Challenge and promise of assessment of structural system reliability

Challenge and Promise of Assessment of Structural System Reliability

Défi de l'évaluation de la fiabilité d'un système structural

Herausforderungen bei der Ermittlung der Zuverlässigkeit von Systemen

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SUMMARY
A brief description of some of the widely used methods for the evaluation of structural system reliability is given in the first part of the paper. An approach, in the form of software for a microcomputer, which uses simulation to compute the probability of failure of a structure is suggested. The method would yield reliable results and pinpoint the critical issues, enabling the engineer to make improvements in design.

RÉSUMÉ
Certaines méthodes largement utilisées pour l'évaluation de la fiabilité des systèmes structuraux sont présentées. Une approche en forme de logiciel pour micro-ordinateur, basée sur la méthode de simulation pour le calcul de la probabilité de ruine d'une structure est aussi présentée. La méthode proposée permet d'obtenir des résultats fiables, indique les paramètres critiques de la stucture et permet à l'ingénieur d'améliorer la conception.

ZUSAMMENFASSUNG
Der erste Teil dieser Arbeit behandelt häufig benützte Methoden zur Bewertung der Zuverlässigkeit von Bausystemen. Eine hier vorgeschlagene Methode macht Gebrauch von Software für Microcomputer, um die Wahrscheinlichkeit des Versagens einer Struktur zu bewerten. Diese Methode ist verlässlich und zeigt die kritischen Punkte, was dem Ingenieur erlaubt, den Entwurf zu verbessern.

1. INTRODUCTION

It has been established that most factors influencing the design and performance of structures are, in reality, uncertain. The structural design codes, consequently, are edging towards probability based methods with a view to rationalize and unify them. Engineers and researchers now recognize that the 'safety' or 'reliability' of a structure is of prime importance and have been formulating methods to quantify the reliability of a structural system. The objective of this paper is to review the methods currently used for the assessment and to suggest an approach which, in the authors' view, is likely to yield more reliable and realistic results by modelling the system with the aid of the rapidly increasing power and speed of microcomputers.

The existing methods used for the assessment of structural system reliability are classified and critically reviewed in the first part of the paper. The implementation of the more 'exact' methods, if not impossible, require the use of considerable statistical skill while the assumptions made in the simpler approaches leave the reliability of the prediction in some doubt; simulation techniques being used at times to assess the reliability and adequacy of the results obtained from these analytical approaches. The most commonly used approach to assessing the probability of failure of a structural system is to determine the upper and lower bounds of the probability. Researchers have suggested many methods for their computations but the reliability of the result depends on the assumptions made in the statistical theories of the idealized world and the narrowness of the margin between the bounds determined.

Monte Carlo simulation has often been used to model the failure mode of structural systems and to verify the results obtained by other methods but seldom as a tool for obtaining the probability of failure directly. The commonly stated reason for the reluctance to use simulation is the perceived computer time required to obtain a fairly reliable estimate. Great strides have been made in the development of computer technology and speed in recent years, presenting the possibility of overcoming this obstacle. The greatest advantage of using simulation techniques is that, unlike other methods, assumptions need not be made regarding the probability distribution of the variables and their correlations. Researchers have, additionally, been discouraged by the perceived difficulty in analysing the effects of even slight changes to the model. The development of specially suited software for use with a microcomputer would remove this difficulty and would not only give an estimate of the probability of failure, but would also be able to study the sensitivity of the objective to variation in the model and that of the model to changes in the variables within each model, enabling the most critical issues to be pinpointed [1].

2. REVIEW OF EXISTING METHODS

2.1 Elementary concepts of reliability

The values of relevant variables which would result in failure, or, in other words, the failure domain, may be separated from the safe region by a failure surface, the equation of which is termed the failure function. In the simplest case of a structural element with resistance R subjected to a load or load effect S (in similar units), the failure function G may be taken as the difference

$$G = R - S \quad ..... \quad (1)$$

Since R and S usually exhibit statistical dispersions, the probability of failure is found as follows if both variables are independent. The probability that the load lies in the range x, x+dx ($f_S(x)$) is found and multiplied by the cumulative probability that the resistance lies below x ($F_R(x)$). This product is summed over all possible values to give the probability of failure P_f as

$$P_f = \int_{-\infty}^{+\infty} f_S(x).F_R(x).dx \quad ..... \quad (2)$$

The reliability index β is defined as

$$\beta = \frac{\mu_G}{\sigma_G} \quad ..... \quad (3)$$

where μ_G - mean value of G
σ_G - standard deviation of G

The methods used for structural reliability assessment were classified into three categories, levels I, II and III, in early research and this grouping is still of some use [2,3]. A detailed description of the classification is given elsewhere [2].

