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Optimization Techniques under Random Loading Effects

Techniques d'optimisation et effets des charges aléatoires

Optimierungstechnik bei Wirkung von Zufallsbelastungen

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1. INTRODUCTION

The developments that have taken place in the last few years in the field of optimization techniques applied to structural problems were restricted mainly to structures subjected to deterministic loadings. The reasons for the lack of research activities towards the analysis of structures under the effects of random loadings could be attributed to the mathematical complication involved in the procedure and the non-availability of sufficient and reliable data regarding the past histories of the random exciting force.

In this paper a simplified approach is reported to deal with the structural optimization problems under non-stationary loadings by making use of the upper bound probability of failure of the structure. The analysis is carried out in two phases:

(A) to obtain an expression for the probability that the response of the structure at a critical zone reaches for the first time an upper limit value with time-dependent control-barriers, interms of their rate of uperossings; and

(B) to seek an approximate solution to the optimization problem, using the result obtained in phase (A), with the probability of failure, the natural frequency of vibration and the frequency response function of the system as restraints.

2. PHASE (A).

The estimation of the upper and lower bound probabilities of failure of a structure in a closed internal of time, has been a field of great interest among engineers dealing with random vibration problems. J.J Coleman[1] for the first time, suggested an approximate solution to estimate the upper bound value interms of the expected rates of the threshold crossings of the response process at positive and negative slopes. However, the process of independent arrivals of failure, as assumed by Coleman, is unacceptable especially for narrow band random process, such as the response of lightly damped dynamic systems. Besides, for low damped structural systems, crossings of response process tend to occur in 'clumps' of dependent crossings and hence the expected rate of threshold crossings should be replaced by the average clumping rate. M Shinozuka[2] has developed a method applicable to stationary and non-stationary cases as well, to estimate the upper and lower bounds for the probability of the first excursion failure within an arbitrary semi-closed time internal (o, t) and constant barriers without the assumption of independent threshold crossings. When the computed values of the upper and lower bounds are sufficiently close to each other, they are just as valuable as the mathematically exact values of the probability as a basis for making engineering decisions. In a paper[3] published later, Shinozuka has further extended his solution to take into account the effects of time-dependent barriers also.

The solution to the above problem with time-dependent barriers, presented in this paper is a modification to Shinozuka's approach with a different interpretation, interms of the expected rate of crossings of the response-barriers.

Following Shinozuka's expression for the upper - bound probability of failure of the structure,

$$P_r[t; -Y_2(t), Y_1(t)] < P_r[t; -Y_2(t), \infty] + P_r[t; -\infty, Y_1(t)] - P_r[\{x(t_1) < -Y_2(t_2)\}\{x(t_2) > Y_1(t)\}] \quad ----(1)$$

where x(t) represents the response of the system at a critical zone and the failure of the system, for the first time, is defined as when $x(t) \geq Y_1(t)$, or $x(t) \leq -Y_2(t)$, in which $Y_1(t)$ and $Y_2(t)$ are positive barriers of response process.

Let $N[Y_1(t), t]$, hereafter referred as N_1, represents a random variable denoting the number of crossings of $Y_1(t)$ from below during the internal (0,t). The probability that $N[Y_1(t), t]$ takes a value 'r' during (0,t), $P_r[N_1 = r]$, can be expressed as:

$$P_r[N_1 = r] = P_r[N_1 = r;\ x(0) \geq Y_1(0)] + P_r[N_1 = r;\ x(0) < Y_1(0)] \quad ----(2)$$

Also,

$$P_r[t; -\infty, Y_1(t)] = P_r[x(0) > Y_1(0),\ N_1 \geq 0] + P_r[x(0) < Y_1(0),\ N_1 \geq 1] + P_r[x(0) = Y_1(0),\ N_1 \geq 0] \quad -------(3)$$

Equation (3) can further be simplified as :

$$P_r[t; -\infty, Y_1(t)] = P_r[x(0) > Y_1(0)] + P_r[x(0) < Y_1(0)]\, P_r[N_1 \geq 1 \mid x(0) < Y_1(0)]$$
$$\leq P_r[x(0) > Y_1(0)] + P_r[x(0) < Y_1(0)] \sum_{s=1}^{\infty} s\, P_r[N_1 = s \mid x(0) < Y_1(0)] \quad -(4)$$

Equation (4) with the help of equation (2) finally reduces to,

$$P_r[t; -\infty, Y_1(t)] \leq P_r[x(0) > Y_1(0)] + E[N_1] - P_r[x(0) \geq Y_1(0)]\, E[N_1 \mid x(0) \geq Y_1(0)] \quad ---(5)$$

in which E denotes the expected value.

