# Optimum design of space trusses

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# Optimum Design of Space Trusses

Projet optimum de treillis spatiaux

Optimaler Entwurf von Raumfachwerken


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## 1. Introduction

The analysis and optimum design of two types of simply supported, double layered space trusses, which are called Takenaka-truss in Japan, is reported herein. The design of the space trusses is usually carried out through the process shown in Figure 1. 1. The design procedure developed here corresponds to the foundamental design stage, and this can be utilized to determine the optimum grid layout and the truss depth for the final design stage. Consequently, the accurate analysis and design procedure should be followed.

The approximate analysis is applied here to treat the following optimum design problem in a mathematically simplified form. And the results obtained by this method are verified, being compared with the accurate results by the stiffness matrix method.

The variables of the optimum design are not only section properties of members of space trusses, but also the depth of the trusses and the spacing. The objective function to be optimized is the cost of the space trusses. Which consists of the costs of members, joints and purlines. The applied design specification is the steel structural standard of Japan (1970) and the deflection limitations. The sequential unconstrained minimization technique is applied to the optimization technique.

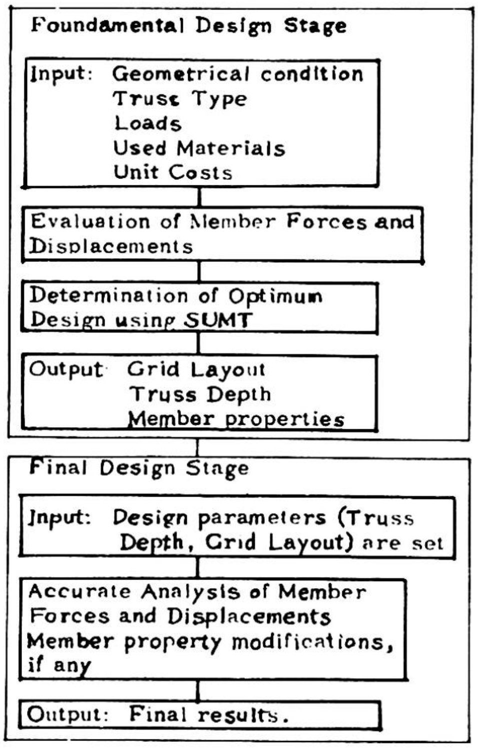



Figure 1.1 Design Process Flow Chart

## 2. Configulation of the Space Trusses

The two types of double layer grid trusses in this study are, Square Pyramid Truss (S.P. Truss) and Star Element Truss (S.E. Truss), which are quite similar each other. S.P. Truss has been utilized often recently, however, S.E. Truss has newly been developed. Both space trusses have diagonal top layer grids and normal lower layer grids, but the relative locations of top and lower chord are different, and the direction of latticed members are also different.

S.P. Truss is composed by arranging the inverted square pyramid elements as a chequered pattern and connecting the neighbouring pyramid apexes with lower layer members (see Figure 2. 1). On the other hand, S.E. Truss is composed by star elements (see Figure 2. 2).

