Considerations on optimum combination of safety and economy

Considerations on Optimum Combination of Safety and Economy

Considérations sur le meilleur compromis entre sécurité et économie

Betrachtungen über den besten Kompromiss zwischen Sicherheit und Wirtschaftlichkeit

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The report |1| has served to emphasize that engineering design requires methods and concepts of probability. The process of selecting the probability of failure of a structure to achieve a stated objective (e.g. make minimum the total expected cost of the structure) is known as probabilistic optimum based-design |7|.

The goal of this discussion is to show that a few useful conclusions can be found in the safety-economy optimization process.

In order to evaluate the total expected cost of a structure C_t, the initial cost C_i, the cost of failure C_f and the probability of failure P_f are to be calculated first. Optimum probabilistic design requires the minimization of C_t :

$$C_t = C_i + C_f \cdot P_f \qquad (1)$$

The central safety factor of a structure is very nearly a linear function of $\text{colog}_{10} P_f$ |2|, |3| ,|6| :

$$\theta = \alpha_1 \cdot \text{colog}_{10} P_f + \alpha_2 , \qquad (2)$$

where α_1 is the increment of central safety factor required to reduce P_f by a factor of 10, and α_2 is a positive constant.

On the other hand, the initial cost C_i may be taken as a linear function of θ |2|, |5|, |8| :

$$C_i = \gamma_1 \cdot \theta + \gamma_2 = \beta_1 \cdot \text{colog}_{10} P_f + \beta_2, \qquad (3)$$

where γ_1, γ_2, $\beta_2 = \alpha_2 \gamma_1 + \gamma_2$ are constants coefficients and β_1 is the increment of the initial cost required to reduce P_f by a factor of 10.

Since the cost of failure C_f is generally independent of the initial cost,

equating the derivative of the total cost to zero gives the optimum probability of failure

$$P_{f_{op}} = \frac{\beta_1}{C_f . \ln 10} \tag{4}$$

and the optimum expected total cost

$$C_{t_{op}} = \beta_1 . \operatorname{colog}_{10} \frac{\beta_1}{C_f . \ln 10} + \frac{\beta_1}{\ln 10} + \beta_2 \tag{5}$$

Therefore the expression for the excess cost ΔC_t (overdesign or underdesign) is the following :

$$\Delta C_t = C_t - C_{t_{op}} = \beta_1 \left(\log_{10} \frac{P_{f_{op}}}{P_f} + \frac{1}{\ln 10} \frac{P_f - P_{f_{op}}}{P_{f_{op}}} \right). \tag{6}$$

Assuming $P_{f_{op}} = 10^{-4}$, a plot of $\Delta C_t / \beta_1$ as a function of P_f is shown in figure 1.

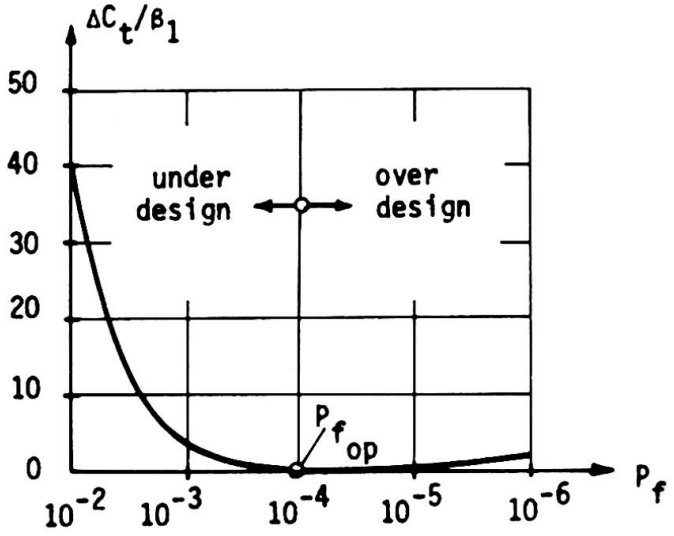

Figure 1 - Excess cost.

It then follows that :

- For the same absolute value of the safety difference

$$D = \operatorname{colog}_{10} P_f - \operatorname{colog}_{10} P_{f_{op}} \tag{7}$$

  the underdesign is more expensive than the overdesign ;
- In the region of overdesign, the excess cost is quite insensitive to the P_f variation ;
- In the region of underdesign, the excess cost is very sensitive to the P_f variation.

In the eventually that damage occurs, the loss C_f (costs of the building, potential loss of human lives and industrial damage caused by the failure) can be expressed as f times the cost C_i |5|.

The value of f gives a good indication of the magnitude of the damage that is caused by failure.

The optimum expected total cost (5) is a linear function of $\log_{10} f$ |4| :

$$C_{t_{op}} = \beta_1 . \log_{10} f + a \qquad (8)$$

where

$$a = \beta_1 . \operatorname{colog}_{10} \frac{\beta_1}{C_i . \ln 10} + \frac{\beta_1}{\ln 10} + \beta_2 .$$

A graph of $C_{t_{op}}$ as a function of $\log_{10} f$ is given in Figure 2.

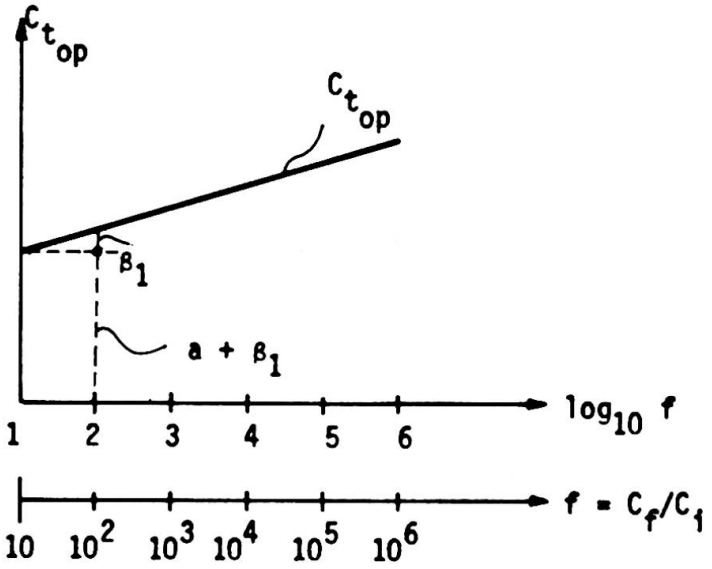

Figure 2 - Sensitivity of the optimal solution.

The foregoing considerations should permit a rational combination of safety and economy, in order to reach a satisfactory approach of an optimum solution.

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SUMMARY

Several independent attempts are made to evaluate an optimal value for the probability of failure and for the expected total cost. The sensitivity of the optimal solution to the magnitude of the damage that is caused by failure is also analysed.

RESUME

Plusieurs tentatives indépendantes sont faites pour obtenir la valeur optimale de la probabilité de ruine et du coût total attendu. La sensitivité de la solution optimale par rapport à l'importance du dommage causé par la ruine est également analysée.

ZUSAMMENFASSUNG

Verschiedene unabhängige Untersuchungen werden gemacht, um den optimalen Wert der Versagenswahrscheinlichkeit mit bezug auf die zu erwartenden Totalkosten festzulegen. Die Abhängigkeit der optimalen Lösung von der durch den Bruch verursachten Schadengrösse wird auch analysiert.