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Autor: Saisho, Motoo

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Mathematical Model of Structural Steel Members

Modèle mathématique des éléments structuraux en acier

Ein mathematisches Modell für Stahltragteile

Motoo SAISHO
Assoc. Prof.
Kumamoto University
Kumamoto, Japan

Motoo Saisho, born 1944, received his D.Eng. from the University of Tokyo in 1973. His primary interests are plastic design and numerical analysis of steel buildings. His current research interests include aseismic structures.

SUMMARY
To calculate the restoring force characteristics of steel members under strong loads, a mathematical structural model is proposed in which the idea of "the equivalent two-flange section" is introduced to make it simple. Although the model has only three degrees-of-freedom of end-deformation, the inelastic and hysteretic behavior of steel member can be analyzed by expressing the accumulated plastic deformation in the closed-form function of the end-deformations. From numerical analysis it is pointed out that the presented model is useful and suitable to carry out the analysis of the detailed behavior of steel buildings and it does not require a large amount of calculation to execute it.

RÉSUMÉ
Un modèle structural mathématique est proposé pour calculer les caractéristiques de récupération d'éléments structuraux en acier sous des charges considérables. Le concept de "section à deux semelles équivalentes" est introduit pour la simplification du modèle. Dans ce modèle, il n'y a que trois degrés de liberté de déformations des extrémités, mais les deux comportements d'inélasticité et d'hystérisis des barres en acier peuvent être analysés en exprimant le déformation plastique accumulée dans une fonction de forme fermée des déformations des extrémités. Utilisant une analyse numérique, on a trouvé ce modèle simple, utile et approprié pour l'exécution d'analyses du comportement détaillé de constructions métalliques.

ZUSAMMENFASSUNG
Um die Rückstelleigenschaften von Stahlteilen unter starken Belastungen zu berechnen, wird hier ein mathematisches Strukturmodell vorgestellt, in dem zur Vereinfachung des Modells das Konzept des "gleichwertigen zweiflanschigen Schnitts" eingeführt wird. Das Modell hat nur drei Freiheitsgrade von End-Deformation, aber das inelastische und hysteretische Verhalten von Stahlteilen kann durch die Analyse der angesammelten plastischen Deformationen als eine geschlossene Funktion der End-Deformationen analysiert werden. Verwendet wird eine numerische Analyse, um zu zeigen, dass dieses Modell nützlich und ohne komplizierte Rechnungen anwendbar ist.

1. INTRODUCTION

The collapse of building frames composed of many members is generally caused by local failure or the failure of a few members among many elements of the frame under intense loads such as seismic load or strong wind force. From this reason the behavior of all members must be analyzed accurately in the structural analysis of building frames composed of many members. To calculate the detailed behavior of steel members under strong loads, the following conditions, which change every moment according to the hysteretic behavior, must be satisfied strictly in the analysis.

- The incremental stress-strain relation.
- The accumulated plastic deformation.
- The plastic zone over the cross section and along the axis of steel member.

To execute the above-mentioned analysis, FEM (Finite Element Method) is the best and most widely used analysis method. However, FEM requires a large amount of calculation. Numerical errors in the analysis are accumulated as the amount of calculation increases. For this reason we must try in the numerical analysis to decrease the calculation as much as possible.

The plastic hinge method has been proposed as a simple analysis method of steel members to decrease the amount of calculation.[1]-[5] With this method, it is difficult to calculate the effect of the plastic zone, which changes every moment over the cross section and along the axis of each member.

Another possibility has been presented to decrease the amount of calculation of FEM by introducing a transfer matrix to combine a few elements.[6] In this way, the size of the matrix to be solved in the analysis becomes smaller than that of the original FEM. But this method also requires a large amount of calculation because the number of freedom in the analysis is basically the same as the original FEM.

In this paper a mathematical structural model is presented which is simple but useful in analyzing the behavior of steel members relatively accurately and can be easily applied to the structural analysis of tall buildings.[7]

2. MATHEMATICAL MODEL

2.1 Assumptions

1) The structural model of a steel member is considered with respect to the cantilever member which is subjected to horizontal load (Fx), vertical load (Fz) and bending moment (Fr) at the free end as shown in Fig.1.

2) The section of the steel member is replaced by a two-flange section. The area and the moment inertia of the replaced section are equal to those of the original section in the elastic range.

When plastic strain is generated the section is replaced by "the equivalent two-flange section" explained in the next paragraph.

