Stochastic optimization methods in collapse load analysis

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Méthodes d'optimisation stochastique dans le calcul de la charge de rupture

Stochastische Optimierungsmethoden für Bruchlastberechnungen

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1. Introduction

The deterministic optimization of statically indeterminate reinforced concrete or steel structures of non-linear behaviour has been worked out in detail e.g. [1, 2, 3]. In contrast to this in the field of the stochastic frame optimization a great number of problems are left insolved.

It is well known [4] that the failure probability of statically indeterminate structures is lower than that of statically determinate ones. This is due to the fact that in the semiprobabilistic design used almost all over the world, the failure probability is associated with one critical cross section /elementary beam length/ only. In reality, the failure of a statically indeterminate structure is not characterized with the failure of one, but of several critical sections /elementary beam lengths/. Obviously, the probability of the simultaneous failure of several critical sections /elementary beam lengths/ is lower than the failure probability of one critical section /elementary beam length/ alone.

In this contribution the increase of the plastic collapse load of a given probability is investigated for statically indeterminate linear plane structures on the basis of the investigations carried out at the Hungarian Institute for Building Science [5, 6, 7].

2. The structural model

The model of the structures investigated is characterized with the following conditions:

/a/ the plane structure is formed of linear bars;
/b/ only one-parametric concentrated static loads are taken into account, with the restriction, that constant moment length cannot appear;
/c/ the influence of shear and normal forces and longitudinal deformations is neglected;
/d/ the collapse mechanism is determined by plastic hinges due to bending only;
/e/ rigid-plastic material behaviour is assumed, i.e. the rotations are concentrated in the plastic hinges and the bars between the plastic hinges are rigid;
/f/ the critical elementary bar lengths /hereinafter referred to as critical sections/ at which, in case of concentrated loads, plastic hinges can be formed are the discontinuity points of the functions or the first derivatives of the bending moments or those of the plastic moment capacities;
/g/ all the quantities influencing collapse load are assumed deterministic but the bending moment capacity is assumed random variable with infinitely divisible distribution function [8] .

As the consequence of conditions /c/ and /d/ the stability problem is not investigated.

Condition /b/ regarding the lack of constant bending moment lengths means that the position of the critical sections is deterministic. If constant bending moment lengths exist, the position of the critical sections should be a random variable and together with the moment capacity can be characterized with an extremal distribution function only.

In accordance with condition /g/ the distribution function among others could be the normal or gamma-type distribution.

3. Formulation and solution of the problem

The problem is solved by the kinematic approach of the plastic analysis to determine the smallest load factor in case of which a collapse mechanism can be formed. For the solution the so called Combinations of Mechanisms method was used in which from a set of independent elementary mechanisms the real collapse mechanism with the smallest load factor is determined from the linear combination of these elementary mechanisms. This method which is well known for the deterministic model [9, 1, 2] was developed for the stochastic model. A related economic problem was independently solved in [10] .

The problem for both models can be formulated as one of mathematical programming, where the objective function is the λ load factor

$$\lambda = \underline{\theta}^* \underline{M} \rightarrow \min \qquad /1/$$

and the constraints are the following system of linear equations

$$\underline{\theta}^* = \underline{t}^* \underline{\underline{\theta}}_f \qquad /2/$$

$$\underline{t}^* \underline{e} = 1 \qquad /3/$$

where $\underline{\theta}$ is the vector of the inelastic rotations at s critical sections;

$\underline{\underline{\theta}}_f$ is the matrix of the inelastic rotations of the set of m independent elementary mechanisms and m = s-n, where n is the degree of statical indeterminacy;

$\underline{e}$ is the vector of external work, done by loads during the formation of elementary mechanisms;

$\underline{t}$ is the vector of constants of the linear combinations forming critical collapse mechanism.

The vector of the inelastic rotations was divided according to [1, 2] as

$$\underline{\theta} = \underline{\theta}^+ - \underline{\theta}^- \qquad /4/$$

and the method was completed with the justification of the uniqueness condition for /4/ in [6,7] as

$$\underline{\theta}^+ \odot \underline{\theta}^- = \underline{0} \qquad /5/$$

where the symbol $\odot$ is the so called logical product. The justification showed for both the deterministic and the stochastic model that the uniqueness condition /5/ is always fulfilled automatically for the extrema of the objective function. Consequently, this non linear condition can be neglected and the remaining constraints are linear.
The vector $\underline{t}$ can be written in the form

$$\underline{t} = \underline{t}' - \underline{t}'' \qquad /6/$$

where $\underline{t}'$ is the new variable vector which in case of subsequent $\underline{t}''$ will always be non-negative,

$\underline{t}''$ is a constant vector.

