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Cylindrical Shells Made of Corrugated Sheets

Coques cylindriques en tôles nervurées

Zylinderschalen aus gerippten Blechen

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INTRODUCTION

The advantages of using light gauge steel sheets in folded plate roofs has been established through studies and practical applications in both Canada and the U.S.A. It is also generally recognized that cylindrical shell roofs have better carrying characteristics, as they translate the applied loads into mainly membrane forces. Furthermore, corrugated sheets with cylindrical curvature are widely available and so far they are employed mainly in non-structural capacities.

This study is to establish methods of analysis and also economical applications of cylindrical shells made of corrugated sheets.

GOVERNING DIFFERENTIAL EQUATIONS

The shells are considered as being made of elastic orthotropic material in which the mechanical properties are equal to the average properties of the corrugated sheets. For the arc-and tangent-type of corrugation, Fig. 1, these properties are [1,3]:

$$D_\phi = \frac{\ell}{c}\, t\, E \tag{1a}$$

$$D_x = \frac{E}{6(1-\mu^2)} \left(\frac{t}{f}\right)^2 t \tag{1b}$$

$$D_{x\phi} = \rho \frac{Et}{2(1+\mu)} \frac{c}{\ell} \tag{1c}$$

$$B_\phi = 0.522\, Ef^2 t \tag{1d}$$

$$B_x = \frac{c}{\ell} \frac{Et^3}{12(1-\mu^2)} \tag{1e}$$

$$B_{x\phi} = \frac{\ell}{c} \frac{Et^3}{12(1+\mu)} \tag{1f}$$

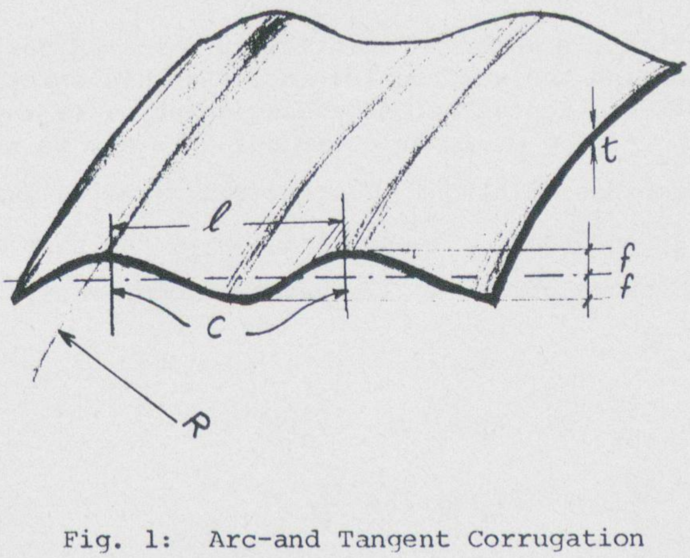


Fig. 1: Arc-and Tangent Corrugation

in which: D_x and D_ϕ = axial ridigity in the x- and ϕ directions respectively; $D_{x\phi}$ = shear rigidity in the $x\phi$-plane; B_x and B_ϕ = bending rigidity in the xz- and ϕz-planes respectively; $B_{x\phi}$ = torsional rigidity; t = average thickness of the sheet;

c = corrugation pitch; ℓ = developed length of corrugation per pitch; f = half depth of corrugation; E = modulus of elasticity of steel; μ = Poisson's ratio; and ρ = a reduction factor to account for the effect of slip at sheet to sheet and sheet to frame connections [3].

The differential equations governing the behaviour of the shell are obtained by using the above mentioned properties together with the equilibrium conditions and geometric relationships of an infinitismal element dx.R$d\phi$. These equations are:

$$D_x u'' + \frac{B_x}{R} w''' + D_{x\phi}\left(\frac{\ddot{u}}{R^2} + \frac{\dot{v}'}{R}\right) + \frac{B_{x\phi}}{2R^3}\left(\frac{\ddot{u}}{R} - \ddot{w}'\right) + p_x = 0 \qquad (2a)$$

$$D_\phi(\ddot{v} - \dot{w}) + D_{x\phi}(R\,\dot{u}' + R^2 v'') + \frac{3B_{x\phi}}{2}(v'' + \dot{w}'') + p_\phi R^2 = 0 \qquad (2b)$$

$$D_\phi(\dot{v} - w) - \frac{B_y}{R^2}(\ddddot{w} + 2\ddot{w} + w) - (B_x R^2 w'''' + B_x R u''') - (2B_{x\phi}\ddot{w}'' + B_{x\phi}\dot{v}''$$

$$- \frac{B_{x\phi}}{2R}\ddot{u}' + \frac{B_{x\phi}}{2}\dot{v}'') + p_z R^2 = 0 \qquad (2c)$$

in which $(\ )^{\bullet} = \frac{\partial(\ )}{\partial\phi}$, $(\ )' = \frac{\partial(\ )}{\partial x}$; p_x, p_ϕ and p_z are the components of the surface (external) loading in the x-, ϕ- and z- directions respectively.

