Structural optimization via penalty methods: a new type of penalty function

Structural Optimization via Penalty Methods: A New Type of Penalty Function

L'optimisation structurale par les méthodes de pénalisation: un nouveau type de fonction de pénalité

Optimierung von Tragwerken durch Strafmethoden: ein neuer Typ von Straffunktionen

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1. Introduction

Sound mathematical idealizations of practical design problems lead as a rule to highly nonlinear, and possibly nonconvex, programming problems.

The main effort in the field of computerized design methods should therefore be concentrated upon the implementation of versatile numerical procedures capable of solving, at least in principle, general mathematical programming problems. It is obvious that particular problems can be solved more cheaply by means of 'ad hoc' techniques exploiting their special properties, but it is the authors' opinion that the general approach should yield the major improvements to structural optimization, at the present stage of its development.

In this note, the attention is focussed on sequential unconstrained minimization techniques, which seem to be among the most interesting approaches for general automated design routines. A new kind of penalty function is introduced, and applied to a typical design problem, with the aim of assessing its capabilities.

2. Mathematical formulation

We consider the following type of problem

$$\begin{array}{ll} \text{minimize} & f(x_i) \\ \text{subject to} & g_j(x_i) \leq 0 \end{array} \qquad (i=1, \ldots, n;\ j=1, \ldots, m) \tag{1}$$

From problem (1) the following parametric problem is derived

$$\text{minimize} \qquad f(x_i) + \sum_{j=1}^{m} < 1 + g_j(x_i) >^{\alpha} \qquad (2)$$

where the symbol $<\cdot>$ has the meaning

$$<\cdot> = \max\,(0,\ \cdot)$$

and the parameter α ranges over the open interval $(1, +\infty)$.

Each inequality constraint $g \leq 0$ is accounted for by a penalty term

$$p(g) = <1+g>^{\alpha} \qquad (3)$$

From fig. 1 it is apparent that function (3) is neither an interior nor an exterior penalty function.

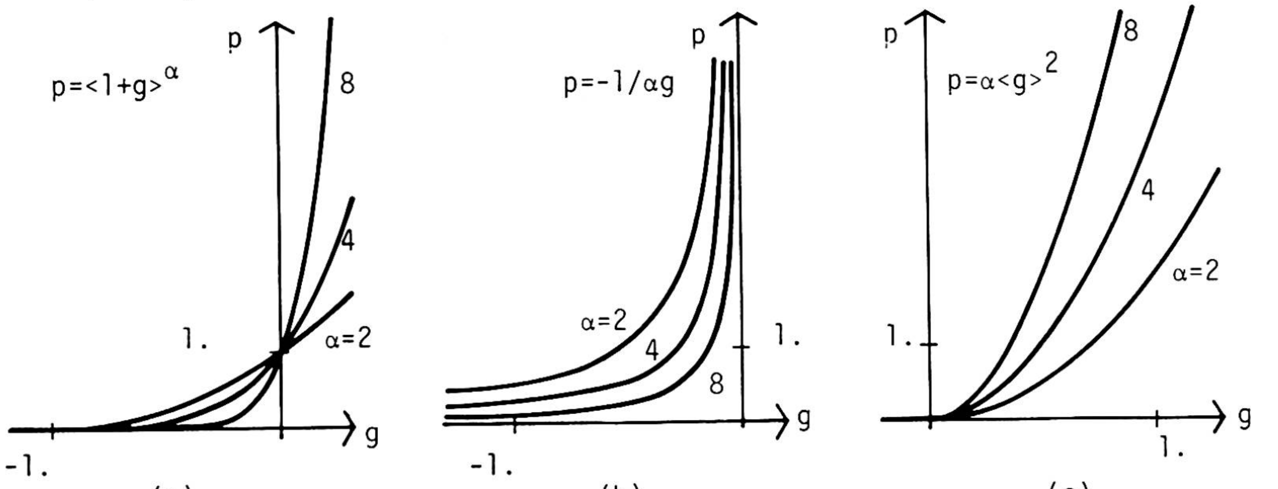

Fig. 1: Proposed penalty function (a) versus interior (b) and exterior (c) penalty functions.