Having /4/ and /6/ the objective function can be written in the following form

$$\lambda = \underset{(2s+m)}{\underline{M}^*} \cdot \underset{(2s+m)}{\underline{x}} \rightarrow \min \qquad /7/$$

and the constraints will be replaced by the following system of linear equations

$$\underset{(2s+m,\, s+1)}{\underline{\underline{A}}} \; \underset{(2s+m)}{\underline{x}} = \underset{(s+1)}{\underline{b}} \qquad /8/$$

where $\underline{M}^* = \left[\underline{M}^{+*}, \underline{M}^{-*}, \underline{0}^*\right]$. $\underline{x}^* = \left[\underline{\Theta}^{+*}, \underline{\Theta}^{-*}, \underline{t}'^*\right]$

$$\underline{\underline{A}} = \begin{bmatrix} \underline{\underline{I}} & -\underline{\underline{I}} & -\underline{\underline{\Theta}}_f^* \\ \underline{0}^* & \underline{0}^* & \underline{e}^* \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} -\underline{\underline{\Theta}}_f^* \cdot \underline{t}'' \\ 1 + \underline{e}^* \underline{t}'' \end{bmatrix}$$

$\underline{M}^+$ and $\underline{M}^-$ are vectors of the positive and negative plastic moment capacities at the critical sections and

$\underline{\underline{I}}$ and $-\underline{\underline{I}}$ are identity matrices of appropriate signs.

The plastic moment capacities for the deterministic model are fixed values, but for the stochastic model they are random variables of known distribution function. The combination of these plastic moment capacities results in the collapse load factor, which, consequently, will also be a random variable of the same type of distribution function.

Any point of the distribution function of the collapse load factor i.e. the collapse load of a given probability can be determined as follows.

It is well known [8] that any linear combination of random variables with infinitely divisible distribution function will be of the same type of distribution function. The mean value, the standard deviation etc. of the resulting distribution can be expressed knowing the mean values, standard deviation etc. of the initial distribution and the combination coefficients as:

$$\bar{\lambda} = (\underline{\Theta}^+)^* \bar{\underline{M}}^+ + (\underline{\Theta}^-)^* \bar{\underline{M}}^- \qquad /9/$$

$$D^2(\lambda) = \left\{\underline{(\Theta^+)^2}\right\}^* \cdot \underline{q}^+ + \left\{\underline{(\Theta^-)^2}\right\}^* \cdot \underline{q}^- \qquad /10/$$

where $\underline{q}^+$ and $\underline{q}^-$ are the variances of the respective plastic moment capacities.

Assume according to [11] that the failure probability of a structure will be $p_o=8{,}2.10^{-5}$. Knowing the distribution function of λ determine that value of λ_s, depending on vectors $\underline{\theta}^+$ and $\underline{\theta}^-$ for which the probability of occurence of the smallest λ will be less than the given p_o. If u_o will be the quantile p_o of the standardized distribution function, then this λ_s value will be

$$\lambda_S = D(\lambda)\, u_o + \bar{\lambda} \qquad /11/$$

Using the previous expressions the value of λ_S can be given as function of rotation vectors as

$$\lambda_S = u_o\sqrt{\underline{x}^* \underline{\underline{Q}}\, \underline{x}} + \underline{\bar{M}}^* \underline{x} \qquad /12/$$

where $\underline{\underline{Q}} = \langle \underline{q}^+, \underline{q}^- \rangle$ is a diagonal matrix, formed of vectors $\underline{q}^+$ and $\underline{q}^-$.

The minimum of this objective function, which in this way is deterministic, will be the collapse load of the given probability according to the stochastic model.

For the deterministic model the objective function is linear and for its solution the simplex method is appropriate. However, for the stochastic model, the objective function is concave as was shown in [6] . This type of problem, with linear constraint can be solved by the cutting plane method [12] well suitable for computer applications [13] .