The system of equations, Eqs. 2a, b, c, is derived without approximation. It encounters a number of terms which have insignificant effect on the results in shallow and/or short shells. These terms may be neglected and the system of equations is simplified as follows:

$$D_x u'' + D_{x\phi}\left(\frac{\ddot{u}}{R^2} + \frac{\dot{v}'}{R}\right) + p_x = 0 \qquad (3a)$$

$$D_\phi(\ddot{v} - \dot{w}) + D_{x\phi}(R\dot{u}' + R^2 v'') + p_\phi R^2 = 0 \qquad (3b)$$

$$D_\phi(\dot{v} - w) - (B_x R^2 w'''' + 2B_{x\phi}\ddot{w}'' + \frac{B_\phi}{R^2}\ddddot{w}) + p_z R^2 = 0 \qquad (3c)$$

METHOD OF SOLUTION

First, a membrane solution is obtained for the governing differential equations considering the surface loading. In this solution the boundary conditions are not satisfied. Thereafter a bending solution of the equations with no surface loading, $p_x = p_\phi = p_z = 0$, is superimposed in order to satisfy the boundary conditions.

A - Membrane Solution: The external load is analysed in its three components p_x, p_ϕ and p_z. As an example, a snow loading in the form of a sine wave, with maximum intensity p at the crown of the shell, has the components:

$$p_x = 0 \qquad (4a)$$

$$p_\phi = \frac{-4p}{n\pi}\cos(\phi_e - \phi)\sin(\phi_e - \phi)\cos\frac{\lambda x}{R} \qquad (4b)$$

$$p_z = \frac{4p}{n\pi}\cos^2(\phi_e - \phi)\cos\frac{\lambda x}{R} \qquad (4c)$$

in which $\lambda = \frac{\pi R}{L}$; L and R = the length and radius of curvature of the shell respectively; ϕ_e = half the central angle.

The membrane solution corresponding to the given snow loading is:

$$N_x = -\frac{12p}{R\pi k^2}\cos 2(\phi_e-\phi)\cos kx \tag{5a}$$

$$N_\phi = -\frac{4pR}{\pi}\cos^2(\phi_e-\phi)\cos kx \tag{5b}$$

$$N_{x\phi} = \frac{6pL}{\pi^2}\sin 2(\phi_e-\phi)\sin kx \tag{5c}$$

$$w = \frac{12p}{\pi}\left[\frac{1}{D_{x\phi}k^2}+\frac{4}{R^2k^4D_x}\right]\cos 2(\phi_e-\phi)\cos kx \tag{5d}$$

$$v = -\frac{6p}{\pi}\left[\frac{1}{D_{x\phi}k^2}+\frac{4}{R^2k^4D_x}\right]\sin 2(\phi_e-\phi)\cos kx \tag{5e}$$

$$u = -\frac{12p}{Rk^3\pi D_x}\cos 2(\phi_e-\phi)\sin kx \tag{5f}$$

in which $k = \frac{\pi}{L}$.

B - Bending Solution: The bending solution can be assumed as follows:

$$w = E^* e^{m\phi}\cos\frac{\lambda x}{R}, \quad u = F^* e^{m\phi}\sin\frac{\lambda x}{R}, \quad v = G^* e^{m\phi}\cos\frac{\lambda x}{R} \tag{6a,b,c}$$

in which E^*, F^* and G^* are constants.

