Structural behaviour and safety criteria

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Structural Behaviour and Safety Criteria

Le comportement et la sécurité des constructions

Verhalten und Sicherheit der Tragwerke

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1. Non-linear Structural Behaviour

In order to enable the non-linear behaviour of plane structures to be analysed considering different moment-curvature diagrams along the bars, a special program was prepared which allows the study of non-linear behaviour in any type of structure for a monotonic increase of the forces [1].

This program considers the moment-curvature diagrams defined by a polygon of twelve sides and, for each bar, by eleven elements, the two extreme ones having a length of $^1/_{20}$ and the middle ones $^1/_{10}$ of the length of the bar. A different moment-curvature diagram may be assigned to each of these elements.

The data given to the computer are: values of the load factor to be considered, length and cross-section types of the different bars, load vectors, displacement transformation matrix and moment-curvature diagrams corresponding to the different types of cross-sections.

The results obtained are displacements (translations and rotations), bending moments and shear forces at the ends of the bars and at the points where concentrated forces are applied. These values being indicated for each value of the load factor, the behaviour of the structure as the load increases can be followed. The computation is carried out by iterative cycles with accelerated convergence.

Using this program, computations are being performed to study the influence of the type of moment-curvature diagrams on the behaviour of different types of structures.

If bi-linear moment-curvature diagrams are considered, these can be reduced to diagrams of the type indicated in Fig. 1.

As an example, the results obtained in reference to a simple structure by the use of this program are presented. The structure, Fig. 2, is made up of two parallel stanchions, of which one has an elastic stiffness and an ultimate moment respectively 4 times and twice those of the other. This simple type

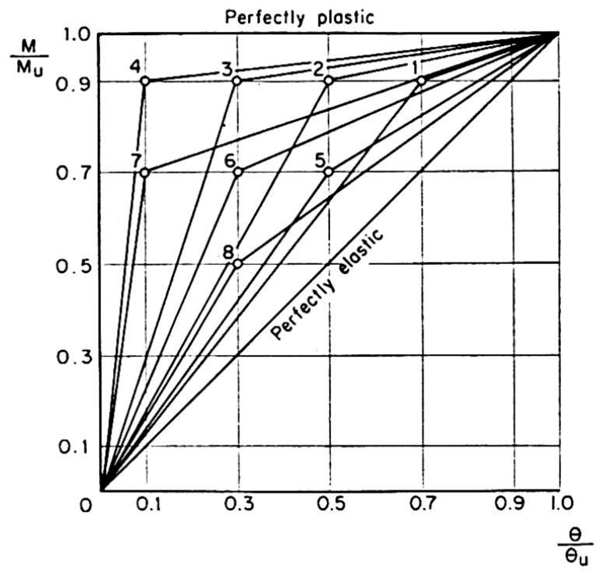

Fig. 1. Bi-linear moment-curvature diagrams.

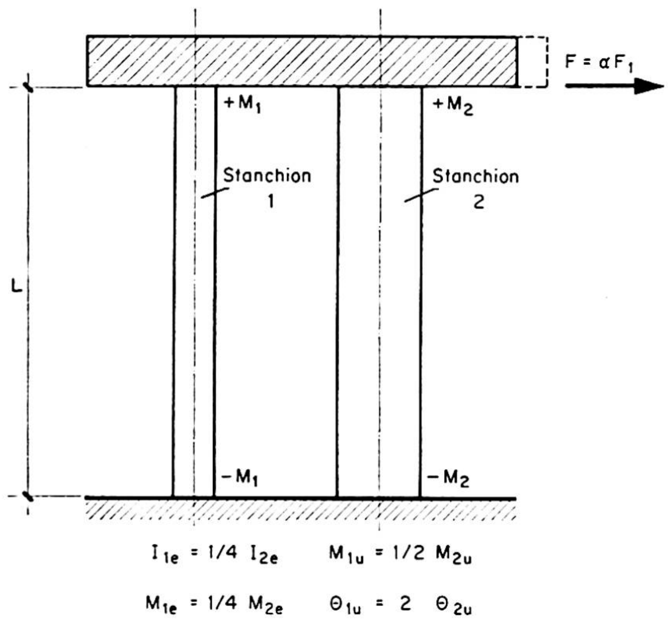

Fig. 2. Structure formed by two parallel stanchions.

of structure was chosen because the accuracy of the results can be easily checked in this case.