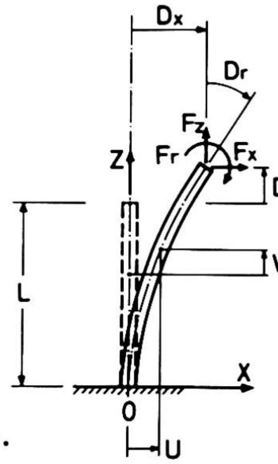

Fig.1
Structural Model

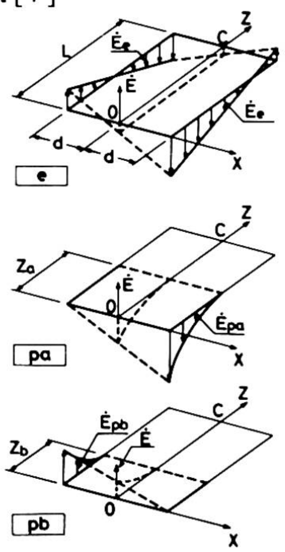

Fig.2
Incremental Strain Distributions

3) The normal stress of the concentrated sections distributes linearly along the axis of the member.

4) The compatibility condition is given by Eq.(1).

$$E=W'+(U')**2/2-U''X \tag{1}$$

in which E : the normal strain at Z-section, ' : the differentiation with respect to Z, U,W : the displacements at Z-section. The notations in this equation are explained in Fig.1. The rate of this equation gives the incremental strain ($\dot{E}$) expressed by Eq.(2).

$$\dot{E}=\dot{W}'+U'\dot{U}'-\dot{U}''X \tag{2}$$

in which the dots mean the increment.

5) The incremental plastic strains ($\dot{E}pa,\dot{E}pb$) are expressed by Eq.(3).

$$\dot{E}pa=Ra(1-Z/Za)\dot{E}e \quad \text{at } X=d\ ,Z=(0,Za)$$
$$\dot{E}pb=Rb(1-Z/Zb)\dot{E}e \quad \text{at } X=-d,Z=(0,Zb) \qquad (3)$$

in which Ra,Rb : the ratio of the incremental plastic strain to the incremental elastic strain, Za,Zb : the length of the plastic zone along the axis, $\dot{E}e$: the incremental elastic strain, d : the half distance between the concentrated sections. The distribution of the incremental strains and the notations are explained in Fig.2. The values of Ra,Rb are decided according to the hysteretic stress-strain relation at the fixed end. The stress-strain relation is expressed by the tri-linear model shown in Fig.3.

6) The shear deformation is neglected.

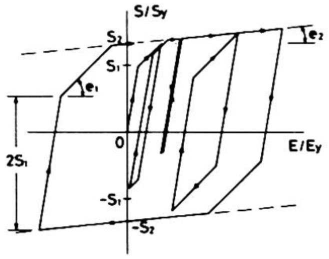

Fig.3 Stress-Strain Relation

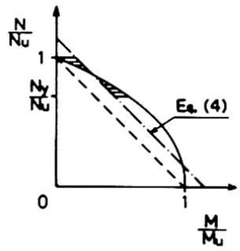

Fig.4 M-N Interaction

2.2 The equivalent two-flange section

Fig.4 shows the moment-axial force (M-N) interaction of the section of a steel member which is defined by the ultimate stress (Mu,Nu). The relation is generally expressed by the curved line as shown by the real line in the figure. However, the M-N relation of the two-flange section explained in the assumption 2) is given by the straight line as shown by the dashed line in Fig.4. In this case the error is too large to analyze bracing members which are subjected to high tensile axial load. To exclude this error, in this study the concentrated areas (Aa,Ab) of the replaced equivalent two-flange section are given under the condition to minimize the sum of the difference between the two curves over the yield axial force (Ny) which is shown by the shaded area in Fig.4. Although the values of Aa,Ab should be changed according to the sectional shape of member, to simplify the calculation, Aa,Ab are given by the representative values shown in Eq.(4).

$$Aa=0.8*A, \quad Ab=0.2*A \qquad (4)$$

where Aa : the sectional area of the higher stress flange, Ab : the sectional area of the other flange, A : the sectional area of the original section.

2.3 The incremental elastic strain

The incremental strain ($\dot{E}$) of the model presented in this study is divided into the three components which are the incremental elastic strain ($\dot{E}e$) and the incremental plastic strains ($\dot{E}pa,\dot{E}pb$). The elastic strain component among them will be expressed by the end-deformations.

