

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 13 (1988)

**Artikel:** Dynamic characteristics of a liquid filled egg-shaped tank

**Autor:** Uno, Kiyoshi / Takanishi, Teruhiko / Naritomi, Masaru

**DOI:** <https://doi.org/10.5169/seals-13065>

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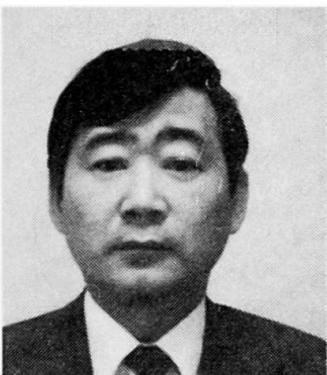
## Dynamic Characteristics of a Liquid Filled Egg-Shaped Tank

Caractéristiques dynamiques d'un réservoir ovoïde plein

Dynamische Einwirkungen einer Flüssigkeit in einem eiförmigen Wassertank

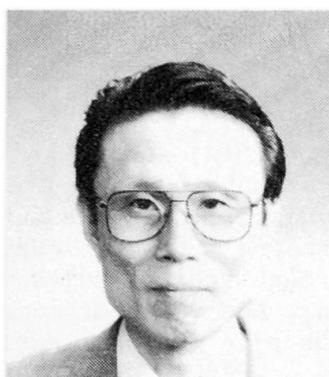
**Kiyoshi UNO**

Assoc. Prof.  
Kyushu Univ.  
Fukuoka, Japan



**Teruhiko TAKANISHI**

Prof. Dr.  
Kyushu Inst. of Tech.  
Kitakyushu, Japan



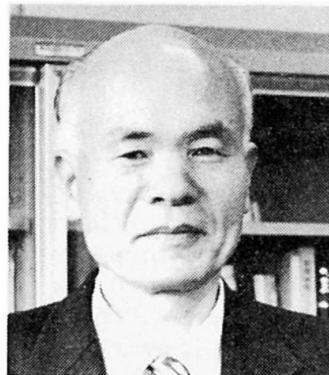
**Masaru NARITOMI**

Dr. Eng.  
Kyushu Univ.  
Fukuoka, Japan



**Seima KOTSUBO**

Prof. Dr.  
Kyushu Kyoritsu Univ.  
Kitakyushu, Japan



### SUMMARY

When an Egg-shaped tank is constructed in Japan, pile-group foundation is used in order to resist the effect of earthquakes. It is difficult to analyse the response of tank, liquid and foundation reaction during earthquakes. We proposed approximate equations for the calculation of dynamic liquid pressures in the Egg-shaped tank during earthquakes.

### RÉSUMÉ

La fondation par groupe de pieux est utilisée pour des réservoirs ovoïdes au Japon, afin de résister aux effets des tremblements de terre. Il est difficile d'analyser le comportement du réservoir, du liquide et de la fondation pendant les tremblements de terre. Les auteurs proposent des équations approximatives pour le calcul des pressions dynamiques du liquide dans le réservoir ovoïde lors de séismes.

### ZUSAMMENFASSUNG

Beim Bau von eiförmigen Wassertanks wird in Japan zur Aufnahme der Erdbebeneinwirkungen auf Pfahlgrundrisse zurückgegriffen. Die Berechnung des dynamischen Verhaltens von Tank, Flüssigkeit und Pfahlgruppe ist relativ schwierig. Es werden Näherungsgleichungen zur Bestimmung der dynamischen Flüssigkeitsdrücke im eiförmigen Behälter vorgestellt.



## 1. INTRODUCTION

The Egg-shaped tank shown in Fig.1 is developed in West Germany and it is more effective on sewage purification than that of a cylindrical tank. Recently many tanks of this type have been constructed in West Germany. When the Egg-shaped tank is constructed in Japan, pile-group foundation is used in order to resist the effect of earthquakes, which in turn makes it difficult to analyse the response of the tank, liquid and foundation system subject to earthquakes.

The purpose of this study is to propose the approximate equations for evaluating the dynamic liquid pressures of the Egg-shaped tank during earthquakes. Through the results of this calculation, the value of moment at the top of the pile can be easily obtained.