The main properties of formulation (2) may be stated as follows:

i) if problem (1) has a (local) solution, a solution of problem (2) will approach it, when α approaches infinity;

ii) in contrast with interior penalty functions, penalty function (3) is defined over the range $-\infty < g < +\infty$;

iii) in contrast with exterior formulations, formulation (2) yields feasible minima for sufficiently large values of α, i.e. the solution of problem (1) is approached from the inside of its feasible region.

Properties ii) and iii) give an obvious advantage to penalty function (3) over interior and exterior penalty functions, respectively.

3. Allowable stress design of a truss with assigned topology

If a minimum weight design is sought, the objective function is easily ex pressed in terms of member cross-sectional areas and joint coordinates. For each member and each load condition, the following constraints are considered

$$g_+ = \sigma/\sigma^+ - 1 \leq 0$$
$$g_- = \sigma/\sigma^- - 1 \leq 0 \quad (4)$$

where σ^+ (σ^-) is the allowable tension (compression) stress of the considered member. If member buckling is accounted for, the compression limit σ^- depends on the (minimum) radius of gyration of the member cross-section. For a given type of cross-section, the radius of gyration can usefully be expressed as a function of the area, thus leaving only one design variable for each member. A second set of constraints will impose a minimum admissible value to each area. Displacement constraints may be obviously included.

The major task is to compute the stress and its gradient (the displacement method of analysis is of course preferable). Special attention must be devoted to the fact that stress constraints (4) are not defined over the entire design space: in fact, there exist (unfeasible) designs for which in one or more members the stress grows to infinity. This difficulty can be cured by introducing suitable modifications of the stress constraints (4) outside the feasible region, and by adopting a careful minimization strategy.

4. Numerical results

An algorithm (AUDE) for the numerical solution of automated design problems, based on the described formulation, has been developed. The minimization (2) is performed, for a sequence of suitably increasing (integer) values of α, using the Davidon-Fletcher-Powell method. Two-point cubic fit for successive unidirectional searches is used. Size and geometry variables are treated simultaneously.

The results obtained for a sample design problem, relative to a steel planar truss, are represented in fig.2. The lower chord is assumed to be straight and made up of six bars, long 5 m each. The total span of the upper chord is 30 m also, but its shape is free. All members are tubular, and their thickness is supposed to be adequately represented by the relationship

$$t = 1.5 + 0.02\,D \qquad (t, D \text{ in mm})$$

D being the diameter. Load conditions are specified by a single 10,000 kg concentrated load, moving along the lower chord. The allowable tension stress is assigned a value of 2,400 Kg/cm^2, and the allowable compression stress is computed in terms of the member slenderness ratio according to the Italian Code requirements.