According to the assumption 4), the incremental elastic strain component can be expressed by Eq.(5).

$$\dot{E}e=\dot{W}e'+U'\dot{U}e'-\dot{U}e''X \qquad (5)$$

The suffix e means the elastic component. As the loads (Fx,Fz,Fr) work only at the free end of the model, the incremental elastic strain at the center of section ($\dot{E}oe$) is constant along the axis and it is given by Eq.(6).

$$\dot{E}oe=\dot{W}e'+U'\dot{U}e' \qquad (6)$$

The incremental curvature ($\dot{U}e''$) which is generated by the incremental elastic strain is the linear function of Z due to the assumption 3). From this condition and the boundary condition at the fixed end, the incremental deflection ($\dot{U}e$) caused only from elastic strain can be expressed by the incremental end-deformations ($\dot{D}xe,\dot{D}re$). The incremental end-displacement ($\dot{D}ze$) in Z-direction due to the elastic strain is derived from Eq.(6) and it is expressed by Eq.(7).

$$\dot{D}ze=\int(\dot{E}oe-U'\dot{U}e')dZ \qquad (7)$$

Substituting $\dot{W}e',\dot{U}e'$ and $\dot{U}e''$ expressed by the incremental end-deformations into Eq.(5), we obtain the following equation.

$$\dot{E}e=[e][\dot{D}e] \qquad (8)$$

where $[\dot{D}e]=[\dot{D}xe/L\ \ \dot{D}ze/L\ \ \dot{D}re]'$, [ ]' means the transposed matrix.

2.4 The incremental plastic end-deformations

The incremental plastic strain is divided into the two components ($\dot{E}pa,\dot{E}pb$) whose distributions are shown in Fig.2. Using the assumption 4), the components can be expressed by Eqs.(9).

$$\dot{E}pa=\dot{E}opa-\dot{U}pa''X \quad \text{at } X=d\ ,Z=(0,Za)$$
$$\dot{E}pb=\dot{E}opb-\dot{U}pb''X \quad \text{at } X=-d,Z=(0,Zb) \tag{9}$$

$\dot{E}opa,\dot{E}opb$ are the incremental plastic strain at the center of the section (X=0). $\dot{U}pa,\dot{U}pb$ are the incremental plastic deflection caused from $\dot{E}pa,\dot{E}pb$ respectively. $\dot{E}pa,\dot{E}pb$ are defined so as to satisfy Eqs.(10) respectively.

$$\dot{E}opa=\dot{E}pa/2, \qquad \dot{E}opb=\dot{E}pb/2 \tag{10}$$

Substituting Eqs.(8),(9) and (10) into Eq.(3), the following relations are derived.

$$Ra(1-Z/Za)[e][\dot{D}e]=-2d*\dot{U}pa'', \qquad Rb(1-Z/Zb)[e][\dot{D}e]=2d*\dot{U}pb'' \tag{11}$$

The functions $\dot{U}pa,\dot{U}pb$ in Eq.(11) are obtained by the use of the boundary conditions at the fixed end.

The incremental plastic end-deformations ($\dot{D}xp,\dot{D}zp,\dot{D}rp$) caused from the incremental plastic strain components ($\dot{E}pa,\dot{E}pb$) are given by Eqs.(12) and (13).

$$\dot{D}xp=\dot{U}pa+\dot{U}pb, \qquad \dot{D}rp=\dot{U}pa'+\dot{U}pb' \quad \text{at } Z=L \tag{12}$$

and

$$\dot{D}zp=\int(\dot{E}opa+\dot{E}opb-U'(\dot{U}pa'+\dot{U}pb'))dZ \tag{13}$$

In the equations $\dot{D}xp,\dot{D}zp$ are the incremental plastic end-displacements in X-direction and in Z-direction respectively. $\dot{D}rp$ is the incremental plastic end-rotation. $\dot{E}opa,\dot{E}opb$ in Eq.(13) can be expressed by $\dot{U}pa,\dot{U}pb$ from Eqs.(9) and (10). Substituting $\dot{U}pa,\dot{U}pb$ into Eqs.(12) and (13), we get Eq.(14).

$$[\dot{D}p]=[T][\dot{D}e] \tag{14}$$

where $[\dot{D}p]=[\dot{D}xp/L\ \ \dot{D}zp/L\ \ \dot{D}rp]'$.

The integrations in the matrix [T] are easily executed and they are expressed in the closed form function of the end-deformations $[\dot{D}e],[\dot{D}p]$ because the deformation included in the integration can be expressed by Eq.(15)

$$U'=\Sigma(\dot{U}e'+\dot{U}pa'+\dot{U}pb') \tag{15}$$

where Σ means the summation of the increments.

2.5 The rate equation

The relation between the end-loads and the end-deformations of a steel member will be derived in this paragraph.