Fig. 3 indicates the redistribution of the moments in function of the increase of the horizontal force. By means of this figure it is possible to determine the bending moment M_1 at the slender bar when the ultimate moment M_{2u} is reached at the stiffer one. For a perfectly elastic behaviour $M_{1e} = M_{2u}/4$ and for a perfectly plastic behaviour $M_{1p} = M_{2u}/2$.

A correspondance can thus be set up between the bi-linear diagrams considered and the values of M_1/M_{1e}. When this ratio equals 1 that means that

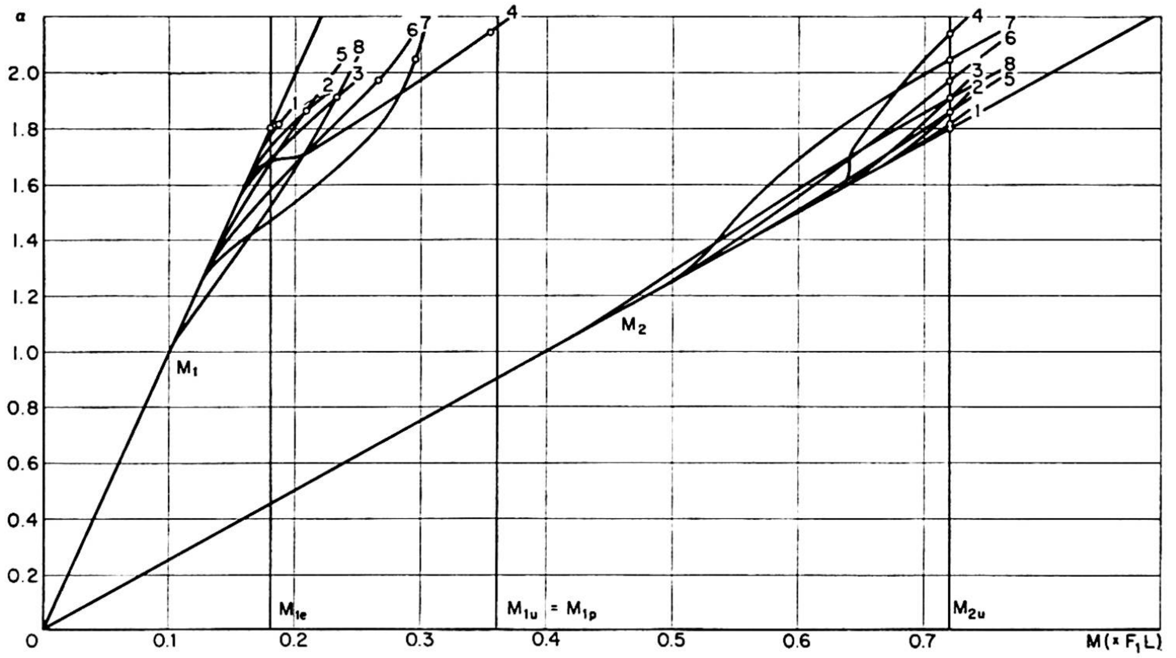

Fig. 3. Distribution of moments for bi-linear behaviour.

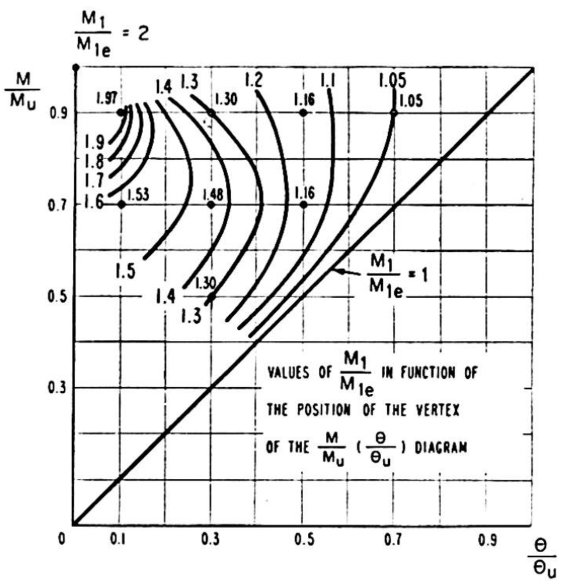

Fig. 4. Ratios M_1/M_{1e} for bi-linear behaviour.

