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## Hysteresis Models for Earthquake Response Simulation

Modèles d'hystérésis pour la simulation de réponse sismique

Hysteresismodelle zur Simulation des Erdbebenverhaltens

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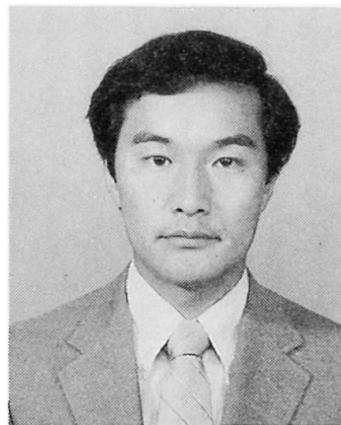
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### **SUMMARY**

This paper presents experimental investigations into the effect of loading rate on the hysteretic behavior of steel components and systems. Steel frames and columns were loaded both dynamically and quasi-statically, and the relationship between the loading rate and the restoring force was examined. A procedure to estimate the hysteresis for dynamic loading based on the hysteresis obtained under quasi-static loading is also proposed.

### **RÉSUMÉ**

Ce document présente les études expérimentales de l'effet du niveau de charge sur le comportement d'hystérésis des éléments et ensembles en acier. Des poteaux et ossatures ont été soumis à des charges dynamiques et quasi-statiques, et la relation entre la charge et la force de restitution a été examinée. Une procédure pour évaluer l'hystérésis due aux charges dynamiques basée sur les résultats des charges quasi-statiques est aussi proposée.

### **ZUSAMMENFASSUNG**

Diese Veröffentlichung befasst sich mit experimentellen Untersuchungen des Einflusses der Belastungsgeschwindigkeit auf das hysteretische Verhalten von Stahlkomponenten und Systemen. Stahlrahmen und Stützen wurden sowohl dynamisch als auch quasi-statisch belastet und der Zusammenhang zwischen Belastungsgeschwindigkeit und Reaktionskräften wurde ermittelt. Eine Methode zur Ableitung der dynamischen Hysteresis von der quasi-statisch ermittelten Hysteresis wird vorgeschlagen.



## 1. INTRODUCTION

To simulate the inelastic response of structural systems under earthquake loading, one should assign to each structural component a hysteresis model that represents the component's restoring force characteristics. Some of the popular models are, for example, the bilinear model and the Ramberg-Osgood model. When using those models, the analyst should assign a certain value to each of the model's individual parameters such as the yield force, the maximum strength, and the elastic stiffness. To do this, one often uses information obtained from the experimental investigations in which the tests were performed quasi-statically (slow loading). Here, a simple but serious question is addressed; whether or not the hysteresis models whose properties are determined based upon the statically loaded test results are valid for the purpose of earthquake response simulation, because the earthquake loading is naturally dynamic. It is well known that the stress-strain relationship of steel is affected by the strain rate. Under a high strain rate, both the yield and maximum stresses increase, while the change in Young's modulus is minimal. Then, the hysteresis of steel structural components and systems should also be affected by the loading rate since their constituent is rate dependent. There are studies that examined the loading rate effect on the hysteresis of steel components and systems. Some studies [1,2] suggested that this effect is not significant as long as the loading rate remains in a level achieved by the earthquake loading, while some other studies [3,4] warned that this effect cannot be neglected by any means. This inconclusiveness in their comments seems to stem from the difficulty to bridge the material's strain rate effect on the stress-strain relationship with the structure's loading rate effect on the hysteresis. Further, this difficulty is given rise to, because (1) as the earthquake loading is nonstationary, the loading rate continuously changes with respect to the time, and (2) as the structural components and systems extend into three dimensions, the strain as well as the strain rate vary from one fiber to another even at a given time.

The goal of the study is to propose simple but systematic procedures to estimate the loading rate effect on the hysteresis of steel components and systems by utilizing the knowledge of the strain rate effect on the steel materials. This paper introduces experimental studies that examined the effect of loading rate on the earthquake response of steel frames and on the hysteresis of steel components. A preliminary proposal to estimate the loading rate effect on the hysteresis of steel components is also included in this paper.