The incremental virtual work equation of the model, shown in Fig.1 and explained in the assumption 1), is given by Eq.(16).

$$[F]'[\dot{D}]=\int\int S.\dot{E}.dZdA/(PyL) \tag{16}$$

in which, $[F]=[Fx/Py\ \ Fz/Py\ \ Fr/(Py.L)]'$, $[D]=[Dx/L\ \ Dz/L\ \ Dr]'$, Py : the yield axial force, S : the normal stress and $\int dZ,\int dA$: the integration along Z-axis and over the sectional area respectively. The notations in Eq.(16) are explained in Fig.1.

The incremental strain $\dot{E}$ in Eq.(16) is the sum of the elastic component and the plastic components and it is shown by Eq.(17).

$$\dot{E}=\dot{E}e+\dot{E}pa+\dot{E}pb \tag{17}$$

The incremental strains in the right side of Eq.(17) can be expressed by the function of $[\dot{D}e]$ by substituting Eqs.(3) and (8) into Eq.(17) and we find

$$\dot{E}=(1+Ra(1-Z/Za))[e][\dot{D}e] \quad \text{at } X=d\ ,Z=(0,Za)$$
$$\dot{E}=(1+Rb(1-Z/Zb))[e][\dot{D}e] \quad \text{at } X=-d,Z=(0,Zb) \tag{18}$$

Since $[\dot{D}]=[\dot{D}e]+[\dot{D}p]$, $[\dot{D}e]$ can be expressed by $[\dot{D}]$.

$$[\dot{D}e]=inv[To][\dot{D}] \tag{19}$$

in which [To]=[T]+[1] and "inv" means the inverse matrix.

The equilibrium equation is derived by substituting Eqs.(18) and (19) into Eq.(16).

$$[F]=inv[To]'\int(\int S(1+Ra(1-Z/Za))[e]'dA+\int S(1+Rb(1-Z/Zb))[e]'dA)dZ/PyL \tag{20}$$

The rate equation of Eq.(20) becomes

$$[To]'[\dot{F}]=[k][\dot{D}e] \tag{21}$$

Substituting Eq.(19) in Eq.(21) we finally obtain the relation between $[\dot{F}]$ and $[\dot{D}]$.

$$[\dot{F}]=[K][\dot{D}] \qquad (22)$$

in which [K]=inv[To]'[k]inv[To]. [K] is the tangent stiffness matrix of the structural model. It is defined by the tangent modulus of the stress-strain relation and the residual deformations. In the presented stiffness matrix in Eq.(22) the influence of the residual deformations is expressed in the closed form function in which only the sum of the incremental end-deformations $[\dot{D}e]$,$[\dot{D}p]$ and the plastic zone length (Za,Zb) are included. This expression of the residual plastic deformation in the stiffness matrix makes the presented method very simple.

2.6 The plastic zone length

Since the loads of the model works only at the free end, it is reasonable to assume that the incremental normal stress distributes linearly along the axis of the member. But the elastic limit stress may not distribute linearly along the axis according to the hysteretic plastic deformation. To simplify the calculation of Za,Zb, the critical stress of elastic range is also assumed to distribute linearly. Under this condition the plastic zone length is easily obtained as the intersection point of the two linear functions of Z.

2.7 Application to steel member and steel frame

If every steel member of building frames is divided into the two parts along the axis, each part can be considered as the cantilever member whose loading condition is the same as shown in Fig.1 and the load-deformation relation is given by Eq.(22). From this reason the relationships between the end-loads and the end-deformations of every steel member can be derived by coupling the two presented equations, shown by Eq.(22), under the continuation conditions at the center along the axis of the member. The derived load-deformation relation is easily applied to the analysis of steel frames.

3. NUMERICAL ANALYSIS

To show the usefulness of the presented analysis method, the numerical analysis of plane steel frames under seismic load has been carried out. The analyzed steel frames, named Frame-1 and Frame-2, are 10-story 3-bay braced frames shown in Fig.5. Frame-1 is designed based on the Japanese Aseismic Design Code and the horizontal strength and the stiffness of every story are perfectly agree with the required criteria of the code. To simulate the collapse behavior, Frame-2 is designed under the half of the seismic load of Frame-1. The ground motion is the N-S component of the well-known El Centro 1940 record amplified by three times. The calculation of the seismic response has been carried out by the use of the linear-acceleration method under the condition that the errors of the energy balance equation [6] does not exceed 0.5 percent of the input energy.