## 2. METHOD OF ANALYSIS

Dimensions of three Egg-shaped tanks constructed in Japan are shown in Table 1. The ratio  $R_\theta/H_\theta$  falls in the narrow range between 0.301 and 0.315 for all three cases so that the shapes of these tanks are considered similar to each other. Since the values  $R_\theta^3$  and  $(H_\theta/2)^3$  are nearly equal to the ratios of volume( $V$ ) of each Egg-shaped tank, it is known that the standard of the shape can be set up from the ratio  $R_\theta/H_\theta$ .  $H_p$  is the height from the bottom of the Egg-shaped tank to the top of the pile, as is indicated in Fig.1. Dynamic liquid pressures can be separated into impulsive part, which depends on short period component, and sloshing part which depends on long period component. Since it is difficult to analyse these two types of dynamic liquid pressures exactly, the transfer matrix proposed by Sogabe is used for calculating the dynamic liquid pressures.

## 3. RESULTS AND PROPOSED EQUATION

### 3.1 Impulsive Liquid Pressures ( $P_i$ )

Since the value of impulsive liquid pressures increases in proportion to the

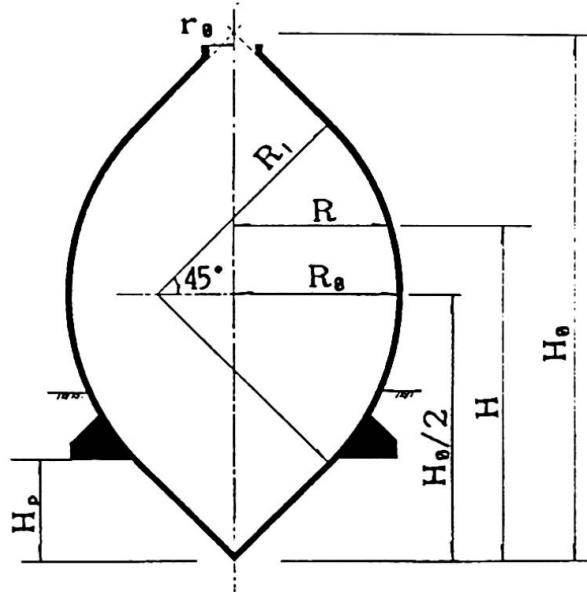


Fig. 1 Egg-shaped tank

Table 1 Dimensions of the Egg-shaped tank

	CASE 1	CASE 2	CASE 3
$R_1$ (m)	10.600	15.550	13.065
$R_\theta$ (m)	6.800	10.900	9.180
$r_\theta$ (m)	1.000	2.500	2.495
$H_p$ (m)	4.000	5.000	4.474
$H_\theta/2$ (m)	11.200	17.340	14.541
$R_\theta/H_\theta$ (m)	0.304	0.314	0.315
$V$ ( $m^3$ )	1748.0	6863.0	4000.0

acceleration and the height of liquid surface( $H$ ), the proposed approximate equations for  $\alpha_i$  and  $H_i/H_0$  are written in the following form

$$\begin{aligned}\alpha_i &= 0.27 (H/H_0) + 0.73 \quad 0.9 \leq H/H_0 \\ &= 1.08 (H/H_0) \quad 0.5 \leq H/H_0 < 0.9\end{aligned} \dots \dots \dots (1)$$

$$\begin{aligned}H_i/H_0 &= 0.25 (H/H_0) + 0.26 \quad 0.8 \leq H/H_0 \\ &= 0.50 (H/H_0) + 0.06 \quad 0.5 \leq H/H_0 < 0.9\end{aligned} \dots \dots \dots (2)$$

where  $\alpha_i$  is the coefficient of added mass and  $H_i$  is the height of the loading point of impulsive liquid pressures. Figure 2 shows  $\alpha_i - H/H_0$  and  $H_i/H_0 - H/H_0$  relationships. Using these values, impulsive liquid pressures( $P_i$ ) and the moment( $M_i$ ) at the top of the pile can be given as follows

$$P_i = \rho \alpha_i V_H \omega_i^2 A_i \dots \dots \dots (3)$$

$$M_i = P_i (H_i - H_p) \dots \dots \dots (4)$$

where  $\rho$  is the density of liquid,  $V_H$  is volume of liquid,  $\omega_i$  is the natural circular frequency and  $A_i$  is amplitude of displacement.

### 3.2 Sloshing Liquid Pressures (Ps)

Since sloshing liquid pressures are under the influence of the first vibration mode, the proposed approximate equations are obtained for the first vibrational mode. The proposed approximate equations for  $\alpha_s$ ,  $H_s$ ,  $T_s$ ,  $\beta_s$ , are written in the following form

$$\alpha_s = 0.37 - 0.40 (H/H_0)^3 \quad 0.5 \leq H/H_0 \dots \dots \dots \dots \dots (5)$$

$$H_s/H_0 = 0.8 (H/H_0) \dots \dots \dots \dots \dots (6)$$

$$T_s/R_0^{0.5} = 1.10 (R/R_0) + 0.40 \dots \dots \dots \dots \dots \dots \dots \dots (7)$$

$$\beta_s = 0.27 / \{1.0 - (H/H_0) + 0.67\} \dots \dots \dots \dots \dots \dots \dots \dots (8)$$

where  $\alpha_s$  is the coefficient of added mass,  $H_s$  is the height of the loading point of sloshing liquid pressures,  $T_s$  is the natural period and  $\beta_s$  is the participation factor.