## 2. EFFECT OF LOADING RATE ON EARTHQUAKE RESPONSE

### 2.1 Pseudo Dynamic Test

The shake table test is known as the most direct experimental procedure to simulate the earthquake response of structural components and systems. Another experimental procedure for the earthquake response simulation, designated as the pseudo dynamic test, has also been devised [5]. This test is a combined numerical analysis and experiment and capable of reproducing the earthquake response of structures under quasi-static loading. The basic algorithm of this test follows; (1) to fabricate the test structure whose earthquake response one wishes to examine, and to set up this structure on the test bed; (2) to assume this structure as a discrete mass-spring system and establish the corresponding discrete equations of motion in a computer; (3) to connect a load applying actuator to the test structure at each assumed discrete mass-position and in the direction of vibration; and (4) to solve the equations of motion by the direct integration method, but, rather than assigning a hysteresis model for each assumed spring, to measure the restoring force directly from the test in which the structure is loaded quasi-statically to the computed displacement positions. By repeating the computation, quasi-static loading, and measurement, the test structure traces a displacement time history as if it were subjected to

the real earthquake loading, but the velocity attained in the test is much smaller than the velocity to be achieved in the real earthquake loading. Details in the concept of the pseudo dynamic test and a summary of the previous applications can be found elsewhere [6].

## 2.2 Comparison Between Shake Table Test and Pseudo Dynamic Test

As the first step to investigate the loading rate effect, a shake table and pseudo dynamic tests were carried out for a steel frame. Comparison of the responses obtained from those two tests makes it possible for us to learn directly the effect of loading rate on the earthquake response, because, from the basics of the pseudo dynamic test, the major difference between the two tests is the loading rate; i.e. dynamic in the shake table test and quasi-static in the pseudo dynamic test. The basic dimensions of the tested steel frame are shown in Fig. 1. Figure 2 shows the test setup for the pseudo dynamic test. Two identical test specimens were fabricated; one for the shake table test and the other for the pseudo dynamic test. Further, the same input ground motion was applied to the two specimens. The obtained displacement time histories and force-deflection relationships are compared in Figs. 3 and 4. Figure 3 shows that the waveforms of the two displacement responses are very similar but there is an offset in the datum line between the two responses. Figure 4 indicates

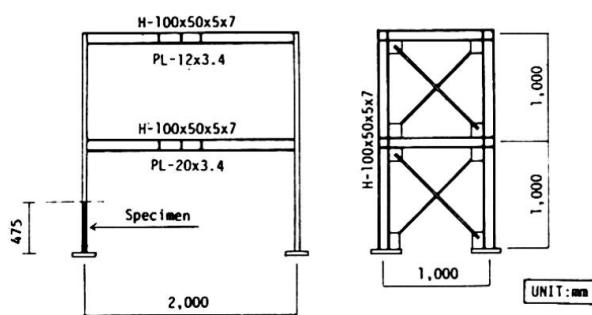


Fig. 1 Specimen for Steel Frame Test

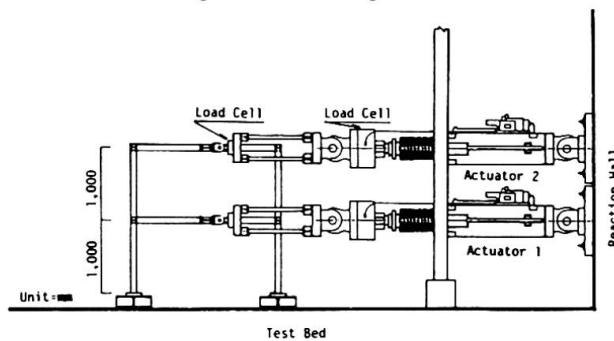


Fig. 2 Test Setup for Pseudo Dynamic Test

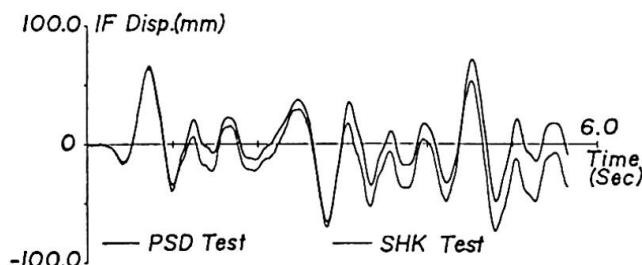
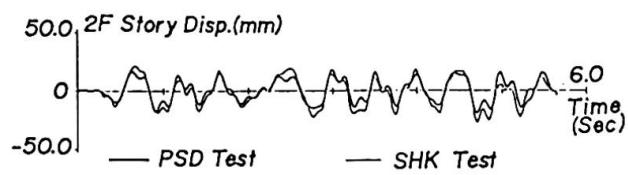


