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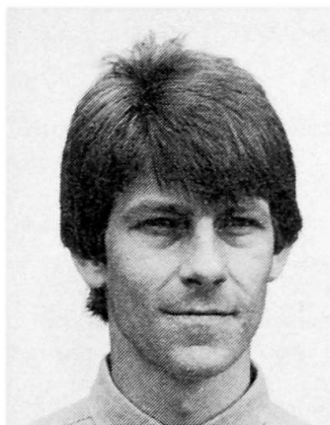
## Structural Information via Modal Testing

Information structurale par des essais modaux

Tragwerksinformation durch Modalversuche

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### SUMMARY

Modal testing of structures is a powerful experimental technique for dynamic problems. It is based on Fourier transformed time records and curve-fitting of experimentally obtained frequency response functions. This paper deals with different possibilities to use this technique as a tool for non-destructive testing. The tests aim at determining static structural properties including buckling loads. A pilot experimental study of columns is summarized as well.

### RÉSUMÉ

L'essai modal de structures est une technique expérimentale efficace pour des problèmes dynamiques. La méthode utilise une transformation de Fourier d'enregistrements des fonctions de temps pour créer des fonctions de réponse de la fréquence. Cet article traite des possibilités d'utilisation de cette technique comme un outil pour des essais non destructifs. Le but des essais est la détermination d'attributs structuraux statiques et les charges critiques. Une étude expérimentale préliminaire de colonnes est aussi présentée.

### ZUSAMMENFASSUNG

Die modale Versuchstechnik für Tragwerke ist ein wirkungsvolles experimentelles Verfahren für dynamische Probleme. Dieses Verfahren geht von fouriertransformierten Zeitaufnahmen und Kurvenanpassungen von versuchstechnisch ermittelten Frequenzantwortfunktionen aus. Dieser Artikel behandelt verschiedene Möglichkeiten der Verwendung dieses Verfahrens als ein Werkzeug für zerstörungsfreie Prüfung. Das Ziel der Versuche ist es, statische Tragwerkeigenschaften einschliesslich der Knicklasten zu bestimmen. Eine experimentelle Vorstudie anhand von Stützen wird kurz zusammengefasst.



## 1. INTRODUCTION

Fourier transform based experimental modal analysis has shown an inherent capability to enable reliable and accurate estimations of modal parameters. Repeated experiments have yielded results with exceptionally small scatter. This is especially the case for eigenfrequency estimates, where almost identical results can be obtained from tests with different kinds of dynamic loading, different location of measurement points and so on. The experience is gained within various research projects related to bridges [6], floors [5] and other civil engineering structures. Modal testing based on impact loading is rather easily performed, at least compared to static tests, which typically need rather heavy equipment and arrangements.

Numerous relevant questions concerning structural properties and loading conditions are very difficult and costly to answer by means of traditional static testing. This is especially true for civil engineering structures in use, but it is also valid for a variety of questions dealing with properties of structural details.

One important class of problems includes the question: "What static stiffness will a specific structural member or joint show under a certain kind of load?" Rotational rigidity of column footings (bolted steel or wooden members) and deflection stiffness of beam members of unknown concrete quality or with unknown amount of cracks are common examples of desired structural information. The unknown degree of semi-rigidity inherent in many connections, which are assumed to act as simple supports, defines another class of problems.

A third type of problems is related to axially loaded slender structures and members, where instability is a major concern. The dynamic characteristics of such members are dependent on the compression force present, cf. [8] and [9]. This fact can be used in the following situations:

- a) The structural properties are known and modal testing can be used to estimate the (unknown) existing axial load.
- b) Provided that the present load and the structural configuration are known, some estimates of the elastic critical buckling load can be made.

The idea to use the modal testing concept to estimate static structural properties has grown out of the above-mentioned observations about the accuracy inherent in the technique. A research project is under development at Chalmers University and the main aim of this paper is to introduce the basic ideas.

## 2. MODAL TESTING

Modal testing is a concept, which here refers to a technique for dynamic structural testing aiming at estimates of one or more of the modal parameters: eigenfrequency  $f_n$ , modal relative damping  $(c/c_{cr})_n$ , mode shape function  $\phi_n(x,y)$  and modal mass  $m_n$ . Such estimates are usually established for a limited number of eigenmodes. Modal testing typically includes the following phases:

- Choice of representative measurement points and directions.
- Dynamic broadband excitation by means of impact or random force applied at one point of the structure.
- Measurement and parallel recording of force and response time histories. The response is typically measured as acceleration.
- Fourier transformation of force and acceleration records respectively yielding complex spectral representations.

