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Autor: Kouhia, Reijo / Paavola, Juha / Tuomala, Markku

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Modelling the Fire Behaviour of Multistorey Buildings

Modélisation du comportement au feu de bâtiments à étages multiples

Modellierung des Brandverhaltens von mehrgeschossigen Gebäuden

Reijo KOUHIA
Research Scientist
Finnish Academy
Espoo, Finland

Juha PAAVOLA
Research Scientist
Helsinki Univ. of Techn.
Espoo, Finland

Markku TUOMALA
Professor
Tampere Univ. of Techn.
Tampere, Finland

SUMMARY
A finite element method is developed for steel framed structures subjected to large loads and high temperatures. Material nonlinearities and large deformations are taken into account. An incremental Lagrangian description is adopted in formulating the equations of motion of structures. An elastic plastic material model within a theory of plasticity is formulated based on the recent studies on the properties of steel at temperatures 20–600°C. An incremental iterative procedure is established to follow the nonlinear response of structures. Emphasis is given to detecting the onset of local nonlinearities, their progress and interaction with global structural response. Beams and frames exposed to fire are analyzed and results are compared with experimental data.

RÉSUMÉ
Une méthode des éléments finis a été développée pour l'analyse des portiques en acier, soumis aux grandes charges et aux hautes températures. Les matériaux déformés du modèle ainsi que les grandes déformations sont pris en compte. Une équation de Lagrange a été adoptée afin d'établir les équations de mouvements des structures. Les récentes études sur les propriétés des aciers soumis aux températures de 20 à 600°C ont permis de formuler un modèle élasto-plastique selon la théorie de la plasticité. Une méthode itérative a été établie pour suivre la réaction non linéaire des structures. L'accent a été mis sur la détection des apparitions de déformées locales sur leur interaction avec la réaction structurale globale. Des poutres et des portiques exposés au feu ont été analysés. Les résultats ont été comparés avec des données expérimentales.

ZUSAMMENFASSUNG
Ein Finite-Element-Verfahren für Stahlrahmentragwerke bei grossen Lasten und hohen Temperaturen wurde entwickelt, bei dem Materialnichtlinearitäten und grosse Deformationen berücksichtigt werden. Die Bewegungsgleichungen basieren auf der inkrementellen Lagrangeformulierung. Eiun elastoplastisches Materialgesetz auf der Grundlage einer Plastisitätstheorie, das sich auf neueste Ergebnisse der Materialforschungen an Stahl bis 600°C stützt, wurde erarbeitet. Das Modell zur Berechnung des nichtlinearen Strukturverhaltens verwendet ein inkrementelles Iterationsverfahren. Der Beginn lokaler Nichtlinearität, ihr Fortschreiten und ihr Zusammenwirken mit der Gesamtstruktur wird besonders beachtet. Die Rechenergebnisse brandbeanspruchter Balken und Rahmen werden mit Versuchsergebnissen verglichen.

1. INTRODUCTION

The behaviour of structures under large loads and high temperatures is complex due to material nonlinearities and large deformations. In fire safety analysis of steel structures analytical calculation methods are applicable only for very simple structures with considerable approximations in the analysing model. Detailed information about structural behaviour can be obtained by the finite element method, FEM. Linear elastic analyses by FEM are nowadays a part of everyday practice in engineering design. In many important applications, however, they do not provide information enough for completely safe but economical design. Nonlinear FEM analyses, on the contrary, require much more experience from the user because material and geometrical nonlinearities are coupled. However, the character of the load-deformation curve is important in assessing the safety of the structure in the post-buckling range.

In the present study a geometrically nonlinear elasto-plastic finite element analysis of steel frames in high temperatures is considered. Finite deformations are taken into account by adopting an incremental Lagrangian formulation of the problem. The temperature dependence of material parameters is modelled in accordance with Refs. [1,2]. The resulting nonlinear equilibrium equations are solved by an incremental iterative procedure based on Newton's method. In constant temperature analyses an arc-length method is used. In transient problems creep type displacement-temperature curves are integrated by an adaptive step-size selection method. The FEM program developed is applicable for three-dimensional beams and frames.