Fig. 3 Displacement Time Histories Obtained from Shake Table and Pseudo Dynamic Tests

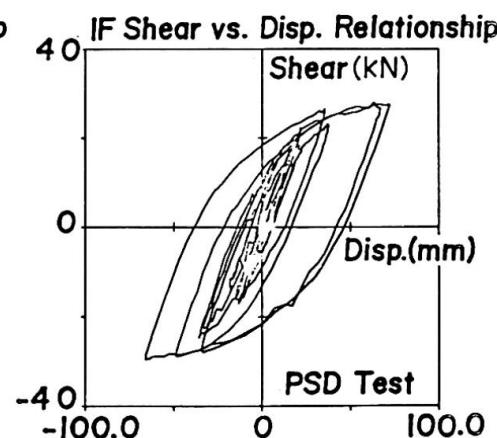
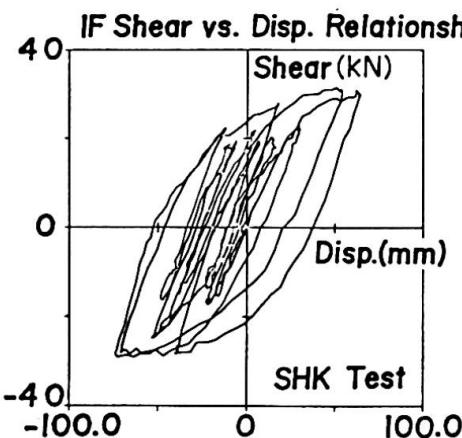


Fig. 4 Hysteresis Curves Obtained from Shake Table and Pseudo Dynamic Tests



that the restoring force in the inelastic range is greater by about 10 to 15 percent in the shake table test. It was found from the post numerical analysis that the difference in the restoring force (Fig. 4) was mainly responsible for the difference in the displacement response (Fig. 3). Details in the shake table and pseudo dynamic tests as well as the post numerical analysis are described in [7].

### 3. EFFECT OF LOADING RATE ON HYSTERESIS

From the results indicated above, it was found that the loading rate indeed affected the hysteresis and accordingly the earthquake response of the steel frame. To further examine the mechanism that changed the hysteretic characteristics, dynamic and quasi-static loading tests were carried out for a cantilever H-shaped steel column (Fig. 5). The test setup is shown in Fig. 6. The tested column had the same material and cross sectional properties as the columns of the steel frame (Fig. 1), and the column length was exactly half the first story's clear span length of the steel frame. Since, in the steel frame, the beam stiffness was significantly larger than the column stiffness, the point of contraflexure of the column remained in the middle of its clear span. Further, the actuator, attached to the free edge of the column, imposed a displacement time history that was exactly half in magnitude the first story displacement response obtained from the pseudo dynamic test (Fig. 3). In those conditions imposed, the cantilever column's hysteresis was representative of the first story hysteresis of the steel frame. Two cantilever specimens were fabricated, and one (designated as the fast test) was tested by imposing the displacement time history dynamically with the velocity that was computed in the pseudo dynamic test, whereas, the other (designated as the slow test) under slow loading in which the velocity was reduced to 1/100 of the velocity in the fast test. Since the two tests sustained the exactly the same displacement time history, comparison in the restoring force between the tests provided us direct information on the loading rate effect.

