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Analytical Model for Vibration of Y-Shaped Structure

Modèle analytique de la vibration de structures en forme de Y

Analytisches Vibrationsmodell für Y-förmige Bauwerke

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SUMMARY
Y-shaped buildings are characteristically non-orthogonal in the directions of structural elements and asymmetric in rigidity. For the Y-shaped building with a semirigid floor system, the longitudinal deformations of its wings, because of their lengths in plan and due to the much more rigid central "core", can be reasonably neglected. The "Spatial Multi-Mass-Point System" is used as the mechanical model to describe the various structural responses under ground motion, i.e. parallel translation, twisting and the deformations of the wings perpendicular to their longitudinal axes.

RÉSUMÉ
Les bâtiments en forme de Y sont, de façon caractéristique, non-orthogonaux dans la direction des éléments structuraux et asymétriques dans leur rigidité. Pour les bâtiments en forme de Y avec un système de plancher semi-rigide, les déformations longitudinales des ailes peuvent être raisonablement négligées en raison de leur longueur en plan et de la trop grande rigidité du noyau. Un modèle mécanique permet de décrire divers comportements structuraux sous l'effet de mouvements au sol, par exemple translation parallèle, torsion et déformation des ailes perpendiculairement aux axes longitudinaux.

ZUSAMMENFASSUNG
Im Grundriss Y-förmige Bauwerke weisen schiefwinklige Anschlüsse der Tragelemente und eine asymmetrische Steifigkeit auf. Bei der Verwendung von halbsteifen Deckensystemen können die Längsverformungen der Y-Flügel verglichen mit den Kernverschiebungen vernachlässigt werden. Ein räumliches Modell mit mehreren Masspunkten wird verwendet, um das Verhalten bei Bodenbewegung, d.h. die Verschiebungen, Verdrehungen und die Verbiegungen der Y-Flügel zu beschreiben.

1. INTRODUCTION

Few actual buildings are entirely symmetric with respect to mass and stiffness distribution about one or more axes of the buildings. This geometric and physical asymmetry is greatly enhanced with irregular shaped high-risers which have come into architectural vogue during the recent decades. Furthere more, floor systems which used to be taken as absolutely rigid diaphragms are now known in some cases that their rigidities are only comparable in magnitude (although often greater than) those of the vertical structural elements, i.e. floors are to some degree deformationable. Damage surveys of major earthquakes clearly reveal the unfavorable effects of asymmetry and floor deformation on some of the damaged buildings. It is therefore incumbent upon the structural engineer to pay serious attention to these facts in his seismic analysis and design of an important building, especially when it is irregular shaped in plan.

2. MATHEMATICAL MODEL

Before discussing the mathematical aspects of the structure's model, it is perhaps necessary to mention briefly the important physical property of the floor system. In recent years inverstigations have been carried out on this subject, some by observing the ambient vibrations of buildings, some by recording the deformed building subject to applied lateral loading. The results may be summarized as: (1) The magnitude of floor stiffness depends on the lenth-width ratio and the material and construction of flooring. (2) The shape of floor deformation is basically of the shear type. (3) The actual values of floor rigidity are difficult to determine and few reliably accurate figures have been given. For the common type of floors of precast concrete, the "basic" shear stiffness of 10^5 kN is often accepted.

Complex-shaped tall buildings are characteristically non-orthogonal in directions of structural elements, asymmetric in rigidity and the lack of coincidence between the dominant direction of ground motion and one of the principal axes of symmetry, thus resulting in strong torsional vibrations during an earthquake. If the building is long and narrow in plan, or the flooring is comparatively flexible, its horizontal deformation is appreciable. An adequate analysis should take all the above facts in consideration.

The establishment of the mathematical model is best illustrated by taking an example. The Y-shaped high-riser is quite popular today. If the wings are short compared to their width, and if the concrete floors are cast-in-situ, the structure may be regarded as a "multiple rigid disk system" for analysis purpose. If , however, the wings are long and narrow, or if the flooring is less rigid, for example, like the precast slab type, or, if the lateral stiffnesses of the vertical structure elements of the wings are much less than that of the central core, then the influence of floor deformation should not be neglected.

For the Y-shaped tall building as shown in Fig.1, the longitudinal deformations of its wings, because of their lengths in plan and due to the much more rigid central "core" can be reasonnally neglected. The "Spatial Multi-mass-point System" (Fig.2) is used as the mathematical model to describle the structural various responses under ground motion, i.e. parallel translation, twisting and the deformations of the wings perpendicular to their longitudinal axes.