2.2 Evaluation of reliability of structural elements

The first step towards the computation of the reliability of a structural system is the evaluation of the probability of failure of the elements in the system. The failure function g(X) in terms of the relevant variables $X_1, X_2, \ldots, X_n$ is formed and the joint probability density function

$$f_{X_1, X_2, \ldots, X_n}(X_1, X_2, \ldots, X_n)$$

of the variables is integrated over the region of failure to obtain the probability of failure P_f of the element.

$$g(X) = g(X_1, X_2, \ldots, X_n) \quad ..... \quad (4)$$

$$P_f = \int\int_{g(x)<0}\ldots\int\int f_{X_1, X_2, \ldots, X_n}(X_1, X_2, \ldots, X_n).dX_1.dX_2\ldots.dX_n \quad ..... \quad (5)$$

Due to the difficulties in forming the joint probability density function, simplifying assumptions are resorted to in determining P_f. Some of the methods used for the evaluation are described in the next few sections.

2.2.1 Numerical integration

The most reliable of the methods available, this involves the computation of P_f using equation (5) by numerical methods. In most circumstances, however, as the joint probability density function cannot be defined, the method would not be of much use.

2.2.2 Maximum entropy distribution method [4]

The first four statistical moments of the basic variables are used to determine the statistical moments of the failure function. A maximum entropy distribution is generated to fit the failure function and P_f is then computed using numerical integration. The method has been shown to yield reliable results under certain conditions.

2.2.3 Second moment methods

A simplification is made by expanding the failure function g(X) in a Taylor series about a point lying on the failure surface ($X_1^*, X_2^*, \ldots, X_n^*$). The series is then truncated at first order terms and approximate values found for the mean and variance of g(X). For uncorrelated variables,

$$\mu_g \simeq - \sum_{i=1}^{n} X_i'^* \left(\frac{\partial g}{\partial X_i'}\right)_* \quad ..... \quad (6)$$

$$\sigma_g^2 \simeq \sum_{i=1}^{n} \sigma_{X_i'}^2 \left(\frac{\partial g}{\partial X_i'}\right)_*^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'}\right)_*^2 \quad ..... \quad (7)$$

The first derivatives are evaluated at the chosen point on the failure surface and X_i' is the standard normal transformation of the variable X_i. The derivation of the equations is discussed at length elsewhere [5].

2.2.4 Monte Carlo simulation

Several trials are performed to model the failure function. Within each trial, a random value of each relevant variable is generated and the value of the failure function determined. The probability of failure can then be estimated by one of two approaches. The first involves dividing the number of trials where $g(X)$ was found to be negative or zero by the total number of trials to give P_f. The second approach uses the values of $g(X)$ generated to find the distribution of $g(X)$ from which the area below zero is computed and taken as P_f. It must be pointed out that often researchers assume each basic variable to follow a stylised probability distribution, for example a normal distribution. Additionally, the basic variables are at times assumed to be independent. If a large number of variables are involved in a purely additive problem it may be acceptable to assume that the distribution of $g(X)$ is normal, but this is seldom the case in reality. Monte Carlo simulation is advantageous in such situations as the probability distributions of the variables need not be assumed to follow a stylised distribution and correlations between variables, if they can be perceived and quantified, could easily be incorporated in the evaluation of the value of $g(X)$ within each trial.

2.3 Evaluation of reliability of structural systems

It is not usually feasible to link the probability of failure of the elements to that of the structure directly as the elements can interact with one another. A simplification is made by classifying systems as either series, where the failure of any one element results in the failure of the system, or parallel, where each element must fail to cause the collapse of the system. In reality most structural systems are a combination of the two. If for example a framed structure exhibiting plastic mechanisms of collapse is considered, the occurrence of each mode may be taken as equivalent to a parallel system as all the plastic hinges necessary to cause the mechanism must occur. The failure of the structure, however, would be equivalent to a series system of different modes of failure i.e. different mechanisms, as the occurrence of just one mode results in system collapse. Since there are complications in deriving the probability of failure of the system, relatively simple methods are used to compute the upper and lower bounds of the probability of collapse P_f. The most commonly used bounds are the simple bounds and the Ditlevsen's bounds. In the former, the bounds are given by

$$\operatorname*{Max}_{i=1}^{m} (P_{fi}) \leqslant P_f \leqslant 1-\prod_{i=1}^{m}(1-P_{fi}) \quad \text{for a series system and}$$

$$\prod_{i=1}^{m} (P_{fi}) \leqslant P_f \leqslant \operatorname*{Min}_{i=1}^{m} (P_{fi}) \quad \text{for a parallel system}$$

where

m - number of modes of failure

P_{fi} - collapse probability of the ith mode

The Ditlevsen's bounds apply for a series system and are more reliable as the range between the bounds is narrower. The expressions for the bounds are

$$P_{f1} + \sum_{i=2}^{m} \operatorname{Max}\Big(P_{fi} - \sum_{j=1}^{i-1} P_{fij};\ 0\Big) \leqslant P_f \leqslant \sum_{i=1}^{m} P_{fi} - \sum_{i=2}^{m} \operatorname*{Max}_{j<i} P_{fij}$$

where P_{fij} is the probability that the failure functions of the ith and jth modes both indicate failure. These bounds require the consideration of all possible pairs of failure modes.