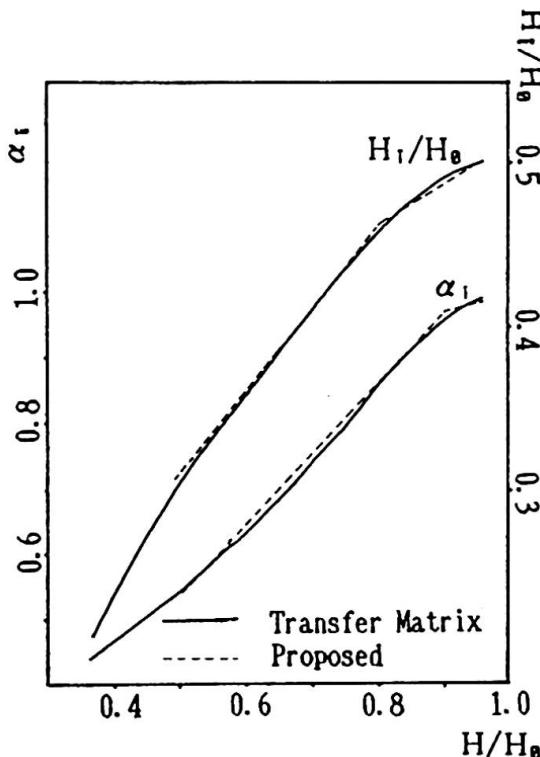


Fig. 2  $\alpha_i - H/H_0$ ,  $H_i/H_0 - H/H_0$  relationships



Figure 3 shows  $\alpha_s - H/H_0$  and  $H_s/H_0 - H/H_0$  relationships.  $T_s/R_0^{0.5} - H/H_0$  relationship and  $\beta_s - H/H_0$  relationship are presented in Fig.4 and Fig.5, respectively.

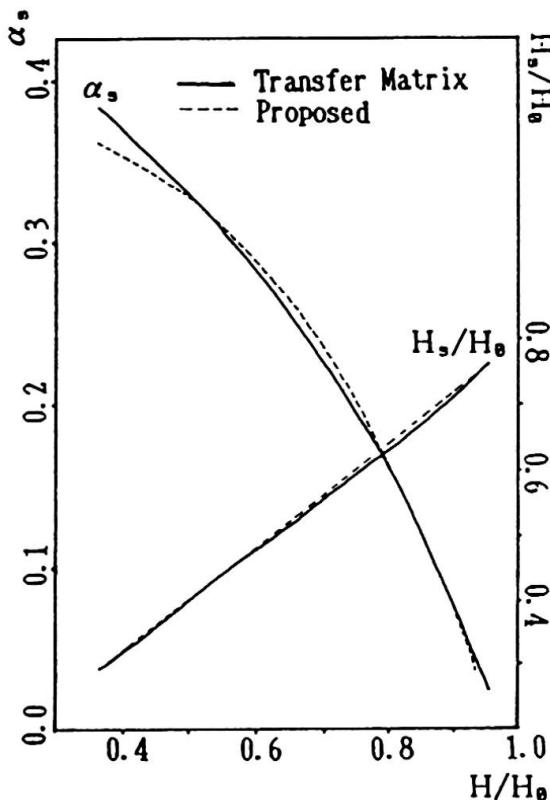


Fig.3  $\alpha_s - H/H_0$ ,  $H_s/H_0 - H/H_0$  relationships

Using these values, sloshing liquid pressures( $P_s$ ), the moment at the top of the pile( $M_s$ ) and the maximum vertical displacement of free-surface( $W_{max}$ ) during earthquakes can be given as follows

$$P_s = \rho \alpha_s V_H \omega_s^2 A_s \quad \dots \dots (9)$$

$$M_s = P_s (H_s - H_p) \quad \dots \dots (10)$$

$$W_{max} = \beta_s S_a / \omega_s^2 = \beta_s A_s \dots (11)$$

where  $\omega_s$  is the natural circular frequency,  $A_s$  is the amplitude of earthquake displacement for  $\omega_s$  and  $S_a$  is the amplitude of acceleration for  $\omega_s$ . Figure 6 shows  $M_i - H/H_0$  and  $M_s - H/H_0$  relationships when the amplitude of input earthquake acceleration is 100  $\text{cm/s}^2$ .