If $N[-Y_2(t), t]$, hereafter referred as N_2, represents a random variable denoting the number of crossings of $-Y_2(t)$ from above during an internal $(0,t)$,

$$\begin{aligned} P_r[t; -Y_2(t), Y_1(t)] < {} & P_r[x(0) < -Y_2(0)] + P_r[x(0) > Y_1(0)] + \\ & P_r[x(0) > -Y_2(0)]\, E[N_2 \mid x(0) > -Y_2(0)] + \\ & P_r[x(0) < Y_1(0)]\, E[N_1 \mid x(0) < Y_1(0)] - \\ & P_r[\{x(0) < -Y_2(0)\}\{x(t) > Y_1(t)\}] \quad \text{- - - -(6)} \end{aligned}$$

Equation (6) in effect represents the best upper bound probability of failure of the structure interms of the rate of crossings of the time-dependent barriers of response process.

In case the response process starts from zero origin, such that $P_r[x(0)=0] = 1$, equation (6) further simplifies to :

$$P_r[t; -Y_2(t), Y_1(t)] < E[N_1] + E[N_2] - P_r[\{x(t_1) < Y_2(t_1)\}\{x(t_2) > Y_1(t_2)\}] \quad \text{..(7)}$$

The approach presented above, to estimate the upper bound value becomes significant in dealing with those problems where a stationary process for a finite time internal is observed, as in certain control system problems.

3. PHASE (B).

An approximate solution to the structural optimization problem is attempted in this phase, making use of the results obtained in phase (A), with the probability of failure of the structure and the system-characteristics as restraints.

Let $Z(d)$ be the objective function to be minimised subject to the condition,

$$P_r\Big[\bigcup_{i=1}^{k} \{S_i(x(d,t)) \geqslant r_i\}\Big]_j \leqslant [p_f]_j \quad \text{. . . . .(8)}$$

and $S_j(x(d,t)) \leqslant r_j$ . . . . (9)

and $\omega_{il} \leqslant \omega_i \leqslant \omega_{iu}$ . . . . (10)

where $S_i(x(d,t))$ is the frequency response function of the system; $x(d,t)$ represents the response (stress, strain or displacement) at a critical zone to random excitation;

ω_{il}, ω_{iu} are the lower and upper limits of the natural frequency of vibration of the structure, respectively;

$[p_f]_j$ denotes the upper limit of the probability of failure under mode j.

Let $P_r[S_i(x(d,t)) \geqslant r_j] = p_i(d)$ . . . . (11)

For example, if the safety of the structure is analysed on the basis of the external load acting on it and its internal resistance, say F and R respectively, both treated as statistically independent normal distributions, then,

$$p(d) = \frac{1}{\sqrt{2\pi}} \int_{\rho}^{\infty} e^{-x^2/2}\, dx, \quad \text{. . . . . (12)}$$

where $\rho = \dfrac{\bar{R} - \bar{F}}{\sigma_R} \dfrac{1}{\sqrt{1 + (\sigma_F/\sigma_R)^2}}$ - - - - - - - - - - - - (13)

in which $\bar{R}$ and $\bar{F}$ are respectively the mean value of the resistance and the load; σ_R^2 and σ_F^2 are their variance.

Equation (8) now reduces to,

$$P_r\left[\bigcup_{i=1}^{k}\{S_i(x(d,t)) \geqslant r_i\}\right]_j = \left[\sum_{i=1}^{k} p_i(d)\right]_j, \quad \ldots\ldots (14)$$

the limit of summation of the time variable being from $-\infty$ to ∞. It follows,

$$\left[\sum_{i=1}^{k} p_i(d)\right]_j \leqslant [p_f]_j, \quad j = 1, 2, \ldots n. \quad \ldots\ldots (15)$$

In the case of non-stationary random excitations, for example, ground acceleration due to earthquakes, the left hand side of equation (15) may be replaced by the upper bound value of the probability of failure of the structure as obtained in phase (A).

4. CONCLUSIONS.

Since a knowledge of the rate of crossings of the time-dependent response-barriers is an essential pre-requisite to the present analysis, a rigorous statistical analysis of the past records of the random exciting force is warranted to achieve a high level of accuracy. A large class of optimization problems in control system engineering could be advantageously studied using this method.

REFERENCES

1. J.J. Coleman, "Reliability of Aircraft Structures in resisting chance failure" - Operations Research, 7, 1959, P640 - 645.

2. M. Shinozuka, "Probability of Structural failure under random loading", Journal of Engg Mechanics, ASCE Pub, 90 EM5, 1964, P 147-170.

3. M. Shinozuka, "On the two-sided barrier problem", Journal of Applied Probability, Vol 2, 1965, P 79-87.

SUMMARY

A general solution to deal with structural optimization problems under non-stationary random loadings is presented, with the upper bound probability of failure of the structure within time-dependent barriers and the system characteristics as restraints.

RESUME

Une technique générale d'optimisation des structures est présentée pour le cas de charges aléatoires. Les caractéristiques du système et les valeurs supérieures de la probabilité de ruine en fonction du temps sont prises en considération.

ZUSAMMENFASSUNG

Es wird eine allgemeine Lösung der Bauoptimierungsprobleme für nicht stationäre Unfallsbelastungen dargestellt, mit der oberen Grenze der Versagenswahrscheinlichkeit innerhalb zeitabhängiger Grenzen und den Systemcharakteristiken als Einschränkungen.