Many researchers have considered different types of structures and suggested different methods by which an estimate of the probability of failure of the system may be found. For example, in the case of framed structures, Stevenson and Moses [6], Kam, Corotis and Rossow [7], Moses [8], Murotsu et. al. [9], Ang and Ma [10], Bennett and Ang [11] and Ranganathan and Deshpande [12] are only a few of the many researchers who have suggested varying methods of estimating P_f. Most of the researchers have assumed the basic variables as following a normal distribution when using examples to illustrate their methods. Some researchers have considered the variables to be correlated while others have assumed them to be independent. A few researchers have used simulation to verify the results obtained by using their proposed methods. It becomes clear that many assumptions can once again be avoided if simulation was used to determine the value of Pf directly.

3. THE CHALLENGE AND THE PROMISE

3.1 The challenge

It can be seen from section 2 that estimating the probability of failure of a structure is a formidable task and has been tackled only by making simplifying assumptions which may not be applicable in real situations. There exists a need, therefore, for developing an approach which is simple to use and yields more reliable results.

3.2 The promise

It has been stated that Monte Carlo simulation is the only method by which a reliable estimate of system collapse probability may be found using the statistics of collapse of each mode [8]. Engineers are often discouraged from using probabilistic analysis to solve a problem but welcome the concept of modelling a structure using Monte Carlo simulation. It is the authors' view that developing computer software using Monte Carlo simulation to predict the probability of failure would therefore not only yield realistic results but would also be more widely accepted. A minimum of statistical knowledge would be necessary for such an approach. The next section gives a brief description of the method.

4. THE RECOMMENDED APPROACH

Figure 1 shows in the form of a flow chart the steps involved in using Monte Carlo simulation for the prediction of the probability of failure Pf. The program 'Venturer' [1] could be used with ease to determine the probability distribution of the basic variables if very little information is available. It is envisaged that the structure would be fed into the program in the form of a network or tree diagram, enabling the computer to determine the various modes of failure that would be possible. The correlation between variables can easily be accommodated in the simulations carried out to determine the probability of failure of each mode of collapse. These would then be combined to estimate the probability of collapse of the structure itself. The software would plot on the screen the possible modes of failure, indicating that which is most likely to occur. By ensuring that the software is user-friendly, it could be used to encourage the user to try a variation in the model or in the variables and observe the effect of these changes. Relatively fewer simulations could be carried out at this stage until the model and variables had been defined to the user's satisfaction. A large number of trials could then be performed to get an estimate for the value of Pf.

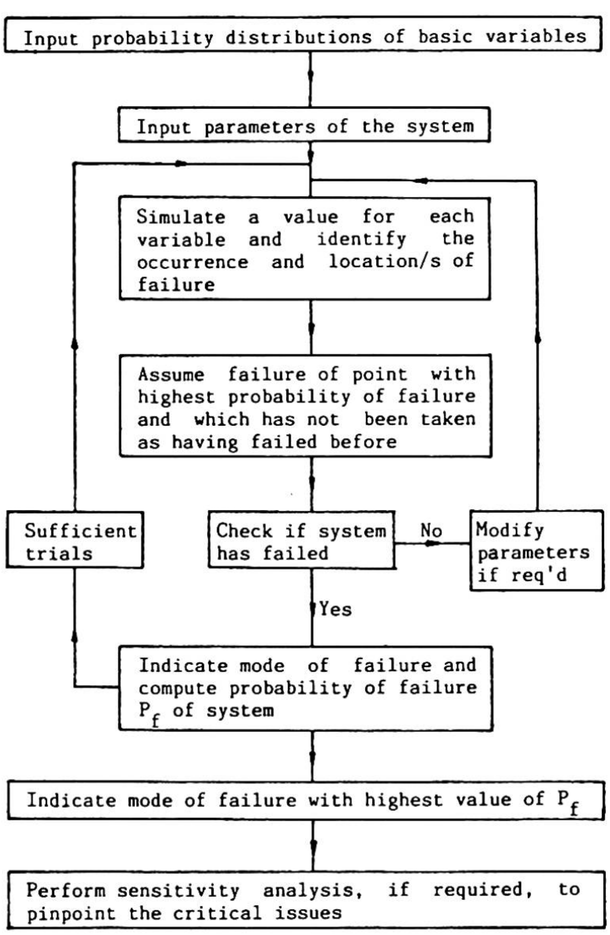

Fig. 1 Flow chart for method

Another advantage in the package would be the sensitivity analysis that could be carried out to determine the sensitivity of the model to changes in the variables. The variables that are most critical would be pinpointed at this stage giving the engineer an opportunity to alter the design. The changes in the value of P_f due to changes in the model could also be analysed and would give the user more insight into the problem. The authors have pursued the approach detailed above and found the method to yield promising results.

5. CONCLUSIONS

It has been pointed out that at present, most methods available for the evaluation of structural reliability or probability of failure involve making assumptions. The suggested approach using Monte Carlo simulation would result in a simple and acceptable software which can be used for a variety of problems and would result in reliable results. The advancement of computer technology has introduced the possibility of using simulation as a viable method for finding the system collapse probability in its own right instead of being used as a calibration tool.

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