2. EQUATIONS OF MOTION

We can assume the dominant components of a very complex earthquake ground motion are the two-directional translations plus rotation. The equations of motion of the system under an earthquake may be written in the familiar form:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{\ddot{U}_g\} \qquad (1)$$

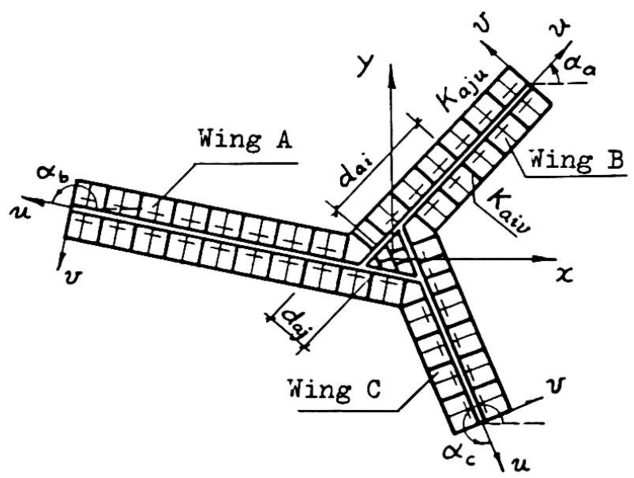

Fig.1 Typical floor plan

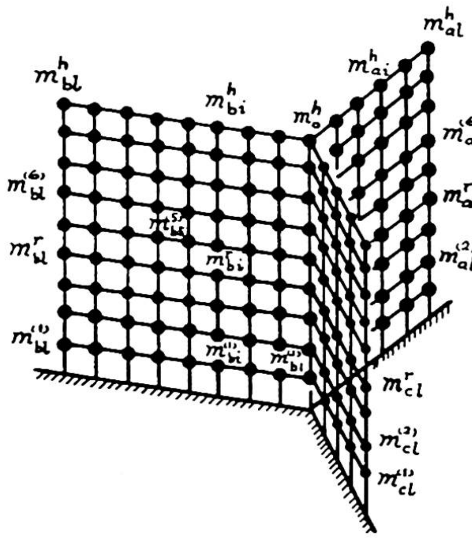

Fig.2 Mechanical model

$$\{U\} = \left[\{x_o\}^T \quad \{y_o\}^T \quad \{\varphi\}^T \quad \{v\}^T\right]^T ,$$
$$\{\ddot{U}g\} = [I][\ddot{x}_g \quad \ddot{y}_g \quad \varphi_g \quad 0\ ]^T ,$$
$$[I] = \mathrm{diag}\left[\{1\}_h \quad \{1\}_h \quad \{1\}_h \quad \{1\}_{(l+\sum_s lh)}\right] ,$$

where $\{U\}$, $\{\dot{U}\}$, $\{\ddot{U}\}$ are respectively the column vectors of the generalized relative displacements, velocities and accelerations. $\{\ddot{U}g\}$ is column vector of the generalized ground acceleration. $\{x_o\}$, $\{y_o\}$ and $\{\varphi\}$ give respectively the x, y, φ directional translations in the global co-ordinates. $\{v\}$ gives the displacements due to floor deformation (Fig.3).

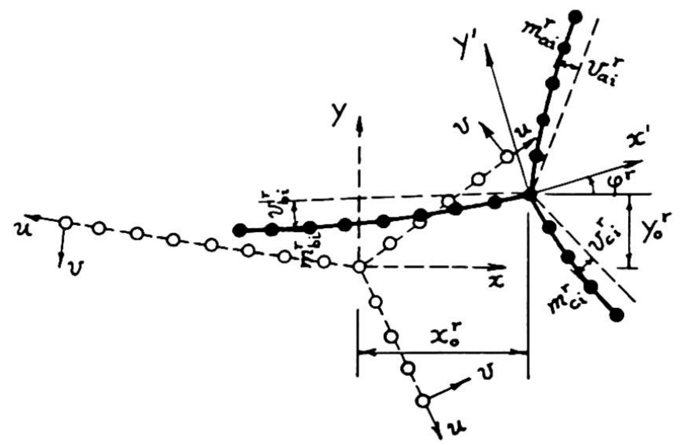

Fig.3 Displacements of the rth floor's mass-points

$[M]$ is the generalized mass matrix, in which $[m]$, $[J]$ are respectively submatrices of mass and mass moment of inertia. $[m_{vx}]$, $[m_{vy}]$ and $[m_{v\varphi}]$ are the coupled mass submatrices between the translations and rotation.