- Dynamic acceleration flexibility functions - *accelerances* - are established by division of spectra. The accelerance (point or transfer) is a complex function of frequency.
- The process described above is repeated for other combinations of loading point and point of response, resulting in a set of accelerances.
- Using theoretical accelerance functions based on the theory of modal analysis of linear systems, a curve-fit procedure is carried out. When minimizing least squares of errors, estimations of the above-mentioned four modal parameters can be made. An example of mode shape estimates is shown in fig. 1. Flexibility type functions are illustrated by fig. 2. Reference [1] is a rather complete presentation of experimental modal analysis.

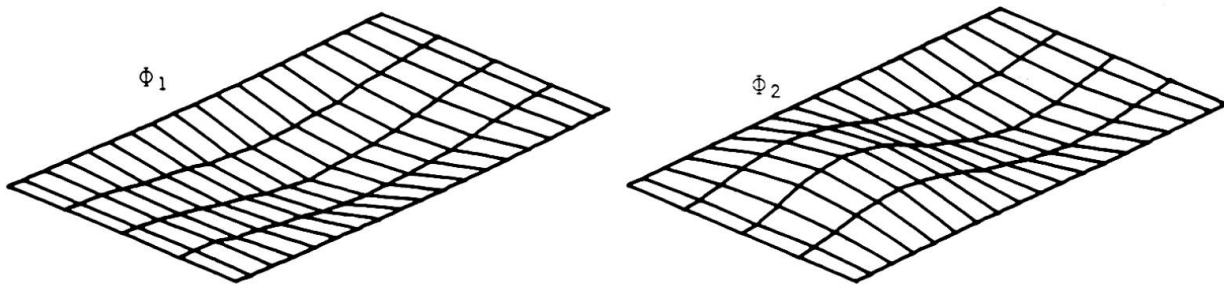


Fig. 1 Measured mode shape functions  $\Phi_n$  for a corrugated web of a steel plate girder. Result from curve-fitting of 94 transfer accelerances and one point accelerance.

### 3. ESTIMATING AXIAL LOAD AND BUCKLING LOAD

#### 3.1 Basic theory

Consider a simply supported beam with constant cross section. The eigenfrequencies  $f_n$  for corresponding bending modes  $\Phi_n(x)$  are calculated from Eq.(1):

$$f_n = (n^2\pi/2) \sqrt{EI/mL^4} \quad (\text{Hz}) \text{ or } (\text{s}^{-1}) \quad (1)$$

where  $n$  is the mode number,  $EI$  is bending rigidity,  $m$  is mass per unit length and  $L$  is span length. If a constant static axial force  $N$  is added, the corresponding eigenfrequencies  $f_n(N)$  of the resulting beam-column can be written:

$$f_n(N) = f_n(N=0) \sqrt{1 - \frac{1}{n^2} \frac{N}{N_{e1}}} \quad (\text{s}^{-1}) \quad (2)$$

where  $f_n(N=0)$  is the eigenfrequency of the unloaded beam as calculated from Eq.(1) and  $N_{e1}$  is the elastic buckling load:

$$N_{e1} = \pi^2 EI/L^2 \quad (\text{N}) \quad (3)$$

Equation (2) can serve as a basis in a non-destructive test approach. Some of the eigenfrequencies  $f_n(N)$  can be measured for a given, but unknown, axial force  $N$ . As apparent from Eq.(2), the influence from axial force on the eigenfrequencies is stronger for lower modes of vibration. The relation between two successive eigenfrequencies can be derived from Eq.(2) as:

$$\left( \frac{f_{n+1}(N)}{f_n(N)} \right)^2 = \frac{(n+1)^2}{n^2} \frac{(n+1)^2 - (N/N_{e1})}{n^2 - (N/N_{e1})} \quad (4)$$



This expression can be used to *estimate the present axial load* as a fraction of the Euler buckling load  $N/N_{e1}$  based on measurements of two successive eigenfrequencies, preferably  $f_1$  and  $f_2$ .