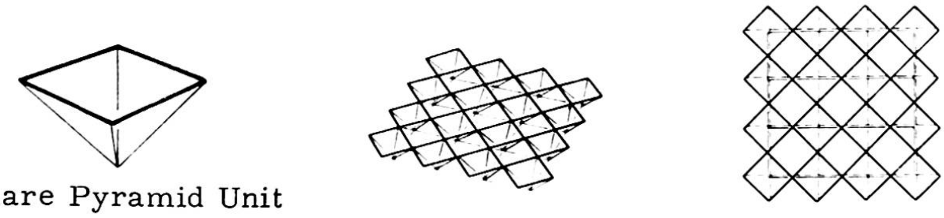



Figure 2.1 S.P. Truss

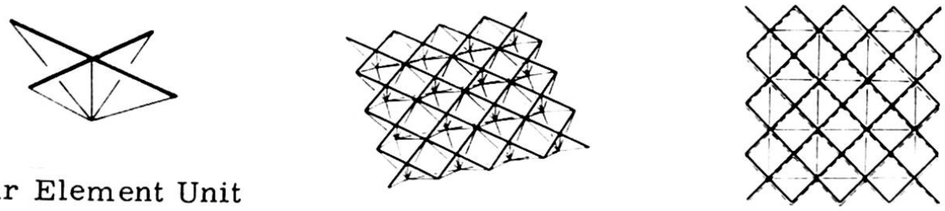



Figure 2.2 S.E. Truss

## 3. Evaluation of Displacements and Member Forces

The rigorous solutions for the axial forces of members and deflections of joints of space trusses under the imposed loading conditions may be obtained by deflection method using an electronic digital computer. In this optimization study, however, much simpler analysing methods are necessarily needed which lead to good approximate solutions and require short time and small core size in computation. One of the most successful ways which seems to satisfy these requirements is to find the equivalent solid plate that has nearly the same force and stiffness distributions.

The S.P. Truss has very small twisting rigidity around x and y axies (Figure 3. 1), that is,

$$M_{xy} = 0 \quad . \tag{3.1}$$

From the wellknown equilibrium equation of solid plate and Equation (3. 1) the following equation can be obtained.

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} \tag{3.2}$$

where w shows the vertical displacement, P the load per unit area and D the rigidity per unit width of the plate. Solving the Equation (3. 2) under the simple support boundary condition by using Fourier series, the displacement is

$$w = \frac{16P}{D\pi^6} \sum_{m=1,3,5\cdots} \sum_{n=1,3,5\cdots} \frac{1}{mn\left(\frac{m^4}{a^4} + \frac{n^4}{b^4}\right)} \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b} \tag{3.3}$$

On the contrary to the S. P. Truss, the S. E. Truss has small twisting rigidity around x' and y' axies,

$$M_{x'y'} = 0 \quad . \qquad (3.4)$$

Equation (3.4) is rewritten as

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{P}{4D} \quad . \qquad (3.5)$$

The rigidity of plate D is expressed as

$$D = \frac{n}{a}\frac{\sqrt{2}\,A_u \cdot A_L}{\sqrt{2}\,A_u + A_L} \cdot E h^2 \, , \qquad (3.6)$$

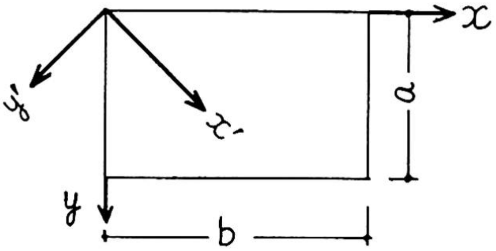



Figure 3.1
Co-ordinate Systems

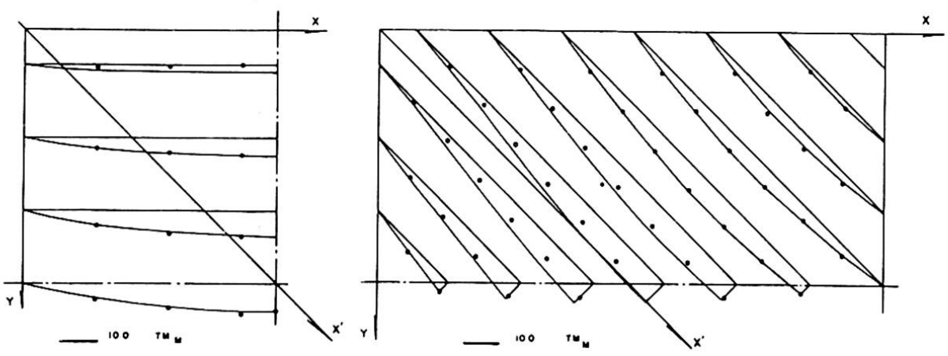
where n is a number of blocks along the y direction. Au and Al are average sectional areas of upper and lower chords members respectively. Member forces are given in Table 3.1. The comparisons between rigorous solutions and plate solutions are shown in Figure 3.2 and 3.3.

In order to minimize the total cost of the space trusses exactly, each member and joint costs are individually to be taken into account, however, this is not practical actually to treat whole members and joints as variables of the objective function.

Therefore some members and joints are to be chosen to represent the structure. Two members for each upper, lower and latticed chord members, and one joint are selected in this study.