$[K]$ and $[K_{UV}]$ are respectively the stiffness matrices in the global and local co-ordinates, in which $[T]$ is the transformation matrix. $[K_U]$, $[K_V]$ and $[K_\varphi]$ are the stiffness matrices refferred to the planar motion of the rigid disk system. $[K_{vv}]$ is the stiffness matrix of the flexible floor system. $[K_v]$ is the stiffness matrix of the vertical structural elements, $[k_U]$ is the stiffness matrix coupling the vertical sub-structures due to stiffness of diaphragms. $[K_{VV}]$ and $[K_{V\varphi}]$ are the stiffness matrices coupling the displacements due to floor deformation and those due to solid-body motion of the floors. $[d_v]$ and $[d_u]$ are the matrices giving the distances between the story mass centers of the wings to the story mass center of the building. $k_{si}^{r,r}$ is the equivalent horizontal shearing stiffness of the floor of the ith bay.

$$\{x_o\} = \left[x_o^{(1)} \quad x_o^{(2)} \cdots x_o^{t} \cdots x_o^{h} \right]^T , \quad \{y_o\} = \left[y_o^{(1)} \quad y_o^{(2)} \cdots y_o^{t} \cdots y_o^{h} \right]^T ,$$

$$\{\varphi\} = \left[\varphi^{(1)} \quad \varphi^{(2)} \cdots \varphi^{t} \cdots \varphi^{h} \right]^T , \quad \{v\} = \left[\{0\}^T \quad \{v_a\}^T \quad \{v_b\}^T \quad \{v_c\}^T \right]^T ,$$

$$\{v_s\} = \left[\{v_{s1}\}^T \quad \{v_{s2}\}^T \cdots \{v_{si}\}^T \cdots \{v_{sl}\}^T \right]^T , \quad (s = a,b,c)$$

$$\{v_{si}\} = \left[v_{si}^{(1)} \quad v_{si}^{(2)} \cdots v_{si}^{t} \cdots v_{si}^{h} \right] , \quad (i = 1,2,\cdots,l)$$

$$M = \begin{bmatrix} [m] & [0] & [0] & [m_{xv}] \\ [0] & [m] & [0] & [m_{yv}] \\ [0] & [0] & [J] & [m_{\varphi v}] \\ [m_{vx}] & [m_{vy}] & [m_{v\varphi}] & [m_v] \end{bmatrix} , \quad \begin{array}{l} [m] = \mathrm{diag}\left[m^{(1)} \quad m^{(2)} \cdots m^{r} \cdots m^{h} \right] , \\ [J] = \mathrm{diag}\left[J^{(1)} \quad J^{(2)} \cdots J^{r} \cdots J^{h} \right] , \end{array}$$

$$[m_v] = \mathrm{diag}\left[[0] \quad [m_a] \quad [m_b] \quad [m_c] \right] ,$$

$$[m_s] = \mathrm{diag}\left[[m_{s1}] \quad [m_{s2}] \cdots [m_{si}] \cdots [m_{sl}] \right] ,$$

$$[m_{si}] = \mathrm{diag}\left[m_{si}^{(1)} \quad m_{si}^{(2)} \cdots m_{si}^{t} \cdots m_{si}^{h} \right] , \quad (s = a,b,c)$$

$$[m_{vx}] = -[m_v][\bar{I}]\{S\} , \quad [m_{vy}] = [m_v][\bar{I}]\{C\} , \quad [m_{yv}] = [m_{vy}]^T ,$$

$$[m_{xv}] = [m_{vx}]^T , \quad [m_{v\varphi}] = [m_v][d_i][\bar{I}]\{1\} , \quad [m_{\varphi v}] = [m_{v\varphi}]^T ,$$

$$\{S\} = \left[\sin\alpha_a \quad \sin\alpha_a \quad \sin\alpha_b \quad \sin\alpha_c \right] ,$$

$$\{C\} = \left[\cos\alpha_a \quad \cos\alpha_a \quad \cos\alpha_b \quad \cos\alpha_c \right] ,$$

$$[\bar{I}] = \mathrm{diag}\left[\{1\}_l \quad \{1\}_l \quad \{1\}_l \quad \{1\}_l \right] , \quad \{1\} = \left[1 \quad 1 \quad 1 \quad 1 \right] ,$$

$$[d_i] = \mathrm{diag}\left[[0] \quad [d_a] \quad [d_b] \quad [d_c] \right] ,$$

$$[d_s] = \mathrm{diag}\left[[d_{s1}] \quad [d_{s2}] \cdots [d_{si}] \cdots [d_{sl}] \right] ,$$