Another way to use Eq.(2) is by repeated measurements of one eigenfrequency  $f_n$ . The first measurement gives a value  $f_n(N=N_0)$  for the existing unknown axial force  $N_0$ . A well-known additional force  $\delta N$  is then applied. The second measurement gives a value for  $f_n(N_0+\delta N)$ . If both sides of Eq.(2) are squared, the resulting relation is a straight line in a diagram like the one in fig. 3. The slope of this line is defined by:

$$df_n^2/dN = (f_n^2(N_0+\delta N) - f_n^2(N_0))/\delta N \quad (5)$$

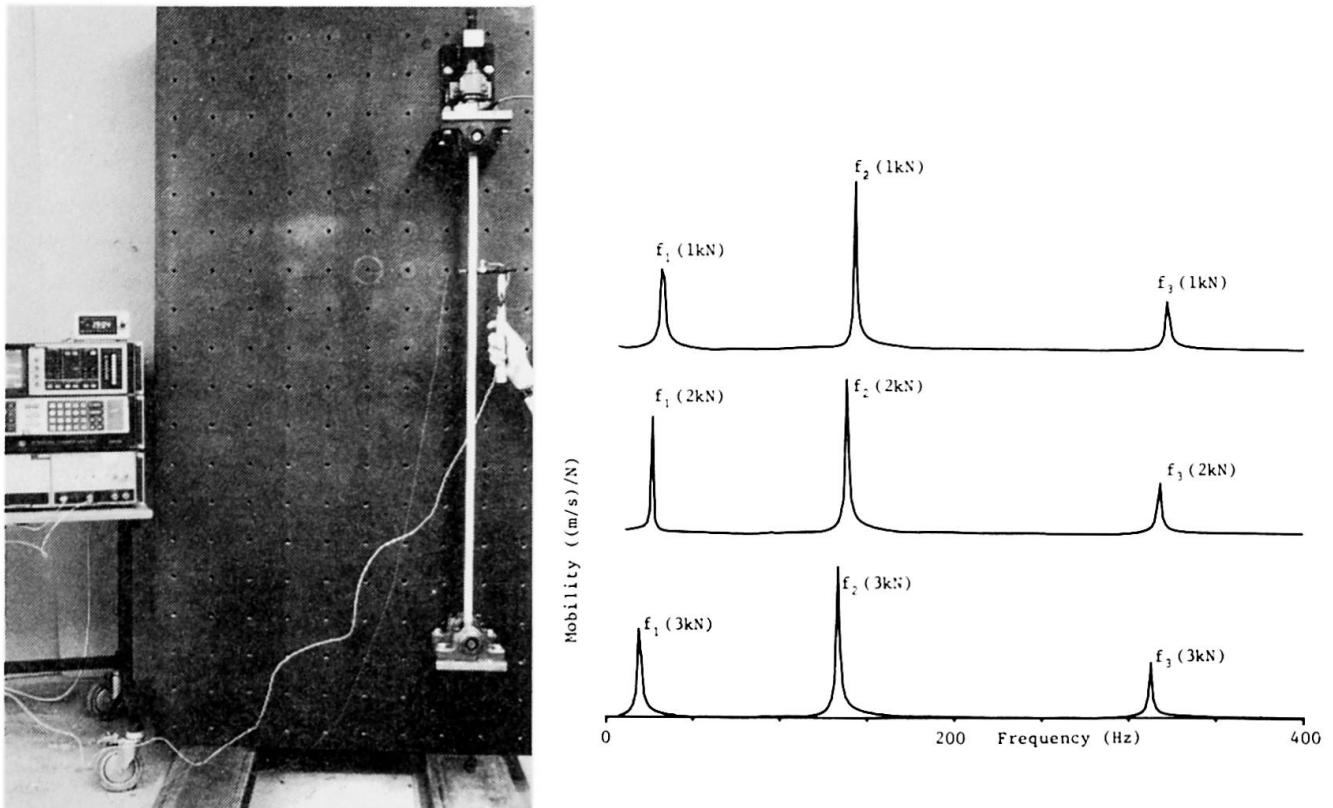
The knowledge of this slope value enables us to estimate the axial load  $N_R$ , which is the difference between the present load  $N_0$  and the buckling load  $N_{e1}$ :

$$N_R = -f_n^2(N_0) \cdot dN/df_n^2 \quad (N) \quad (6)$$

The additional force  $\delta N$  could in a real situation be applied by means of an added mass on top of a column. The fundamental mode eigenfrequency is the most sensitive to a given small additional force.

### 3.2 Laboratory tests

This section is a presentation of a pilot study. The aim is to study how accurately the load estimates discussed in the previous section can be made. The test specimen is an aluminium column with rectangular hollow cross section, which is simply supported with span length of 1.20 m. One of the supports is equipped for application and measurement of axial static force, cf. fig.2. Accelerances are established by means of hammer impact modal testing for 13 different levels of



**Fig. 2** Test set-up (left) and measured point mobilities related to three different levels of axial force  $N = 1, 2$  and  $3$  kN respectively (right).