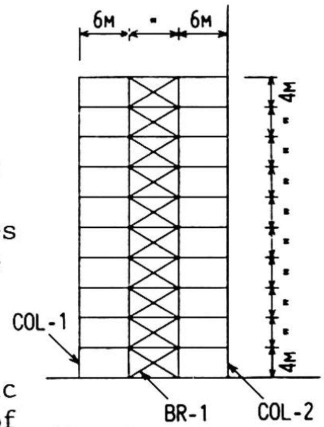

Fig.5
Calculated Frame

Numerical results are shown in Figs.6-9. From these figures we can say the inelastic hysteretic behavior and the collapse behavior of the frames are analyzed fairly well. It is also shown that the restoring force characteristics of the columns, which effect strongly on the response of the frames, are remarkably complicated and different mutually and the deformation of the frames tends to be concentrated only in one story. These results emphasize that in the analysis of

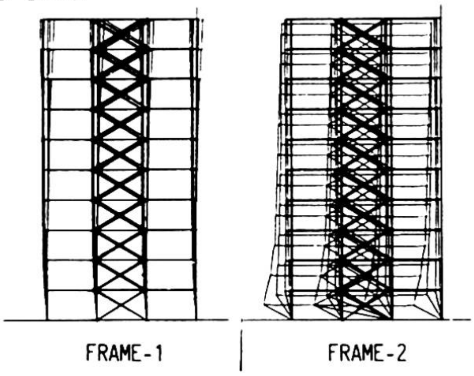

Fig.6 Deflected Shape
(in Real Proportional Scale)

steel frames the detailed behavior of all members should be analyzed.

4. CONCLUSIONS

i) The proposed analysis method is simple and does not require so much calculation but it can simulate the hysteretic inelastic behavior of steel members under intense loads relatively accurately. Since the presented method is applicable to columns, beams and braces in the same manner, it is useful and suitable for the detailed structural analysis of building frames without a large amount of calculation.

ii) The collapse of building frames is caused from the failure of a few members whose restoring force characteristics are very complicated. Accordingly it is necessary for the reliable structural analysis of tall buildings, which are composed of many members, to analyze the detailed behavior of all members accurately.

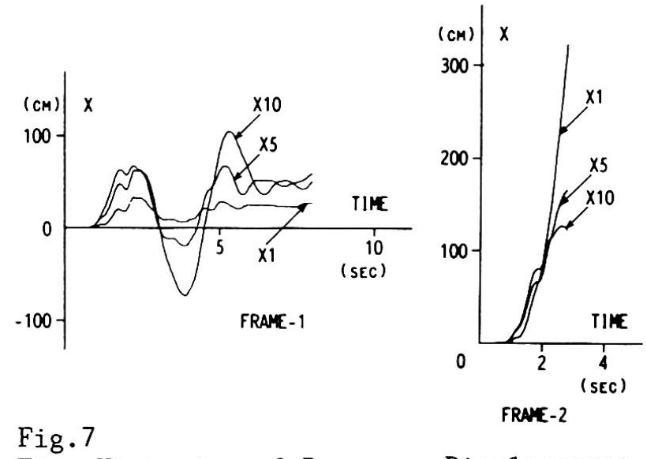

Fig.7
Time Histories of Response Displacement

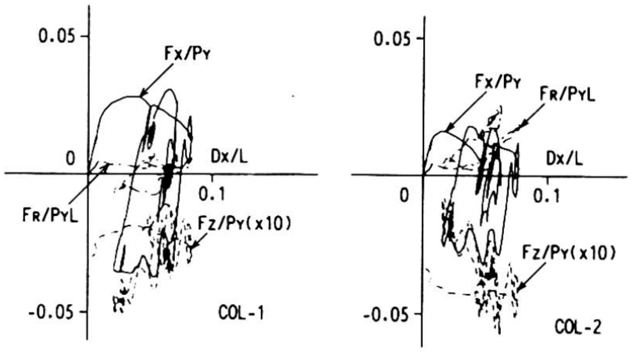

Fig.8
Hysteretic Behavior of Columns

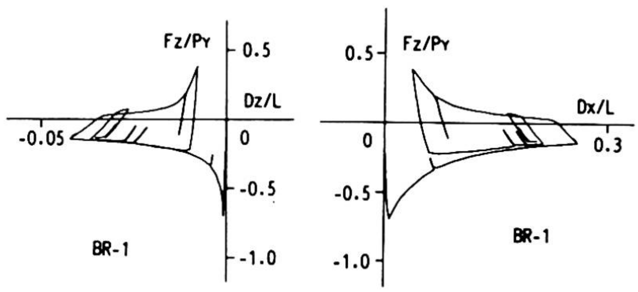

Fig.9
Hysteretic Behavior of Brace

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