The major findings obtained from those tests can be stated as follows. (1) The two obtained hysteresis curves are compared in Fig. 7, in which the restoring force obtained from the fast test is greater in the inelastic range by 10 to 15 percent. This difference agreed with the difference observed in the steel frame test. A closer look of those curves, however, indicated that, in the fast test, the restoring force was constantly higher in the inelastic range, but decreased rapidly as shown in Fig. 8 when the displacement approached its extreme value (the maximum displacement in Fig. 8). In fact, the force almost matched with the restoring force obtained in the slow test at its extreme position, where the velocity was reduced to zero. This observation supports our intuition that, at the extreme displacement where the velocity is reduced to zero, the restoring force should be the same in both the fast and slow tests. (2) The stiffness in the elastic range was almost identical between the two tests, indicating that the loading rate did not influence the elastic behavior. This observation also agrees with the known fact that Young's modulus of steel remains unchanged regardless of the strain rate. (3) In the fast test, the yielding region was more confined near the clamped edge (the most stressed section). Since the displacement at the free edge was taken to be the same, in turn, this confined yielding region resulted that the curvature at this region was greater in the fast test. (4) Figure 9 shows the relationship between the curvature rate and the restoring force ratio at the cross section 20 mm inside the clamped edge (designated as SEC1 in Fig. 5). Here, the curvature rate was estimated from the strain rates measured at SEC1 during the fast test, and the restoring force ratio was taken as the restoring force in the fast test relative to that in the slow test. Furthermore, the plots were made only for the region where SEC1 yielded significantly. The figure shows that the maximum curvature rate and strain rate was about 0.01 (1/mm/sec) and 0.2 (1/sec) respectively, and the

restoring force ratio scattered between 10 to 15 percent.

#### 4. ESTIMATION OF EFFECT OF LOADING RATE ON HYSTERESIS

From the above experimental observations, the relationship between dynamically and quasi-statically loaded hysteresses can be stated quantitatively as follows. (1) The elastic stiffness remains unchanged regardless of the loading rate. (2) Under dynamic loading, the restoring force increases in the inelastic range, and this increase is relatively constant until the displacement approaches its extreme value (Fig. 8). (3) Under dynamic loading, the restoring force drops rapidly near the extreme displacement and merges with the restoring force obtained under quasi-static loading. Since, as shown in Fig. 8, this drop occurs only very near the extreme displacement, one may reasonably neglect this drop and assume that the restoring force is constantly higher up to the extreme displacement.