Table 3.1 Member Forces

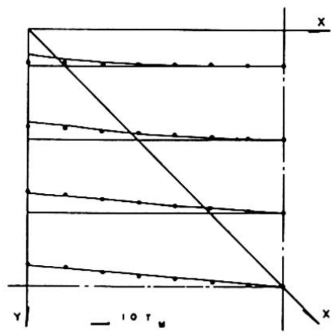
| Member | Sectional area | Direction | S. P. Truss | | S. E. Truss | |
|---|---|---|---|---|---|---|
| | | | Member force | Member length | Member force | Member length |
| Upper chord member | Aue | x' | $M_{x'} = M_{y'} = M$ | $\frac{1}{\sqrt{2}}\frac{a}{n}$ | $\frac{1}{\sqrt{2}}\frac{a}{n}\frac{M_{x'}}{h}$ | $\frac{1}{\sqrt{2}}\frac{a}{n}$ |
| | Auc | y' | $\frac{1}{\sqrt{2}}\cdot\frac{a}{n}\cdot\frac{M}{h}$ | | $\frac{1}{\sqrt{2}}\frac{a}{n}\frac{M_{y'}}{h}$ | |
| Lower chord member | Ale | x | $\frac{a}{n}\frac{M_x}{h}$ | $\frac{a}{n}$ | $M_x = M_y = M$ | $\frac{a}{n}$ |
| | Alc | y | $\frac{a}{n}\frac{M_y}{h}$ | | | |
| Latticed member | Awep | x | $\frac{a}{n}Q_x\sqrt{1+\frac{a}{4n^2h^2}}$ | $\sqrt{h^2+\frac{a^2}{4n^2}}$ | | |
| | Awem | y | $\frac{a}{n}Q_y\sqrt{1+\frac{a}{4n^2h^2}}$ | | | |
| | Awc | x' | | | $\frac{a}{n}Q_{x'}\cdot\sqrt{1+\frac{a^2}{2n^2h^2}}$ | $\sqrt{h^2+\frac{a^2}{2n^2}}$ |
| | | y' | | | $\frac{a}{n}Q_{y'}\cdot\sqrt{1+\frac{a^2}{2n^2h^2}}$ | |
| Number of joints | | | $3\frac{b}{a}n^2 + (\frac{b}{a}+1)\,n$ | | $3\frac{b}{a}n^2 + (\frac{b}{a}+1)\,n + 1$ | |




Lower Chord

Upper Chord

Figure 3.2
Comparison of Bending Moments




o Rigorous Solutions
—— Plate Solutions

Figure 3.3
Comparison of Shearing Forces

## 4. Optimum Design of Space Frames

### 4.1 Mathematical Model

Design variables are the sectional areas of members, Aue, Auc, Ale, Alc, Awep, Awem, Awc, the number of blocks, n, and the truss depth, h, which are described in the previous section.
The objective function f of the structure to be minimized is the total cost of steel skeltons.

$$f = \sum_{i=1}^{m} C_m \rho A_{mi} + \sum_{i=1}^{n} C_{mp} \rho A_{pi} + C_j \cdot N \quad , \tag{4.1}$$

where Cm, Cmp, Cj are the unit cost of members, purlins and joints; m, n, N are the number of members, purlines and joints, respectively.

Tubular sections and wide flange sections are used for the members of trusses, and purlines respectively. Empirical relationships between section properties are obtained by plotting section properties commercially provided.

(i) For steel tubes

$$I = (0.625 A)^{\frac{19}{8}} \,. \tag{4.2}$$

(ii) For wide flange sections

$$A = 0.58\, I^{\frac{2}{4}} \,, \tag{4.3}$$

$$Z = 0.58\, I^{\frac{3}{4}} \,. \tag{4.4}$$

The diameter of the spherical joint is assumed three times of the diameter of the largest members and the thickness is assumed twice of that of the largest members.