Eqs. 6a, b, c are substituted in the governing equations, Eqs. 2a, b, c, after replacing p_x, p_ϕ and p_z by zero. A non-trivial solution of the resulting homogeneous system of equations is governed by the following characteristic equation:

$$\begin{aligned} m^8 &+ m^6\left[2-\lambda^2\left(\frac{D_x}{D_{x\phi}}+\frac{2B_{x\phi}}{B_\phi}\right)\right] + m^4\left[\lambda^4\left(\frac{D_x}{D_\phi}+\frac{2D_xB_{x\phi}}{B_\phi D_{x\phi}}+\frac{B_x}{B_\phi}\right) - \lambda^2\left(\frac{2D_x}{D_{x\phi}}+\frac{2D_\phi B_x}{D_{x\phi}B_\phi}\right)+1\right] \\ &+ m^2\left[-\lambda^6\left(\frac{D_xB_x}{D_{x\phi}B_\phi}+\frac{2D_xB_{x\phi}}{D_\phi B_\phi}\right) + \lambda^4\left(\frac{2D_x}{D_\phi}-\frac{2B_x}{B_\phi}+\frac{3D_xB_{x\phi}}{D_{x\phi}B_\phi}\right) - \lambda^2\left(\frac{2B_{x\phi}}{B_\phi}+\frac{D_x}{D_{x\phi}}\right)\right] \\ &+ \left[\lambda^8\left(\frac{D_xB_x}{D_\phi B_\phi}\right) + \lambda^4\left(\frac{D_xR^2}{B_\phi}+\frac{3D_xB_{x\phi}}{2D_{x\phi}B_\phi}+\frac{D_x}{D_\phi}\right)\right] = 0 \end{aligned} \tag{7}$$

A simplified characteristic equation can be obtained in a similar way by substituting Eqs. 6a, b, c in the simplified set of equations Eqs. 3a, b, c:

$$\begin{aligned} m^8 &+ m^6\left[-\lambda^2\left(\frac{D_x}{D_{x\phi}}+\frac{2B_{x\phi}}{B_\phi}\right)\right] + m^4\left[\lambda^4\left(\frac{D_x}{D_\phi}+\frac{2B_{x\phi}D_x}{B_\phi D_{x\phi}}+\frac{B_x}{B_\phi}\right)\right] \\ &+ m^2\left[-\lambda^6\left(\frac{2B_{x\phi}D_x}{B_\phi D_\phi}+\frac{B_xD_x}{B_\phi D_{x\phi}}\right)\right] + \left[\lambda^8\left(\frac{D_xB_x}{D_\phi B_\phi}\right)+\lambda^4\left(\frac{D_xR^2}{B_\phi}\right)\right] = 0 \end{aligned} \tag{8}$$

If isotropic properties are considered for the shell, Eq. 7 and Eq. 8 yield the well known characteristic equations of Flügge and Donnell respectively.

The roots of either Eq. 7 or Eq. 8 can be written as follows:

$$m = \pm\alpha_1 \pm i\beta_1 \quad \text{and} \quad m = \pm\alpha_2 \pm i\beta_2 \tag{9}$$

and the deflection, w:

$$w = \{e^{\alpha_1\phi}[A_n \cos\beta_1\phi + B_n \sin\beta_1\phi] + e^{-\alpha_1\phi}[C_n \cos\beta_1\phi + D_n \sin\beta_1\phi]$$
$$+ e^{\alpha_2\phi}[E_n \cos\beta_2\phi + F_n \sin\beta_2\phi] + e^{-\alpha_2\phi}[G_n \cos\beta_2\phi + H_n \sin\beta_2\phi]\} \cos \frac{\pi}{L} x \qquad (10)$$

The values of a set of roots, Eq. 9, are considered to be exact when calculated from Eq. 7 and approximate when calculated from the simplified Eq. 8. The deviations between these sets of roots increase with the increase of the ratios L/R. The average error in the 8-roots is taken as a base to determine the ratio L/R within which the simplified system of equations can be used with a reasonable degree of accuracy in the final results. Fig. 2 shows the percentage of the error versus L/R. It also shows a similar curve for the percentage of errors in the 8-roots when using the simplified equations for concrete shells (Donnell equations of isotropic shells).

The simplified Donnell equations are generally accepted for isotropic shells when $L/R < 1.6$ [5]. Fig. 2 shows that for $L/R < 3.9$, the same degree of approximation is not exceeded by using the simplified equations, Eqs. 3a, b, c for shells made of corrugated sheets.