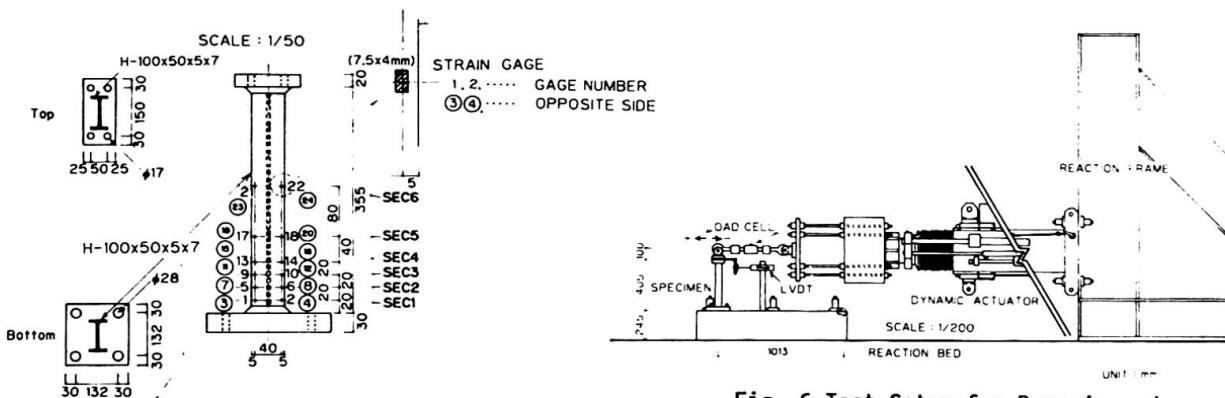


Fig. 5 Specimen for Steel Cantilever Column Test

Fig. 6 Test Setup for Dynamic and Quasi-Static Loading Tests

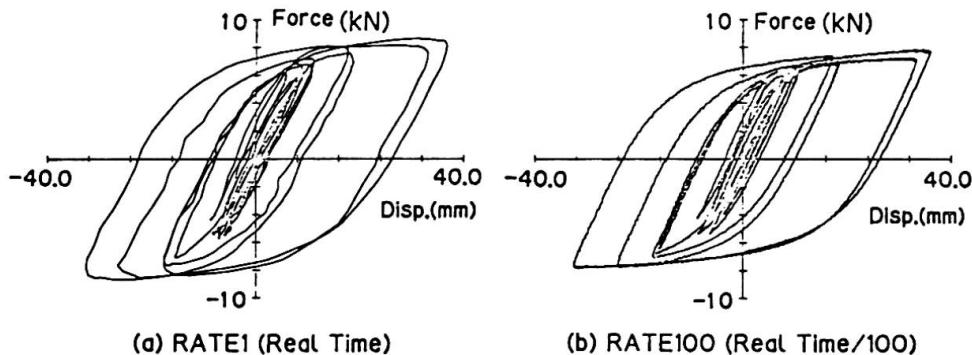


Fig. 7 Hysteresis Curves Obtained from Dynamic and Quasi-Static Loading Tests

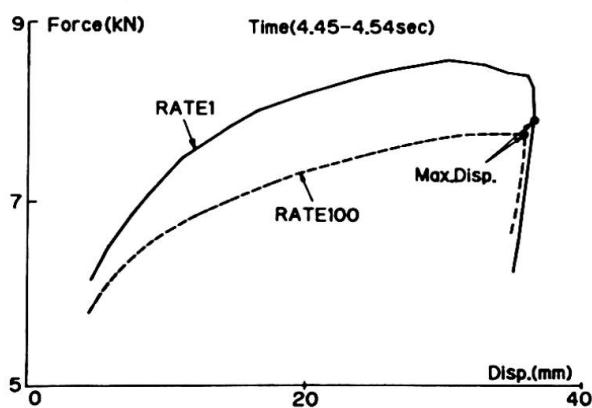


Fig. 8 Comparison of Dynamic and Quasi-Static Hysteresis Curves

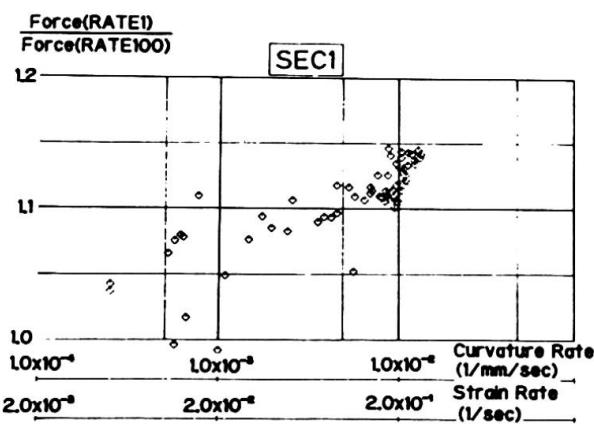


Fig. 9 Curvature Rate vs. Reactional Force Ratio Relationship



Considering the statements (1) to (3), the hysteresis under dynamic loading can be estimated reasonably but without much elaboration by simply adding, in the inelastic range, some restoring force to the restoring force obtained under quasi-static loading. Then, the question that remains is how to estimate this additional restoring force. A procedure to estimate the additional restoring force for cantilever steel beams (like the one in Fig. 5) is proposed below; (1) To estimate the curvature rate at the clamped edge from the velocity at the free edge. In fact, this estimate is made possible if the moment vs. curvature relationship is specified for the cross section. Further, it is presumed that the velocity is a quantity that can be obtained in the numerical computation; (2) to bridge the curvature rate with the strain rate by assuming that the plane section remain plane after deformation; (3) To estimate the moment increase based upon the strain rate at the section. Here, the stress increase in relation to the strain rate is taken to be known through our previous knowledge; (4) To estimate the restoring force increase by dividing the moment increase by the length of the beam. The validity of this procedure was calibrated for the cantilever steel column (Fig. 5). According to the test result, at 4.45 sec, the velocity at the free edge was 277.5 mm/sec, and the restoring force increase was by 945 N. On the other hand, the computation using the proposed procedure gave a value of 1,010 N for the restoring force increase. The estimated value is only 7 percent larger, demonstrating the appropriateness of this procedure. Details in the procedure proposed as well as the fast and slow test results are depicted in [8].

## 5. CONCLUSIONS

This paper presented experimental studies in which steel frames and columns were loaded both dynamically and quasi-statically, and demonstrated how the loading rate affected the hysteresis of those structures. This paper also described a procedure to estimate the hysteresis for dynamic loading based upon the corresponding hysteresis obtained under quasi-static loading.

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