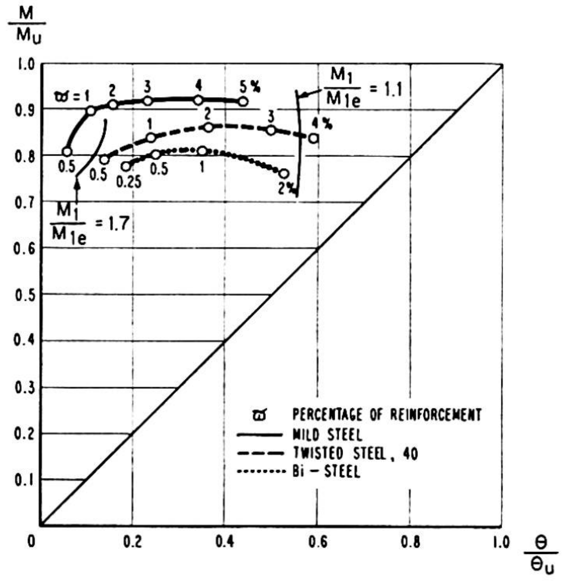

Fig. 5. Loci of the vertices of bi-linear diagrams for reinforced concrete beams.

the distribution of moments corresponds to perfect elastic behaviour, when the ratio equals 2 it corresponds to a perfect plastic behaviour.

Fig. 4 indicates the lines of equal value of the ratio M_1/M_{1e}. It is interesting to note that high values of this ratio are only obtained if the vertices of the diagrams are very near the vertex corresponding to perfect plasticity.

If bi-linear diagrams are used to represent the mechanical behaviour, it is of interest to study the correspondence between the quality of the materials and the position of the vertices of the bi-linear diagrams.

Considering for instance rectangular reinforced concrete beams and supposing that the ultimate moment is attained when the strains at the concrete or at the steel reach respectively the values of $3.5\,^0/_{00}$ or $10\,^0/_{00}$, the corresponding moment-curvature diagrams may be represented by bi-linear diagrams with the vertices at the points indicated in Fig. 5. The position of the vertices changes in accordance to the percentage of reinforcement and the quality of steel. These diagrams were obtained as described in [1].

Combining Fig. 4 and 5 it is seen that, for instance in the case of deformed twisted steel 40 for percentages of reinforcement going from 0.5% to 4%, the corresponding values of M_1/M_{1e} change from about 1.7 to 1.1.

This shows that if elastic design was adopted it would indicate for the slender stanchions ultimate moments, on the safe side, with errors between 70 and 10%. On the contrary, plastic design would indicate moments on the unsafe side with errors between 15 and 45%.

2. Randomness of the Structural Behaviour

The randomness of the behaviour of the structures can only be analysed if the usual relations between forces and deformation are replaced by relations statistically defined.

For instance, the moment-curvature diagrams considered as certain have to be replaced by a statistical distribution of diagrams, Fig. 6a. Each diagram corresponds to the behaviour of an element of length ΔL. Considering a population of different elements, for each value of the bending moment M_0 or of the curvature θ_0 it is possible to define a statistical distribution of the θ and M respectively. The cumulants of these distributions are represented by $P_r\{\theta' > \theta/M_0\}$ and $P_r\{M' < M/\theta_0\}$. The mean values, standard deviations and coefficients of variation are represented by $E(\theta/M)$, $D(\theta/M)$, $C(\theta/M)$ and $E(M/\theta)$, $D(M/\theta)$, $C(M/\theta)$.

The diagram of Fig. 6b indicates the values of $E(M/\theta)$ and $D(M/\theta)$ corresponding to the diagram 6a.