4. Practical application of the method

The effectiveness of the more exact stochastic model was checked on some practical examples of different parameters.

The deterministic and stochastic models can be compared by prescribing similar failure probabilities for critical sections using the deterministic model /p_i/ and for the whole structure using the stochastic model /p_o/ and determining how much the load bearing capacity computed according to the deterministic model will be exceeded by the one computed according to the stochastic model.

It was proved [7] that for this condition the deterministic load bearing capacity will be a lower bound solution of the stochastic load bearing capacity. In [6, 7] two simple upper bound solutions were also given.

Simple one span, one storey frames were analysed in case of 7 loading schemes, consisting of vertical and horizontal concentrated loads. The possible distributed loads were modelled by a system consisting of an odd number of concentrated loads.

The distribution function of the plastic moment capacities of the critical sections was assumed to be of normal distribution.

The span /l/ to height /h/ ratio was assumed as $l/h=2,4,1/2$.

The assumed ratios of the plastic moment capacities of the girder $/M_l/$ and the column $/M_h/$ are shown in the Table 1.

Table 1

Plastic moment capacity type		1	2	3
M_l/M_h	+	3/2	1	3
	-	1	2/3	3/2

Signs + and - indicate moments, producing tension at the inner and outer side, respectively, of the bars. The coefficient of variation of the plastic moment capacities was assumed as r=0.015, 0.05, 0.15 and 0.25. Of course for the latter and small failure probabilities the assumed normal distribution gives a considerable error. The convergence of the solution was very slow in case of high coefficients of variations, too.

Altogether 3o frames were investigated using both the deterministic and the stochastic model.

The results of the calculation for the frame shown in Fig.1 are given in Table 2.

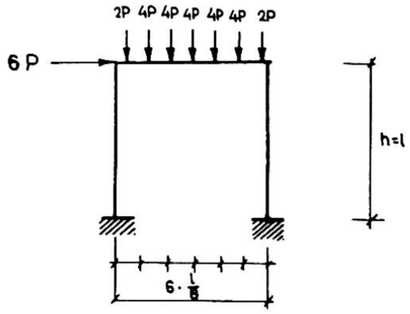

Fig.1

Table 2

parameters / number of example	l/h	plastic moment capacity type	$\frac{\lambda_{so}}{\lambda_{do}}$	$\frac{r_\lambda}{r}$	p_i
1o	2	1	1.1o4 1.o51	o.553 o.782	$1.9.10^{-2}$ $1.6.10^{-3}$
13	4	1	1.o78 1.o26	o.663 o.887	$6.1.10^{-3}$ $4.1.10^{-4}$
16	1/2	1	1.116 1.o61	o.5oo o.738	$3.o.10^{-2}$ $2.7.10^{-3}$
22	2	3	1.o82 1.o32	o.648 o.862	$7.4.10^{-3}$ $5.8.10^{-4}$

where λ_{so} and λ_{do} are the collapse load factors for the stochastic and for the deterministic model, respectively,

r_λ is the coefficient of variation of the collapse load factor for the frame,

r is the coefficient of variation of the plastic moment capacity at the critical sections,

p_i is failure probability of the plastic moment capacity at the critical sections, assuming the failure probability of the whole frame $p_o = 8{,}2 \cdot 10^{-5}$.

The two values in each box in Table 2 correspond to the lower and upper bound values after iterations consuming prefixed computer time.

5. Discussion of the results

/a/ From the results it became clear, that a substantial difference is observed between the load bearing capacity of the deterministic and the stochastic structural models. This difference is given in Table 3.

Table 3

r	0.015	0.05	0.15
$\lambda_{so} / \lambda_{do}$	2-3 %	3-12 %	22 %

/b/ The different analyses according to the deterministic and stochastic models give not only different collapse load factors, but in some cases different failure mechanisms too, as is shown in Fig.2.

