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Discussion - Discussion - Diskussion

Generalised Approximate Method of Assessing the Effect of Deformations on Failure Loads (M. R. Horne)[1]

Méthodes d'approximation généralisées pour l'évaluation de l'effet des déformations sur les charges de rupture

Allgemeine Näherungsmethoden zur Bestimmung des Einflusses von Verformungen auf die Bruchlasten

A. HRENNIKOFF[2]
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Determination of the load factor of a known structure is a basic problem of plastic theory. The author approaches it in an indirect way by the use of a Rankine type formula, in which the required load factor is expressed through two other load factors: the rigid plastic, in determination of which the change in shape of the structure is ignored, and the elastic. The discusser wishes to comment on some aspects of this development.

1. The Rankine formula, which is used also in other areas of structural engineering, is empirical in nature, and the arguments in its favour contained in the author's earlier papers do not prove it but merely show its plausibility. This does not disqualify the formula, but rather makes its acceptance dependent on experimental or theoretical confirmation in a wide range of special cases.

2. In view of the difficulty of determination, the elastic load factor entering the Rankine formula, is replaced by pseudo-elastic factor based on special assumptions of deformability of the material. The members are assumed to remain totally undeformed except at the sites of plastic hinges where they deform in accordance with a special "rigid-plastic-rigid" scheme. The extent of the strain range k in plastic section of this scheme is left unspecified. With several other assumptions and a reasoning, which the discusser has found difficulty in following, a formula for pseudo-elastic load factor is derived in terms of the strain range k.

The pseudo-elastic approach is applied to buckling of a pin-ended strut, and the value of the unknown strain range k is determined by making the

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[1]) See "Publications" — voir «Mémoires» — siehe «Abhandlungen», vol. 23, p. 205.

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result conform to the Euler's critical load. The pseudo-elastic formula and the value of k so found are then used in analysis of a two-legged frame. In spite of good results with the frame the author increases k by 50%, as a measure of safety, thereby effecting a proportional decrease in the value of the pseudo-elastic load factor. The increased k is apparently intended for use in all types of structures under all loading conditions.

The pseudo-elastic analysis being the essence of the proposed method, deserves a thorough critical examination.

a) There is no physical resemblance between the localized hinge deformations of the model in the pseudo-elastic analysis and the distributed deformation of the structure itself.

b) Although the strain at the pseudo-elastic hinge is described on p. 207 as exceeding the one at the beginning of strain hardening, its computed value with $k = 4/\pi^2$ corresponds to less than one half of the *elastic* range, i. e. something less than $^1/_{40}$ of the value which it is supposed to possess.

c) The 50% increase of the computed value of k, devoid of any theoretical justification, appears to be solely a device to place the load factor neatly within the range of some predetermined results.

d) The paper gives an impression of identity of the mechanisms used for determination of the rigid plastic and the pseudo-elastic load factors. Yet in the example of the four-storey frame the two mechanisms are different. This makes uncertain the basis for selection of the mechanism to be used in the pseudo-elastic analysis.

e) The two-legged frame example referred to above will produce odd results if the loading is modified as follows: two equal compressive forces are applied horizontally opposite each other to the ends of the beam and the two vertical loads are placed symmetrically on the beam instead of the columns, forming a symmetrical four-hinge beam mechanism. If now constant plastic moments are assumed in the columns, in other words if the horizontal reactions under the columns are taken as zero, which is consistent with statics and deformations, the critical load becomes zero because the quantity $h_1 + h_2$ is infinite. On the other hand, if these reactions are assumed distinct from zero, which is also legitimate, the critical load becomes finite and of a variable magnitude depending on the arbitrary value of the assumed reactions. It is felt that ambiguities of this kind are hardly legitimate in a rational method.

3. The results obtained by the use of Rankine formula with pseudo-elastic load factor are compared with what the author calls "the accurate solutions" in application to two four-storey one-bay building frames and several one-storey pitched roof frames. The agreement is close in some cases, not so close but safe in others, and unsatisfactory in cases involving one-sided failures of pitched roof buildings. These results, inconvincing to the writer, apparently satisfy the author.

With regard to the author's reference to "the accurate solutions", the writer knows of no rigourous workable method for determination of the true load factor in the non-rigid plastic theory, not because proper methods of mechanical calculation are not available but because the exact theory of elasto-plastic analysis is non-existent. The author mentions two non-rigourous assumptions in his brief reference to the "accurate solution" by Wood: the absence of strain hardening and an inexact manner of moment distribution at a joint. The effect of residual stresses, ignored in English plastic literature, is viewed as important in America.

4. The load factor determined by the Rankine formula is apparently not intended to provide for possible failure by lateral-torsional buckling and local crippling, and for this reason cannot be considered as the final or true load factor. This insufficiency of the proposed analysis points to a basic defect of plastic theory. The early promise by its pioneers, of easy determination of the failure intensity of the load was not fulfilled and merely led to an idealized fictional quantity — the rigid-plastic load factor, followed later by further abstractions: the factors in lateral-torsional instability and the overall frame instability. Other varieties of load factors were discussed in the author's earlier papers. Actually of course there is only one load factor, influenced by all the tendencies implied in the partial factors, and this factor so far has eluded the efforts of the plasticians. In the mean time, in refutation of the early promises of simplicity, a formidable literature has been evolved, which only few apart from the authors may find lucid.