If different structures are built of a material with the assumed mechanical properties and if α is the load factor affecting the forces applied to the structures, a correspondance between the displacements, δ, of the structure, and the load factor, α, as indicated in Fig. 6c, shall be obtained. In this case it is also possible to define the behaviour of the structure by the $P_r\{\delta' > \delta/\alpha\}$ and $P_r\{\alpha' < \alpha/\delta\}$.

The safety of the structure may then be judged by the condition $P_r\{\delta' > \delta/\alpha\} < \epsilon$.

As before the mean values, standard deviation and coefficients of variation can be considered for these distributions (Fig. 6d).

For simplifying the analysis some assumptions are made.

It is admitted that the transformation of the mean diagrams defining the mechanical properties, $E(M/\theta)$, into those defining the mean structural behaviour $E(\alpha/\delta)$ can be done in accordance with the usual structural theories.

As the contribution of the deformation of the different elements to the deformation of the whole structure has a linear character, the central limit theorem applies and a normal distribution shall be usually obtained.

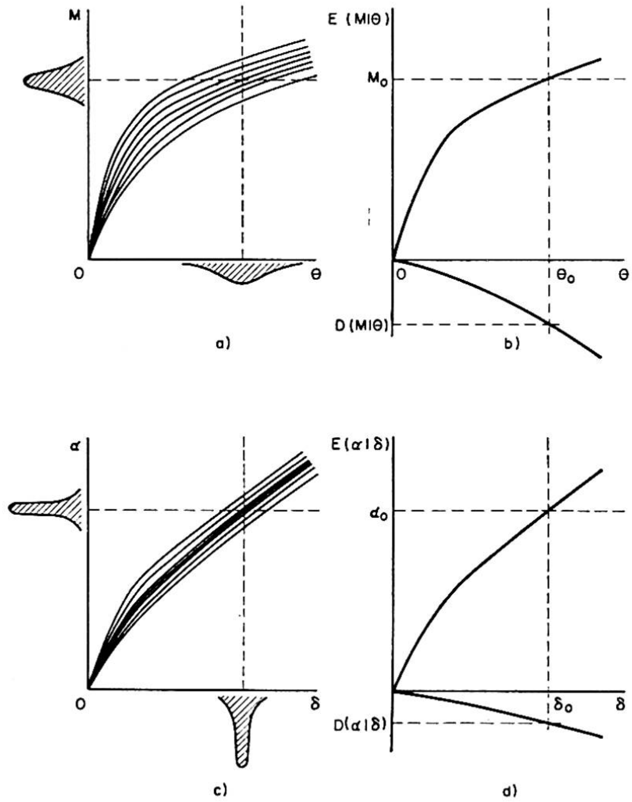

Fig. 6. Statistical representation of mechanical properties and structural behaviour.

If so the object of the statistical theories of structures shall then be to compute the standard deviations corresponding to the behaviour of the structure from those that correspond to the mechanical properties.

A general theory of this type is not yet established. For the present a numerical analysis can easily be performed and computations are being made following the program indicated in 1.

For these computations different $M(\theta)$ diagrams are distributed by a random process to different elements of the bars. The results concerning the behaviour of the different structures thus obtained are statistically analysed. It is intended to present at the Congress results of these computations.

Although a general statistical theory of structures is not available it is hoped that useful results shall be obtained from the analysis of numerical experiments and the application of particular theories, such as the theory of similitude [2].

3. Combining the Randomness of the Loads with the Randomness of the Structural Behaviour

According to the hypotheses presented, structural behaviour may usually be described by normal distributions. If so the probability of collapse for a load factor, α, is given by the normal distribution

$$\varphi(\alpha) = \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}\, c\, \overline{\alpha}}\, e^{-\frac{(\alpha-\overline{\alpha})^2}{2 c^2 \overline{\alpha}^2}}\, d\alpha .$$

$\overline{\alpha}$ being the mean value and c the coefficient of variation of the distribution considered.

On the other hand the variability of the loads is much influenced by their character.

Dead loads and some types of live loads are so well defined that they can almost be considered as certain or as normal with a small coefficient of variation.