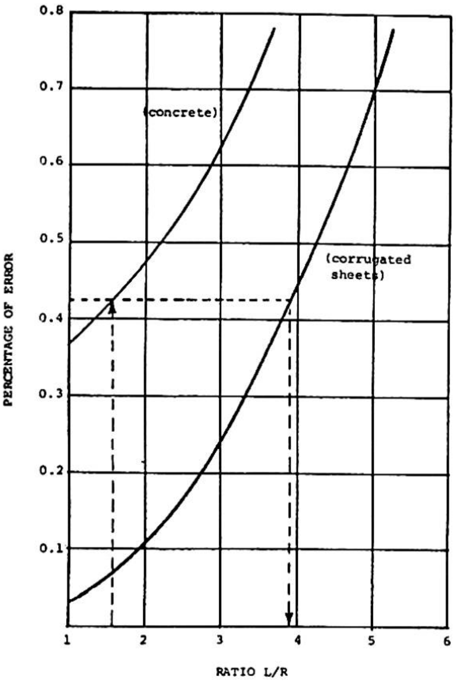
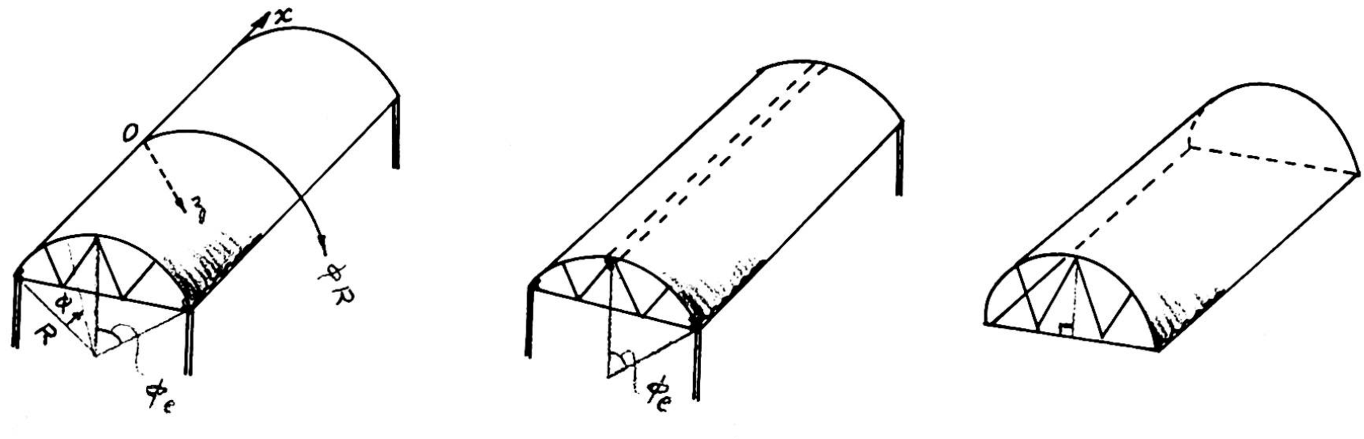


Fig. 2: Percentage of error in roots vs. the ratio L/R

BOUNDARY CONDITIONS

Three practical types of shells are analysed: I - shells with longitudinal stiffeners in the valleys only; II - shells with longitudinal stiffeners in valleys and crowns; III - half barrels supported along their four edges. Figs 3a, b, c show these three shells with the boundary conditions to each one of them.



SHELL I	SHELL II	SHELL III
At $\phi = 0$ and $\phi = 2\phi_e$;	At $\phi = 0$, same as case I	At $\phi = 0$ and $\phi = 2\phi_e$
$M_\phi = 0$	At $\phi = \phi_e$;	$w = 0$
$Q_\phi = 0$	$Q_\phi = 0$	$M_\phi = 0$
$N_\phi = 0$	$\theta = 0$	$v = 0$
$\varepsilon_{x,shell} = \varepsilon_{x,stiffener}$	$v = 0$	$\varepsilon_x = 0$
	$\varepsilon_{x,shell} = \varepsilon_{x,stiffener}$	

Fig. 3: Types of Shells and Their Boundary Conditions

THEORETICAL RESULTS

The membrane and bending solutions are superimposed and the integration constants $A_n, B_n, \ldots$ of Eq. 10, are calculated for each type of shell satisfying the boundary conditions. In a similar way the displacements u and v are found and the components of internal forces are calculated and arranged in tables for practical use. These tables will be reported in reference [4].