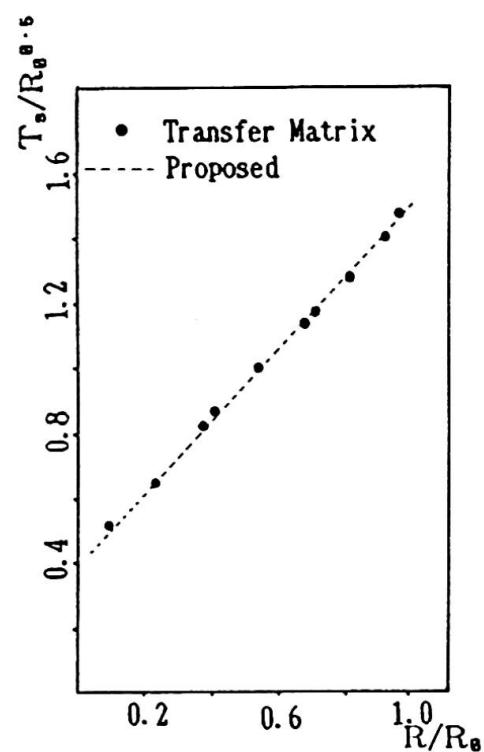


Fig.4  $T_s/R_0 - R/R_0^{0.5}$  relationships

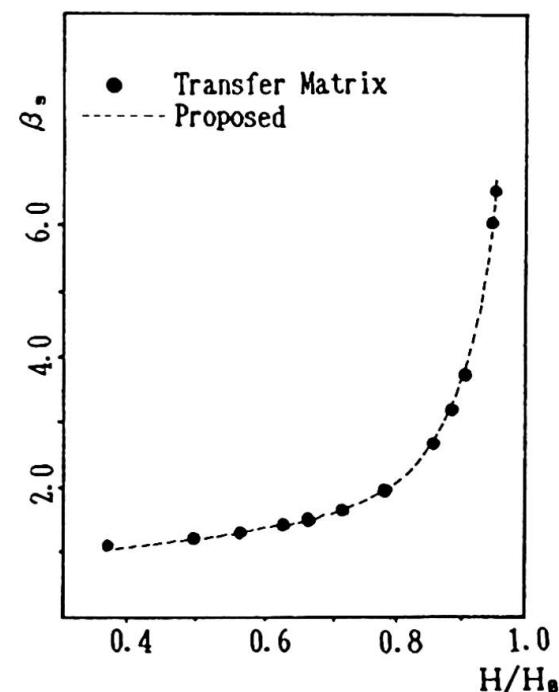


Fig.5  $\beta_s - H/H_0$  relationships

#### 4. CONCLUSIONS

The results are useful for aseismic design of the pile-group foundation. When comparing our proposed results with those of the transfer matrix, the accuracy is quite satisfactory, and the dynamic liquids' characteristics of the Egg-shaped tank can be easily given.

It is essential for aseismic design of the Egg-shaped tank to evaluate the moment at the top of the pile-group foundation. The proposed equations are readily used to estimate this moment.

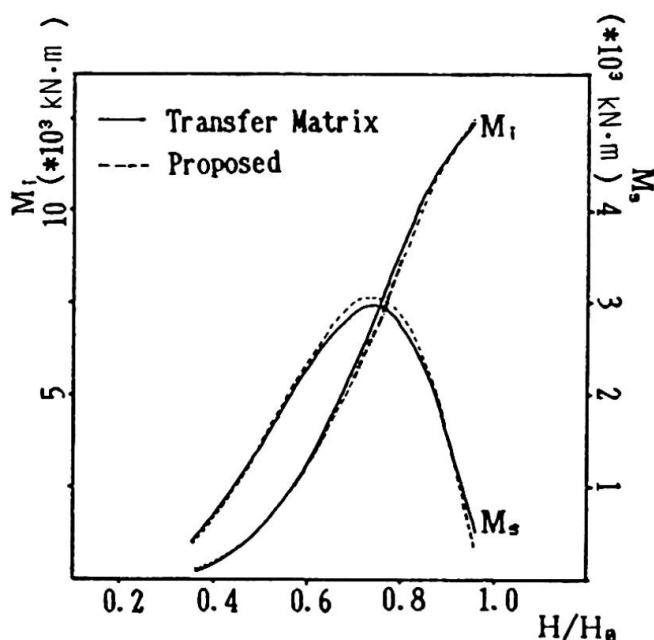


Fig. 6  $M_1 - H/H_0, M_3 - H/H_0$   
relationships

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