Conclusions

1. The pseudo-elastic theory has no place in rational structural analysis as arbitrary and ambiguous.

2. There is no objection in principle to the use of Rankine formula in determination of the load factor, provided it allows for all types of failure and is adequately confirmed over exhaustively wide range of conditions. Unfortunately such confirmation either by rigourous theoretical analysis or large scale tests appears impossible.

Summary

The discusser feels that the proposed method for determination of failure loads is unsatisfactory because of the following characteristics:

1. Arbitrariness and ambiguity of the pseudo-elastic theory.
2. Incompleteness in ignoring the effects of lateral-torsional buckling and local crippling.
3. Insufficiency of substantiation of the Rankine type empirical formula.

Résumé

L'auteur estime que la méthode proposée pour la détermination de la charge de ruine n'est pas satisfaisante pour les raisons suivantes:

1. La méthode pseudo-élastique est arbitraire et ambiguë.
2. Il n'est pas tenu compte du déversement et du voilement local.
3. La formule de Rankine, de caractère empirique, n'est pas suffisamment établie.

Zusammenfassung

Der Autor findet die vorgeschlagene Methode zur Bestimmung der Traglast aus folgenden Gründen anfechtbar:

1. Zweideutigkeit und Willkür der Pseudo-Elastizitätstheorie.
2. Unvollständigkeit wegen der Vernachlässigung des seitlichen Torsionsknickens und lokalen Beulens.
3. Unzulässigkeit einer allgemeinen Begründung der empirischen Formel von Rankine.

Reply - Réponse - Antwort

M. R. HORNE

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In reply to Professor HRENNIKOFF, it is not true to say that the Rankine load is purely empirical. It has been shown [1] that there are strong theoretical reasons for believing that the Rankine load is an approximate lower bound to the failure load of an elastic-pure plastic structure. More significantly, there is also a not inconsiderable amount of experimental and empirical evidence in its favour [1, 2, 3, 4].

The treatment proposed in the paper is intended to deal with overall frame instability and its effect on the failure load predicted by simple plastic theory. It would be nothing short of miraculous if the same simple method sufficed for all forms of instability. It came, in fact, as a complete surprise to the author that the method worked as satisfactorily as it appears to in relation to frame instability. The examples have not been chosen merely because they suit the method, and in a range of realistic frames examined, the use of the recommended value of k gives answers for the critical load that are within the range 60% to 120% of the correct value. The great majority of frames lie within the range of 80% to 110%. For those structures which would not in pratice prove unsatisfactory because of excessive deflexions at working loads, the elastic critical load is likely to be at least six times the simple plastic collapse load. This being so, and taking the extreme cases of a 40% underestimate and a 20% overestimate in the critical load, the extreme errors in calculating the Rankine load become an underestimate of 8½% and an overestimate of 2½%. The great majority of frames would show extreme errors of 3½% underestimate and 1½% overestimate. These errors hardly seem serious in the light of the many other approximations.

The calculation of elastic-plastic collapse loads by digital computer programme, with full allowance for frame instability and change of geometry, is now a routine procedure in at least three Engineering Departments in Britain, namely in the Faculties of Science and of Technology at Manchester and in the Engineering Department at Cambridge. Some particulars of the type of programme involved were given some years ago by LIVESLEY [5, 6].

The author is well aware of the many criticisms that may be levelled at the proposed method — this is hardly surprising in view of its simplicity. There is however sufficient evidence that, considering this simplicity, the method is of some value. The author would be very interested to learn of other semi-empirical methods of comparable simplicity and generality that are equally or more successful. Details of such methods would be of great interest for comparison, both with the method proposed in the paper and with the results of accurate solutions that are available.

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Summary

Professor HRENNIKOFF, in his criticism of the author's use of the Rankine load, makes no comment on the evidence referred to in the the paper and

bibliography. The method contained in the paper is put forward simply as an empirical means of estimating failure loads in relation to overall elastic-plastic failure, and no claim is made that all the other factors mentioned in the discussion are allowed for.

Résumé

En critiquant l'emploi de la formule de Rankine, le professeur HRENNIKOFF ne discute pas les preuves contenues dans l'article et la notice bibliographique. La méthode présentée ne veut être qu'un procédé empirique d'estimer les charges limites en rapport à la ruine élasto-plastique d'ensemble, sans prétendre couvrir les autres facteurs mentionnés dans la discussion.

Zusammenfassung

In seiner Kritik gegenüber der Anwendung der Rankineschen Formel geht Professor HRENNIKOFF nicht auf den Aufsatz und in den Literaturangaben enthaltenen Beweise ein. Die vorgeschlagene Methode ist als empirisches Verfahren für die Bestimmung von Traglasten im Zusammenhang mit dem elastisch-plastischen Gesamtversagen anzusehen. Auf eine Mitberücksichtigung der andern in der Diskussion erwähnten Faktoren wird hier kein Anspruch erhoben.