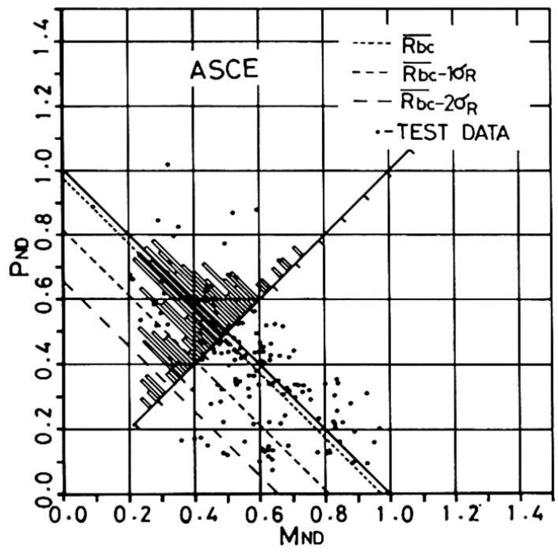

Fig.10 Test data based on ASCE spec.

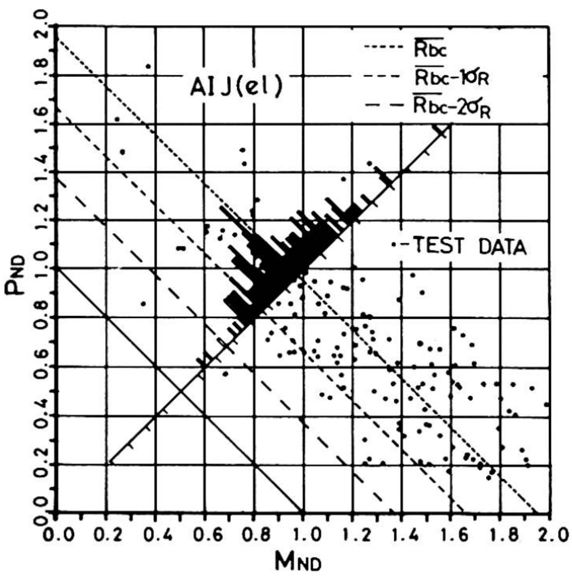

Fig.11 Test data based on $AIJ_{(el)}$ spec.

Fig. 7 presents on a comparative basis the characeristic resistance curves by these formulas and the strength by the present design formula. AIJ(el) and ECCS curves are close to $\bar{R}_b$ curve and LRFD curve is close to $\bar{R}_b - 1 \cdot \sigma_{Rb}$ curve.

4.3 Resistance of Beam-Columns

It is diffecult to formulize the resistance of beam-columns based on regression analysis of test data. In this paper, therefor, the resistance of beam-columns is discussed by test data [12]~[15] which are ploted on the correlogram based on various design formulas. When based on the statistic arrangement method, the resistance is evaluated by making reference to a section of coordinate which consists of the vertical axis presenting the axial force and of the horizontal axis presenting the bending moment (refer to Fig. 8). First, the data is plotted in a coordinate, and then, triangular subsections are selected by lines starting at the origin so that each subsection includes 20 dots of data. Distance of a dot measured from the origin is taken as L. The variation of 20 test data are evaluated in the line which have the mean value $\bar{\psi}$ of an angle ψ as measured from the horizontal axis to each L line. In this case, it is important that the mean value $\bar{R}_{bc}$ and standard deviation σ_{Rbc} of R_{bc} are uniform in every subsections with angle $\bar{\psi}$.

A port of numerical results based on seven design formulas are shown in Figs. 9 to 11. From these results, Eq. (21) is proposed as the best-ballanced form of correlation. The resistance R_{bc} is given with average value of $\bar{R}_{bc}$ and maximum value of σ_{Rbc}.

$$\frac{P_r}{\bar{R}_c \cdot p_y} + \frac{C_m \cdot M_r}{\bar{R}_b \cdot m_p} = 0.95 \pm 0.23K \qquad (21)$$

Where, $C_m = 0.6 + 0.4\beta \geqq 0.4$ = moment ratio
p_y and m_p = nominal yield axial force and nominal full plastic moment which correspond to nominal value of p_u and m_u in Eq. (10)
K = standardized value

Fig. 9 indicates the results of this formulas. The data flock a line of average value of 0.95. To illustrate the comparison results of the existing design formulas and test data in the coordinate, Fig. 10 and 11 present the results of ASCE and AIJ(el). In case of ASCE, the average values of data substantially coincide with those of design strength. In case of AIJ(el), the average values of data are two times more distant as compared with the design strength, which fact denotes that two times large marginal allowance is included in the design strength.

5. CONCLUSION

As above, the present study has introduced a design method which is based on the standardized value evaluation. It has been developed with an aim to formulate an approach to the more reliable design system for structural steel members. Also, by the statistical processing of test data, the statistical value of resistance have been formulated, the resulting values being compared with the design formulas now available. The results may be summarized as follows.

The design method, which utilizes the standardized values, consists of two parameters, namely the mean value and standard deviation of the resistance of members and load effects. These parameters directly defines the probability of failure; therefore, designers can incorporate the design level of safety into their design in a numerical way. The statistical values of resistance are calculated by the moment, up to the secondary order, of the mean values and standard deviation from the test data and are resolved into the formula. In this connection, the resistance based on test data and the design strength according to the design criteria now available are compared. In the succeeding stage, it will be required to make more realistic the statistical values including the load effects for the practicable application of the design method.

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