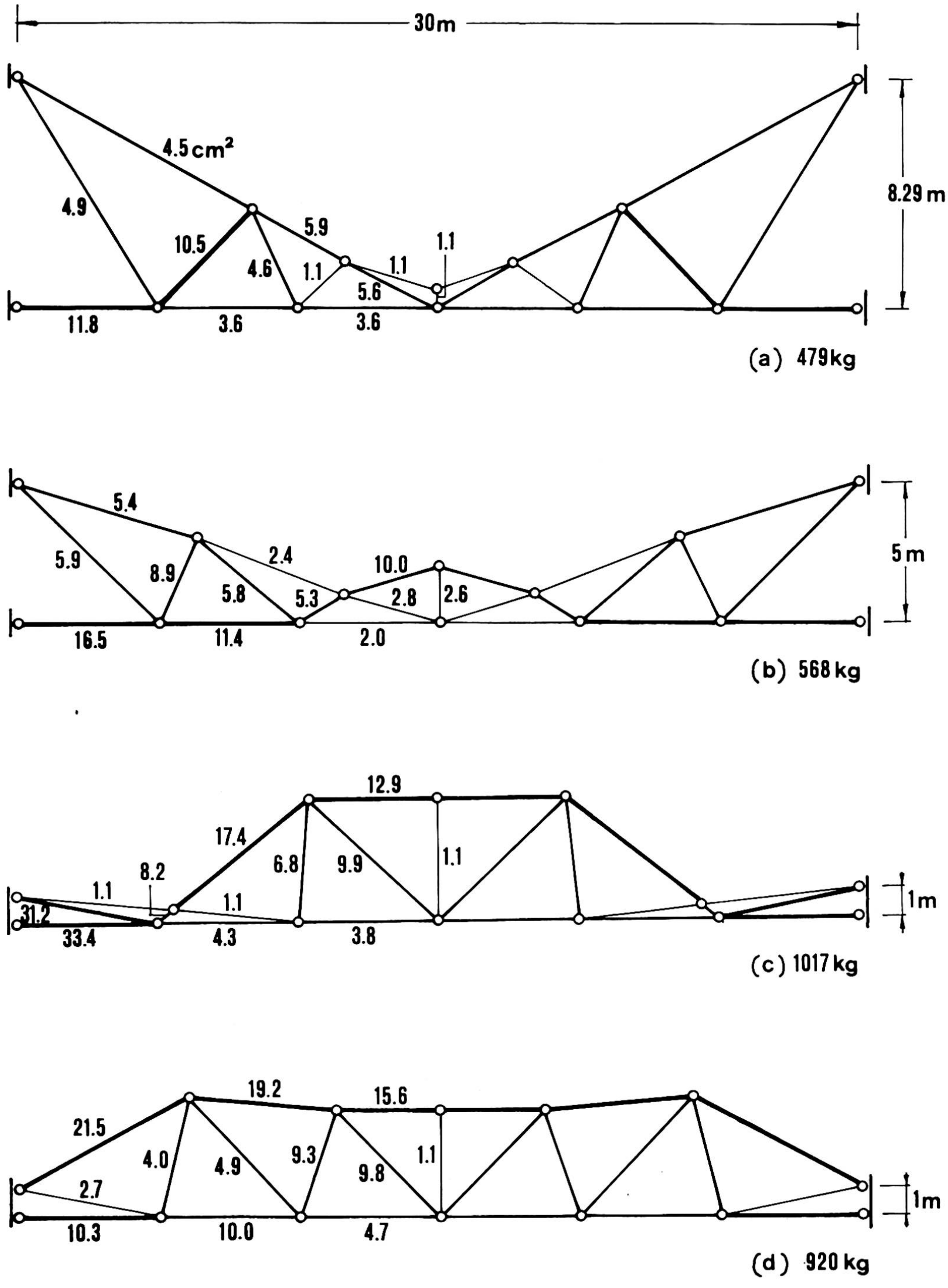

Fig. 2: Truss design

The lower chord should not undergo vertical displacements greater than 1/800 of the span. The truss should be designed for minimum weight.

Taking into account the obvious symmetry of the optimal solution(s), the above stated problem can be treated with 12 size variables, 6 geometry variables, and 3 load conditions. The optimal design obtained by AUDE is depicted in fig. 2a, where the member areas (in cm^2) are also reported. Note that 1.1 cm^2 was the minimum allowable area used in the computation. The weight of the optimum truss is 479 Kg, its height 8.29 m.

If now the distance H between the supports if given a fixed value, the geometry variables reduce to 5, and the optimum weight should obviously increase. Fig. 2b shows the solution obtained for H = 5 m. For H = 1 m two local optima have been detected (figs. 2c, d), the second one being a good candidate for the global solution.