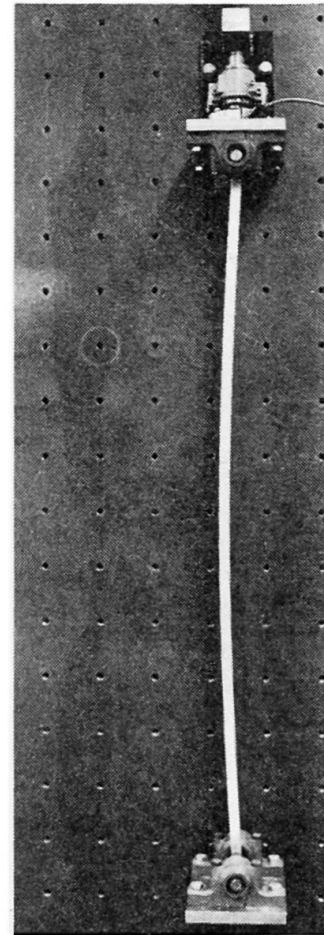
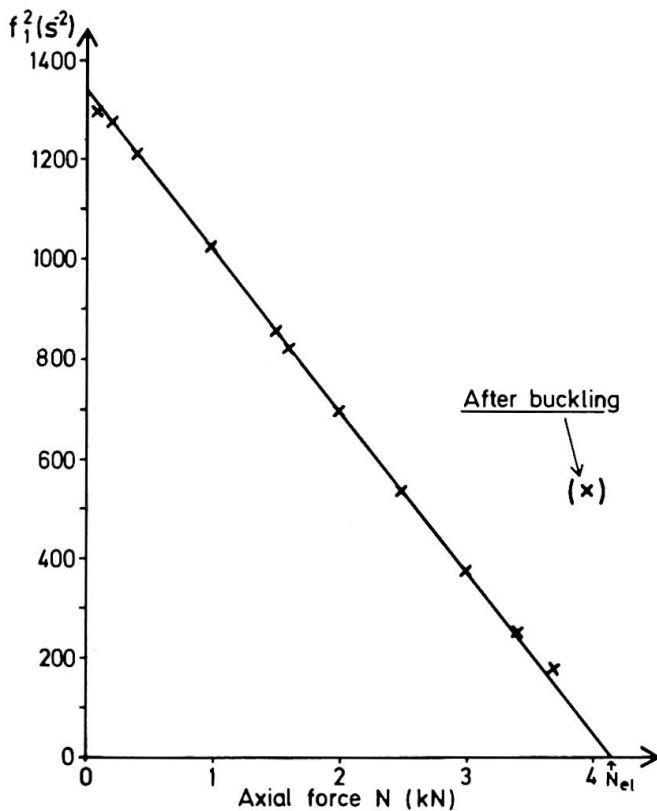
axial force  $N$ . The accelerances are transformed to mobilities (vibration *velocity*/force) and some are shown in fig.2. The three lowest eigenfrequencies are determined via curve-fitting for each load level. The fundamental frequencies are plotted in fig.3. The straight line approximation results in an Euler buckling force  $N_{e1} = 4.16$  kN. The two load levels  $N_0 = 1.50$  and  $N_0 + \delta N = 1.60$  kN can be used to test the approach related to Eq.(5) and (6). Eq.(5) gives:

$$df_1^2/dN = (28.76^2 - 29.34^2)/100 = -0.3370 \quad (7)$$

and Eq.(6) gives:

$$N_R = 29.34^2/0.3370 = 2554 \text{ N} \quad (8)$$

This estimate could then be compared with the difference  $N_{e1} - N_0 = 4160 - 1500 = 2660$  N. The estimate is then appr. 4% lower than this value, which must be considered as promising.



**Fig.3** Plot of experimentally determined fundamental mode eigenfrequency (squared) versus axial force  $N$  (left) and illustration of buckled specimen (right). Experimental buckling load is approximately 4.0 kN.

#### 4. CONCLUDING REMARKS

The approach to column force estimation problems illustrated in chapter 3 only included one of the four modal parameters - the eigenfrequency. If mode shapes and modal masses are taken into account as well, more complex structures may be handled and with still better accuracy. This holds also for the other kinds of problems, where bending rigidity or the degree of end rotational rigidity are sought. The references illustrate some practical problems that may be handled with non-destructive modal testing. One important possibility is to use





repeated tests over time, aiming at detection of structural changes due to ageing, fatigue or material deterioration, cf. Ref. [3] and [10].

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