Therefore, the objective function f of the structure for S. P. Truss,

$$\begin{aligned} f = \rho \Big[ \Big\{ & 2\sqrt{2}\, na \tfrac{5}{9} A_{ue} + 2\sqrt{2}\, na \tfrac{4}{9} A_{uc} + \left(2na \tfrac{5}{9} - 2a\right) A_{le} \\ & + 2na \tfrac{4}{9} A_{lc} + na\sqrt{1 + \tfrac{4n^2h^2}{a^2}} \cdot \tfrac{4}{9} \left(A_{wep} + A_{wem}\right) \\ & + 2na\sqrt{1 + \tfrac{4n^2h^2}{a^2}} \cdot \tfrac{5}{9} A_{wc} \Big\} \cdot C_m \quad + 0.58^{\frac{1}{8}} \left(\tfrac{1.5}{16}\right)^{\frac{3}{8}} \left(\tfrac{P a^5}{F l_p^{\frac{1}{2}}}\right)^{\frac{3}{8}} \cdot n^{-\frac{4}{8}} \cdot C_{mp} \\ & + \left(3n^2 + 2n\right) \cdot C_5 \cdot A_{lc}^{\frac{27}{16}} \cdot C_j \Big] \quad , \end{aligned} \tag{4.5}$$

where lp is the spacing of purlines (cm), $C_5$ is the coefficients obtained by the relationship between member properties of tubular sections.

### 4.2 Sequential Unconstrained Minimization Technique

The optimum problem, mentioned in the previous section, may be obtained by several mathematical techniques. Here, the sequential unconstrained minimization technique developed by Davidon is adapted. The objective function to be minimized is converted to the following equation F,

$$F = f + R_k \sum_{j=1}^{n} \frac{1}{1 - \sigma_j} \quad , \tag{4.6}$$

therefore, the optimum design problem with constraints is changed to the unconstrained optimum design problem. The macro flow chart is shown in Fig. 4.1.

### 4.3 Parametric Study

Using the developed programming, a parametric study was carried out. The observed results tell the interesting behaviors of the optimum designs.

(i) Parameter: Span
Constants given:

$P = 200\ kg/m^2$
$F = 2.4\ t/cm^2$
$E = 2.1 \times 10^6\ kg/cm^2$
$C_j = 308{,}000$ yen (1,000 dollars)
$C_m = 123{,}000$ yen ( 400 dollars)
$l_p = 180$ cm

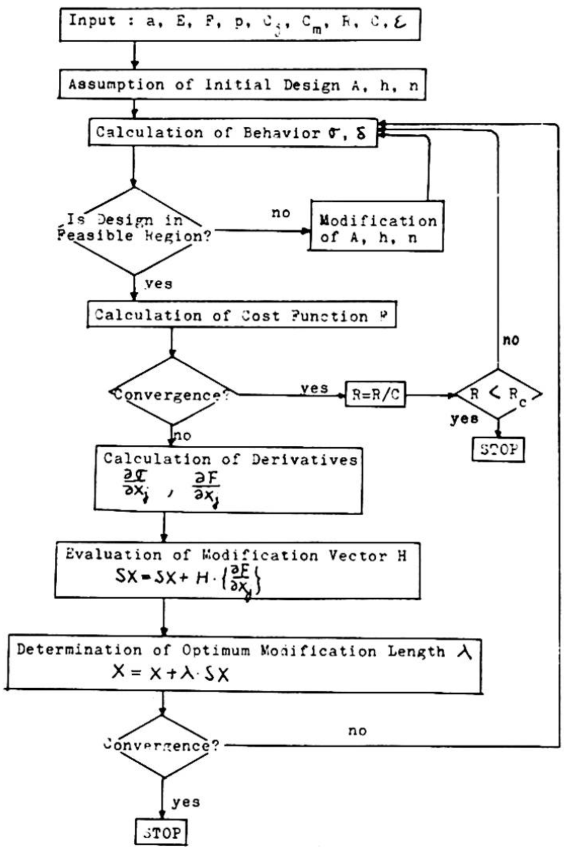



Figure 4.1

Macro Flow Chart of S.U.M.T.

At the optimum design, all stresses of members are fully constrained, however, the deflection limitation p = a/300 does not dominate at all. The cost per unit area increases almost linearly with the span length of the whole structure. Span per depth is scattered between almost 8 - 12, which is coincident to the usually adapted value in the actual design. Moveover, the optimum number of blocks seems to be obtained so that the angle of the diagonal member is almost $45^o$ (actually $42^o$-$49^o$).

(ii) Parameter: Loading
Constant given:
Span length 72 meters, F, E, Cj, Cm, lp are same as 4. 1.

The cost per unit area increases almost linearly with loading amount.