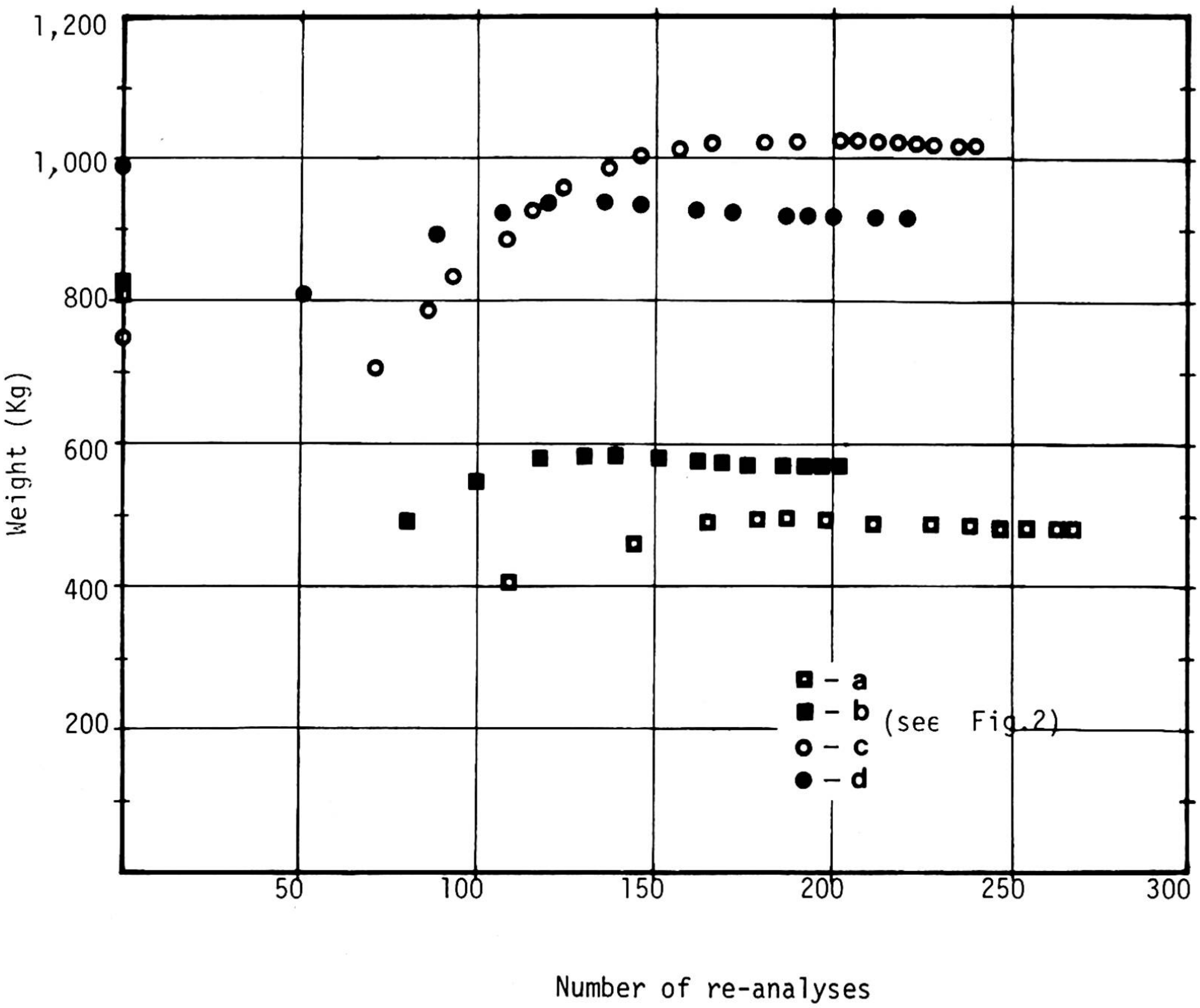

Fig. 3: Minimization trend

In each of these calculations, the parameter α was increased until a value of about 8000, and 200 ÷ 250 re-analyses were performed. Fig. 3 shows the sequences of minima relative to the four cases of fig. 2. As it is seen, after a drastic change on the first response surface, the objective function approaches rather smoothly its asymptotic value.

SUMMARY

An exponential penalty function is introduced and applied to a typical non-linear and nonconvex design problem. Some results on geometry optimization of plane trusses are presented and discussed.

RESUME

On introduit une fonction de pénalisation exponentielle, et on l'applique à un problème typiquement non linéaire et non convexe d'optimisation structurale. On présente et on discute quelques résultats relatifs à l'optimisation géométrique de structures réticulées planes.

ZUSAMMENFASSUNG

Eine exponentielle Straffunktion wird auf ein typisch nichtlineares und nichtkonvexes Tragwerksproblem angewandt. Einige Ergebnisse über die Optimierung der Geometrie von ebenen Fachwerken werden angegeben und besprochen.