$$[d_{si}] = \mathrm{diag}\left[d_{si}^{(1)} \quad d_{si}^{(2)} \cdots d_{si}^{t} \cdots d_{si}^{h} \right] , \quad (s = a,b,c)$$

$$[K] = \begin{bmatrix} [K_{xx}] & [K_{xy}] & [K_{x\varphi}] & [K_{xv}] \\ [K_{yx}] & [K_{yy}] & [K_{y\varphi}] & [K_{yv}] \\ [K_{\varphi x}] & [K_{\varphi y}] & [K_{\varphi\varphi}] & [K_{\varphi v}] \\ [K_{vx}] & [K_{vy}] & [K_{v\varphi}] & [K_{vv}] \end{bmatrix} = [T]^T[K_{UV}][T] ,$$

$$[K_{UV}] = \begin{bmatrix} [K_U] & [0] & [K_{U\varphi}] & [0] \\ [0] & [K_V] & [K_{V\varphi}] & [K_{Vv}] \\ [K_{\varphi U}] & [K_{\varphi V}] & [K_\varphi] & [K_{\varphi v}] \\ [0] & [K_{vV}] & [K_{v\varphi}] & [K_{vv}] \end{bmatrix} , \quad [T] = \begin{bmatrix} \{C\}_4 & \{S\}_4 & \{0\} & [0] \\ \{S\}_4 & -\{C\}_4 & \{0\} & [0] \\ \{0\} & \{0\} & \{1\}_4 & [0] \\ \{0\} & \{0\} & \{0\} & [I]_{4x4} \end{bmatrix}$$

$$[K_U] = \mathrm{diag}\left[[K_{ou}] \quad [K_{au}] \quad [K_{bu}] \quad [K_{cu}] \right] ,$$

$$[K_V] = \mathrm{diag}\left[[K_{ov}] \quad [K_{av}] \quad [K_{bv}] \quad [K_{cv}] \right] ,$$

$$[K_\varphi] = \left[[d_v] \quad [d_u] \right] \begin{bmatrix} [K_U] & [0] \\ [0] & [K_V] \end{bmatrix} \begin{Bmatrix} [d_v] \\ [d_u] \end{Bmatrix} , \quad \begin{array}{l} [K_{\varphi U}] = -[d_v][K_U] , \\ [K_{\varphi V}] = [d_u][K_V] , \end{array}$$

$$[d_v] = \mathrm{diag}\left[[d_{ov}] \quad [d_{av}] \quad [d_{bv}] \quad [d_{cv}] \right] ,$$

$$[d_u] = \mathrm{diag}\left[[d_{ou}] \quad [d_{au}] \quad [d_{bu}] \quad [d_{cu}] \right] ,$$

$$[d_{sv}] = \mathrm{diag}\left[d_{sv}^{(1)} \quad d_{sv}^{(2)} \cdots d_{sv}^{t} \cdots d_{sv}^{h} \right] ,$$

$$[d_{su}] = \mathrm{diag}[\, d_{su}^{(1)} \quad d_{su}^{(2)} \cdots d_{su}^{t} \cdots d_{su}^{h} \,] \ , \quad (s = 0, a, b, c)$$

$$[K_{\varphi U}] = -[d_v][K_U] \ , \quad [K_{\varphi v}] = [d_u][K_V] \ , \quad [K_{U\varphi}] = [K_{\varphi U}]^T \ ,$$

$$[K_{VV}] = [K_V] + [k_V] \ , \quad [K_V] = \mathrm{diag}[\, [0] \quad [K_{av}] \quad [K_{bv}] \quad [K_{cv}] \,] \ ,$$

$$[K_{sV}] = \mathrm{diag}\, [[K_{s1v}] \quad [K_{s2v}] \cdots [K_{siv}] \cdots [K_{slv}]] \ , \quad (s = a, b, c)$$

$$|K_{siv}| = \begin{bmatrix} K_{siv}^{1,1} & K_{siv}^{1,2} & \cdots & K_{siv}^{1,t} & \cdots & K_{siv}^{1,h} \\ K_{siv}^{2,1} & K_{siv}^{2,2} & \cdots & K_{siv}^{2,t} & \cdots & K_{siv}^{2,h} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ K_{siv}^{r,1} & K_{siv}^{r,2} & \cdots & K_{siv}^{r,t} & \cdots & K_{siv}^{r,h} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ K_{siv}^{h,1} & K_{siv}^{h,2} & \cdots & K_{siv}^{h,t} & \cdots & K_{siv}^{h,h} \end{bmatrix} ,$$