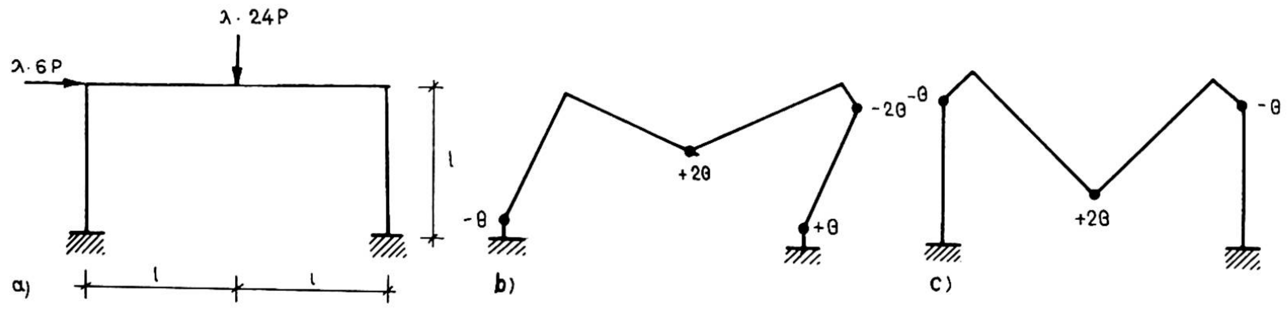

Fig.2

a - the frame scheme; b - failure mechanism according to the deterministic model; c - failure mechanism according to the stochastic model.

/c/ The coefficients of variation of the collapse load factor of the frame for the stochastic model are much lower than for the deterministic model, as can be seen in Table 2. The ratio of r_λ / r was between 0,5 and 0,78.

/d/ There is another way of comparison of the results obtained according to the two models. This is the determination of the failure probabilities of the plastic moment capacities at the critical sections p_i at a given failure probability of the whole frame $p_o = 8{,}2 \cdot 10^{-5}$ according to the stochastic model. These values of p_i in case of examples of good convergence were in the range of 10^{-3} - 2.10^{-2}, which is much higher than in case of the deterministic model, where in each critical section $p_i = 8,2.10^{-5}$ should be maintained.

6. Conclusions

The stochastic structural model for statically indeterminate plane structures formed from linear bars gives considerably higher load bearing capacity, lower coefficient of variation, higher failure probability in each critical section, than the deterministic structural model. In some cases the failure mechanisms can also be different for stochastic and deterministic models.

It is planned to investigate distributions more realistic than the normal one taking the elastic-plastic material behaviour and the randomness of the critical section position into account. Examples of more complicated structural schemes are planned to be analysed by applying computational methods of better convergence.

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SUMMARY

The increase of the plastic collapse load of a given probability is investigated for statically indeterminate linear plane structures, assuming the plastic moment capacities at the critical section to be random variables of infinitely divisible distribution. The Combinations of Mechanisms method was developed for the stochastic structural model. The mathematical and computational problems were solved and 30 simple frame examples were investigated. The results showed higher plastic collapse load, lower coefficient of variation and higher possible critical section failure probabilities for the stochastic model as compared to the deterministic one.

RESUME

L'augmentation de la charge plastique de rupture pour une probabilité donnée est examinée pour des systèmes de barres hyperstatiques en plan, sous la condition que les capacités de moment plastique sont des variables probables d'une distribution infiniment divisible. La "combinaison des mécanismes" est développée pour le cas du modèle stochastique. Les problèmes mathématiques et d'ordinateur sont résolus et 30 portiques simples examinés. Les résultats ont montré pour le modèle stochastique une charge de rupture plastique élevée, un moindre coefficient de variation et une plus grande probabilité de rupture possible comparé au modèle déterministique.

ZUSAMMENFASSUNG

Die Erhöhung der plastischen Bruchlast gegebener Wahrscheinlichkeit wurde bei statisch unbestimmten ebenen Stabwerken unter der Bedingung geprüft, dass die plastische Momenten-Tragfähigkeit in den kritischen Querschnitten eine unbegrenzt dividierbare Zufallsvariante ist. Die Methode der "Kombination der Mechanismen" wurde im Fall eines stochastischen Konstruktionsmodells weiterentwickelt. Mathematische und rechnungstechnische Fragen wurden gelöst und das Zahlenmaterial von 30 einfachen Rahmen geprüft. Die Ergebnisse zeigen eine höhere plastische Bruchlast, kleinere Variationskoeffiziente und grössere mögliche Wahrscheinlichkeit der Zerstörung im Falle des stochastischen Modells gegenüber dem deterministischen Modell.