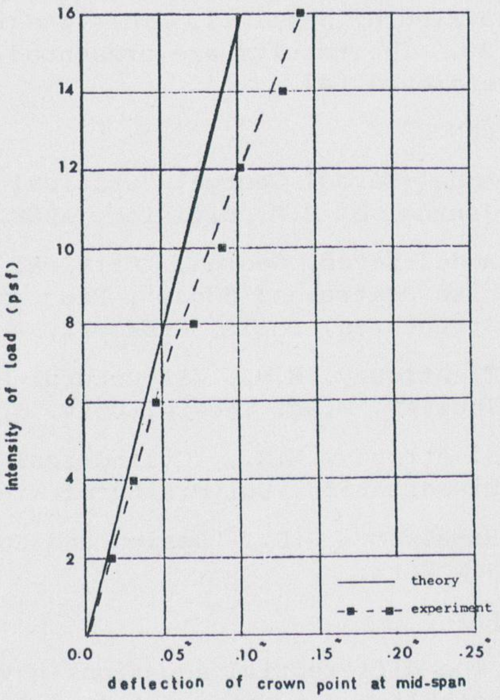
COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

An experimental program was undertaken with full scale tests for the shells I and II, Fig. 3a, b. The experimental results show good agreement with those obtained theoretically. Fig. 4a, b, c are taken as a sample from the experimental program given in reference [4].

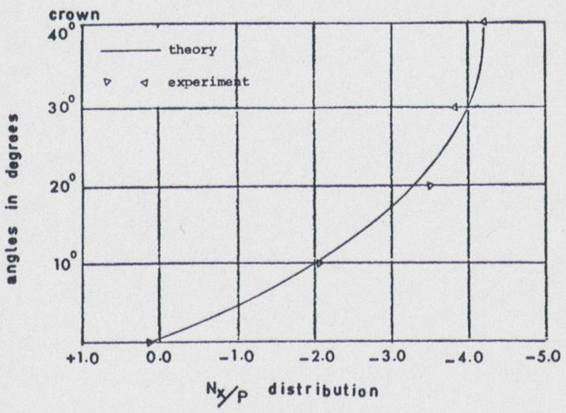
OBSERVATIONS AND CONCLUSIONS

1 - The theoretical analysis is verified experimentally. This proves that treating the corrugated sheets as orthotropic shells is a valid approach which adequately considers the main features of response of the shells.

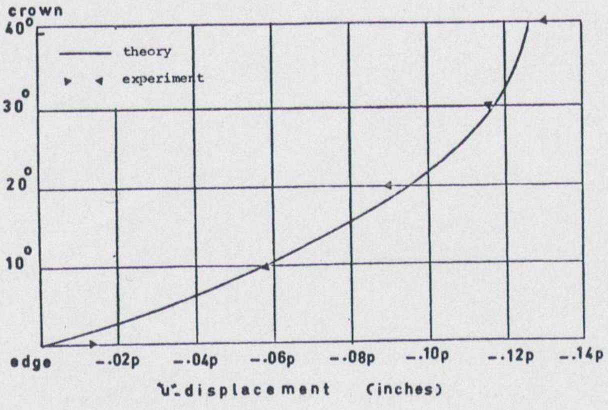
2 - Design formulas and tables are prepared for practical use [4]. These formulas are based on the simplified governing equations, Eqs. 3a, b, c, which yield results with sufficient degrees of accuracy for shells with



a - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL LOAD-DEFLECTION CURVES OF CROWN POINT AT MID-SPAN.



b - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF N_x-DISTRIBUTION AT MID-SPAN.



c - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE DISPLACEMENT "U" AT SUPPORT.

Fig. 4: Comparison Between Experimental and Theoretical Results

the ratio $L/R \leq 3.9$. Shells with higher L/R ratios are handled as follows: a - Shell I undergoes too large deflections and becomes of no practical use; b - Shell II can be analysed as a beam. This approach yields results that are reasonably in agreement with the present analysis when $L/R \simeq 3.5$. Furthermore, the beam approach is expected to lead to better results for shells with higher ratios of L/R; c - The analysis of Shell III with $L/R > 3.9$ requires the use of the exact equations, Eq. 2a, b, c. This problem is now under investigations.

3 - The local shear buckling is a prime factor in determining the ultimate load that can be carried by Shell II. This shear buckling was examined theoretically and experimentally [1, 2]. The results are presented in tables to supplement the design ones referred to in reference [4].

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SUMMARY

The differential equations governing the behaviour of shells made of corrugated sheets are established in an exact as well as in a simplified form. The simplified equations yield acceptable results when the ratio of length to radius of the shell is less than 3.9. Membrane and bending solutions are superimposed to satisfy the governing equations as well as the boundary conditions.

An experimental program verified the theoretical results and showed that treating the corrugated sheets as orthotropic shells is an acceptable approach.