The optimum designs of the S. E. Truss were almost same as those of the S. P. Truss when Cj = 0, that is, the weight is minimized, the results show almost same tendency. Moreover, almost the same results were obtained for the change to the purline spacing, too.

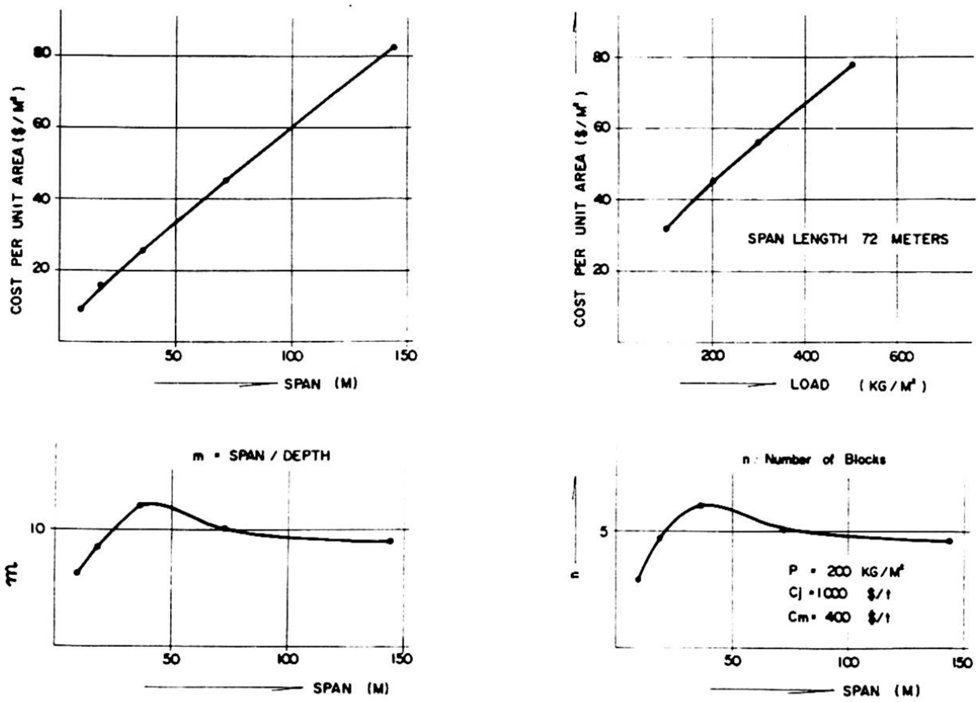



Figure 4.2 Results of Parametric Study

## 5. Conclusions

Through this study, the following conclusions have been atained.

(1) Comparisons with the results obtained by the stiffness matrix method confirm the theoretical analysis presented.

(2) The sequential unconstrained minimization technique works very effectively in the optimum design of the double layered space trusses, and shows good convergence. Computer time for one case is approximately two minutes using IBM 360/65.

(3) Through a parametric study, structural characteristics of the Takenaka-truss have been obtained. The cost per unit area is almost proportional to the span.
The optimum ratio of the truss depth to the span is approximately 8 - 12.
At the optimum design, the angle of the latticed members are approximately 45°. The results for S.P. Trusses and S.E. Trusses are almost same.

The optimum design with truss depth limitation, and the rectangular plan can be readily developed in the future.

## Bibliography

1. Kato B. and Hirata Y. "The Analysis of a space truss composed of square pyramid units", Space Structure, pp201 - 212, September 1966.
2. Renton, J.D., "The Formal Derivation of Simple Analogies for Space Frames", IASS Pacific Symposium Report October 1971.
3. Geiger, D.H., "A cost evaluation of space trusses of large span", AISC Engineering journal April 1968.
4. Architectural Institute of Japan, "The steel structural design code", May 1970 (in Japanese).
5. Fiacco, A.V., McCormick, G.P., "Nonlinear Programming: Sequential Unconstrained Minimization Techniques", 1968.

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## Summary

The analysis and optimum design of simply supported, double layered space trusses is presented here. The approximate analysis using a transformation method to a continuous equivalent plate is good enough for design use. The optimum design by a sequential unconstrained minimization technique insures good convergence. Through a parametric study of approximately twenty cases, the structural characteristics of the Takenaka-truss have been studied.