Other types of loads such as those due to earthquakes and wind have high dispersions and in general their distribution cannot be considered as normal. In this case extreme distributions [3] may be reasonably assumed. For wind action [4] the probability of a load intensity higher than β may be taken as an extreme distribution of type II,

$$\psi(\beta) = 1 - e^{-(k\beta)^{-\gamma}} .$$

For other types of loadings, such as exceptional live loads on highway bridges, it is impossible to assume that their intensity is random [5]. In this case the load factor can only be considered as strategic, depending on decisions.

If both loadings and structural behaviour are considered random, their randomness has to be combined in order to compute the probability of collapse.

The collapse shall correspond to the probability of $\beta < \alpha$ or, what is the same, $\gamma = \alpha - \beta < 0$.

Considering distributions $\varphi(\alpha)$ and $\psi(\beta)$ the probability of $\gamma < 0$ is given by

$$\int_{-\infty}^{+\infty} \psi(\alpha)\, \varphi'(\alpha)\, d\alpha .$$

Table I indicates the probabilities of collapse for the distributions presented in Fig. 7.

For the loading distribution two hypotheses were considered, one corresponding to a normal distribution of mean $\bar{\beta} = 1$ and coefficient of variation $c_\beta = 0.1$ and the other corresponding to the extreme distribution $\psi(\beta) = 1 - e^{-(2\beta)^{-5}}$. The structural behaviour was considered as certain $\alpha = 2$ and with normal distribution $\overline{\alpha} = 2$, $c_\alpha = 0.1$ and 0.2.

Table I shows that a change in the dispersion of structural behaviour has a strong influence on the probability of collapse if the load has a normal

Table I. Probabilities of Collapse

Structural behaviour $\varphi(\alpha)$ $\bar{\alpha}=2$	Load distribution $\psi(\beta)$	
	Normal $\bar{\beta}=1$ $c_\beta=0.2$	Extreme, Type II $\psi(\beta)=1-e^{-(2\beta)^{-6}}$
Certain $c_\alpha=0$	0.3×10^{-6}	10×10^{-4}
Normal $c_\alpha=0.1$	5×10^{-6}	13×10^{-4}
Normal $c_\alpha=0.2$	200×10^{-6}	20×10^{-4}

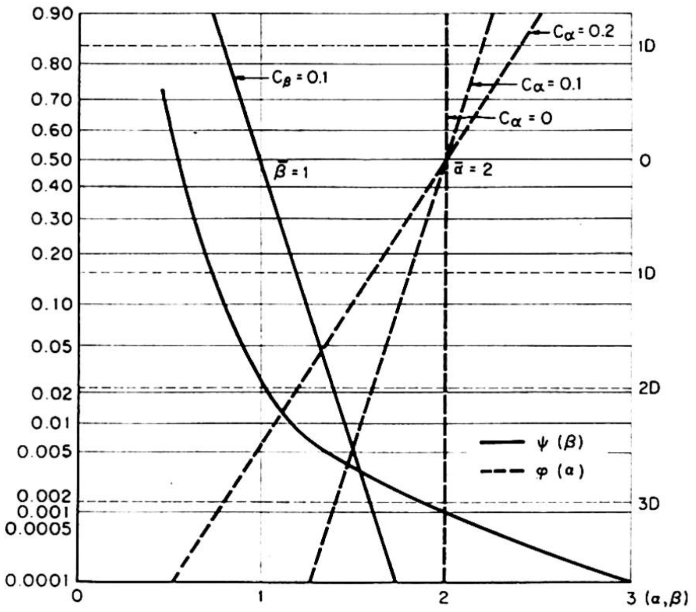

Fig. 7. Exemples of load factor and mechanical behaviour distributions.

distribution. On the other hand for load distributions with a long wing, the probability of collapse is almost independent of the dispersion of structural behaviour and the only way to reduce the probability of collapse is to increase the mean value of α.

The consequence of this fact are important for structural design and attention was already called to this problem regarding earthquake actions [6].

4. Conclusions

The object of the present paper is to discuss some fundamental criteria of structural design.

From the results presented the following conclusions can be drawn.

4.1. Non-linear analysis of structures has become pratical and not even too expensive by the use of present computational means, notably electronic computers. This analysis enables the behaviour of the structure under increasing loads to be completely followed and so to establish limit conditions on cracking, deformation and rupture on satisfactory bases.