$$[k_v] = \mathrm{diag}\left[\, [0] \quad [k_a] \quad [k_b] \quad [k_c] \,\right] ,$$

$$[k_s] = \begin{bmatrix} [k_{s1}]+[k_{s2}] & -[k_{s2}] & & & & & \\ -[k_{s2}] & [k_{s2}]+[k_{s3}] & -[k_{s3}] & & & 0 & \\ & \cdots & \cdots & \cdots & & & \\ & & -[k_{si}] & [k_{si}]+[k_{s(i+1)}] & -[k_{s(i+1)}] & & \\ & & & \cdots & \cdots & \cdots & \\ & 0 & & & -[k_{s(l-1)}] & [k_{s(l-1)}]+[k_{sl}] & -[k_{sl}] \\ & & & & & -[k_{sl}] & [k_{sl}] \end{bmatrix}$$

$$[k_{si}] = \mathrm{diag}\left[\, k_{si}^{1,1} \quad k_{si}^{2,2} \quad k_{si}^{r,r} \quad k_{si}^{h,h} \,\right] , \quad (s = a, b, c)$$

$$[K_{vV}] = [K_v][\bar{I}] \ , \quad [K_{v\varphi}] = [K_v][d_i][\bar{I}] \ ,$$

$$[K_{Vv}] = [K_{vV}]^T \ , \quad [K_{\varphi v}] = [K_{v\varphi}]^T \ ,$$

3. ANALYSIS AND COMPUTATION

The solution of the dynamic equation

$$[U][A] = [\lambda][A] \qquad (2)$$

where $[U]$ stands for $[K]^{-1}[M]$, $[A]$ the total modal matrix and $[\lambda]$ the eigen value diagonal matrix, is trivial. $[\lambda]$ and $[A]$ give all the natural periods of vibration and modal shapes, while the participation factors of modes can be determined by simply inverting $[A]$, i.e.

$$[\Gamma] = [A]^{-1}[I] \qquad (3)$$

$$\begin{bmatrix} \gamma_{x1} & \gamma_{y1} & \gamma_{\varphi 1} & \gamma_{v1} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{xj} & \gamma_{yj} & \gamma_{\varphi j} & \gamma_{vj} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{xN} & \gamma_{yN} & \gamma_{\varphi N} & \gamma_{vN} \end{bmatrix} = [A]^{-1} \begin{bmatrix} \{1\}_h & & & \\ & \{1\}_h & & 0 \\ & & \{1\}_h & \\ & 0 & & \{1\}_{(1+\sum_s l_s h)} \end{bmatrix}$$

$$[A] = \left[\, \{A_1\} \quad \{A_2\} \cdots \{A_j\} \cdots \{A_N\} \,\right] , \qquad N = 3h + 1 + \sum_s l_s h$$

$$\{A_j\} = [\, \{X_j\}^T \quad \{Y_j\}^T \quad \{\Phi_j\}^T \quad \{V_j\}^T \,]$$

$$= [\, X_o^{(1)} \quad X_o^{(2)} \cdots X_o^h \mid Y_o^{(1)} \quad Y_o^{(2)} \cdots Y_o^h \mid \Phi^{(1)} \quad \Phi^{(2)} \cdots \Phi^h \mid$$

$$0 \quad V_{a1}^{(1)} \quad V_{a1}^{(2)} \cdots V_{a1}^h \cdots V_{ai}^{(1)} \cdots V_{ai}^h \cdots V_{al}^{(1)} \cdots V_{al}^h \quad V_{b1}^{(1)} \cdots$$

$$V_{b1}^h \cdots V_{bl}^{(1)} \cdots V_{bl}^h \cdots V_{c1}^{(1)} \cdots V_{c1}^h \cdots V_{cl}^{(1)} \cdots V_{cl}^h \,]$$

The resultant sideways of the jth mode of the vertical structural elements in the local (or member) co-ordinates are derived from the generalized displacements of the structure by means of co-ordinate transformation. The jth earthquake forces acting on the vertical elements as isolated members are given by multiplying the displacement matrices by the corresponding stiffness matrices. The members' design internal forces are finally obtained by superposing all the modal contributions according to the CQC (complete quadratic combination) method.

4. CONCLUSIONS

When torsion and floor deformation are considered the foundamental period of the vibration of the irregular shaped building is about 5 % greater, the increase of the internal forces of the verticle elements far away from the central core may amount to as much as 20~90 % . For buildings of large foundamental periods , the first 15 modes should be taken in to account. For this reason, the CQC method is more accurate than commonly used SRSS method and the discrepency between the two is known to be approximate 10 % .

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