4.2. Although non-linear analysis may be applied to solve practical problems, it is deemed more useful to employ it for defining the corrections to be introduced in the usual methods (elastic or plastic) to make them more accurate.

4.3. A more convenient formulation of safety problem requires that the statistical behaviour of the structures be taken into account. The best way to establish statistical theories of structures seems to be the following: 1. to use current theories to define the mean behaviour of the structure in function of the mean mechanical properties of the materials; 2. to study how to transfer the randomness of mechanical properties to the randomness of structural behaviour. For this last purpose numerical experiments seem to be particularly useful, but experimental and analytical methods have also to be considered.

4.4. In most cases collapse is attained not by rupture but because displacements are much too high. If displacement values are taken as the ultimate condition, useful simplifying assumptions can be introduced (normal distribution of displacements) which would be incorrect for rupture. This justifies the choice of limit conditions (collapse) with respect to displacements for the usual static problem.

4.5. According to 4.3 mechanical properties diagrams corresponding to small probabilities are inadequate to study the structural behaviour, mean diagrams having to be used. Randomness must be introduced by affecting the mean structural behaviour with the variability deriving from the material properties and geometry.

4.6. To compute the probability of collapse of a structure under random loads, it is necessary to combine this randomness with the one deriving from structural behaviour. When the variability of the load is small, the probability of collapse depends mainly on the wing of the distribution of the mechanical behaviour alone. For highly variable loads (earthquakes, wind) the probability of collapse depends mainly on average values of mechanical behaviour, being not much affected by the dispersion of this behaviour.

These facts have important consequences for safety criteria.

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Summary

Possible improvements in methods of structural design are discussed.

A method for studying the non-linear behaviour of plane linear structures by electronic computation is described and used. An example is presented of the influence of the type of diagram that represents the mechanical properties of the materials on the structural behaviour of a simple structure.

Convenient ways to establish statistical theories of structures which allow to transfer the randomness of mechanical properties to the randomness of structural behaviour are discussed.

For the case of random loads their randomness is combined with that of structural behaviour in order to compute the probability of collapse. The influence of the load randomness on safety criteria is studied for some simple cases.

Conclusions are drawn concerning the most convenient bases for methods of structural design.

Résumé

L'auteur discute quelques perfectionnements possibles des méthodes de calcul statique.

Il décrit d'abord une méthode itérative pour le calcul non-linéaire de portiques plans, à l'aide des calculateurs électroniques. Cette méthode est utilisée pour étudier l'influence du type de diagramme qui représente les propriétés mécaniques sur le comportement d'une structure simple.

Il discute ensuite les moyens les plus convenables pour établir des théories statistiques de structures. Ces théories doivent permettre de passer du caractère aléatoire des propriétés mécaniques à celui du comportement statique.

Pour terminer, il combine les distributions aléatoires des charges et du comportement statique de façon à calculer la probabilité de ruine et juge les résultats obtenus du point de vue de la sécurité.

Il tire des conclusions sur les bases les plus convenables pour le dimensionnement des constructions.

Zusammenfassung

Der Autor behandelt mögliche Verbesserungen der Bemessungsmethoden von Tragwerken.

Für die Untersuchung des nicht linearen Verhaltens von ebenen Tragwerken mit Hilfe elektronischer Rechengeräte wird eine Iterationsmethode besprochen und angewendet. Als Beispiel wird der Einfluß untersucht, den die Form der Kurve, die durch die mechanischen Eigenschaften des Werkstoffes gegeben ist, auf das Verhalten eines einfachen Tragwerkes ausübt.

Es werden zweckmäßige Wege für die Festlegung von statistischen Theorien behandelt, die es erlauben sollen, vom Wahrscheinlichkeitscharakter der mechanischen Eigenschaften zu demjenigen des Verhaltens des Tragwerkes überzugehen.

Die stochastische Verteilung der Lasten wird mit derjenigen des statischen Verhaltens kombiniert und daraus die Einsturzwahrscheinlichkeit bestimmt. Der Einfluß der Ergebnisse auf die Sicherheitskriterien wird für einige einfache Fälle untersucht.

Es werden Schlüsse über die zweckmäßigsten Grundlagen für die Bemessung von Tragwerken gezogen.