The numerical examples calculated consist of steel beams and frames for which experimental data exist. Solutions obtained by different material models are compared with the test results of Refs. [2,4]. In addition, an ECCS calibrating multistorey frame [3] exposed to a local fire is studied.

2. FINITE ELEMENT FORMULATION

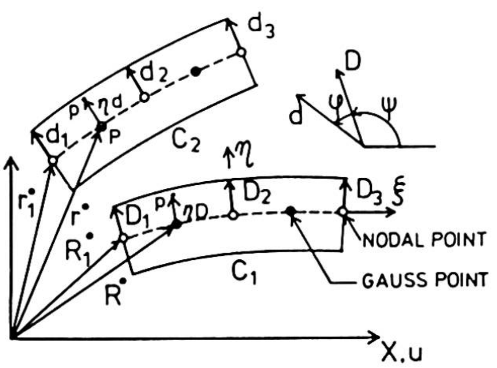

Fig. 1 Geometry of plane frame element

In the finite element method the displacement vector $\mathbf{u}$ of a material point $\mathbf{X}$ is interpolated within an element by shape functions $\mathbf{N}$ and nodal point displacements $\mathbf{q}$

$$\mathbf{u}(\mathbf{X}) = \mathbf{N}(\mathbf{X})\mathbf{q} \tag{1}$$

Inserting (1) into an equation of incremental virtual work gives the following equilibrium equation [5]

$$^1(\mathbf{K}_1 + \mathbf{K}_g)\mathbf{q} = ^2\mathbf{Q} - ^1\mathbf{F} \tag{2}$$

where $\mathbf{K} = \mathbf{K}_1 + \mathbf{K}_g$ is the tangent stiffness matrix, $\mathbf{K}_g$ the geometrical stiffness matrix, $\mathbf{Q}$ the external force vector and $\mathbf{F}$ the vector of internal forces. The left superscript 1 refers to the reference configuration at $\mathbf{q}(t_i)$ and the index 2 to the configuration at $\mathbf{q}(t_i + \Delta t)$, where t is a load parameter. In the incremental Lagrangian formulation the matrix $\mathbf{K}_1$ is also dependent on incremental deformations between configurations 1 and 2.

An isoparametric curved beam element is shown in Fig. 1. The position vector of an arbitrary point P in the reference configuration is

$$\mathbf{R} = \mathbf{R}^o + \eta \mathbf{D}_1 + \varsigma \mathbf{D}_2 \tag{3}$$

where $\mathbf{D}_k$ are director vectors and η, ς coordinates in the cross section. $\mathbf{R}^o$ and $\mathbf{D}_k$ are interpolated by polynomials N_i

$$\mathbf{R}^o = \textstyle\sum_{i=1}^{n} N_i(s)\mathbf{R}_i^o, \quad \mathbf{D}_k = \sum_{i=1}^{n} N_i(s)\mathbf{D}_{ki} \tag{4}$$

where n is the number of nodes in one element. In the deformed configuration the position vector of material point P is

$$\mathbf{r} = \mathbf{r}^o + \eta \mathbf{d}_1 + \varsigma \mathbf{d}_2 \tag{5}$$

where $\mathbf{r}^o$ and $\mathbf{d}_k$ are interpolated by polynomials N_i, correspondingly. The director vector $\mathbf{d}$ in the deformed configuration 2 is obtained from $\mathbf{D}$ in configuration 1 by the formula $\mathbf{d} = \mathbf{QD}$, in which $\mathbf{Q}$ is a rotation matrix. The displacement vector is then

$$\mathbf{u} = \mathbf{r} - \mathbf{R} \quad \textit{or} \quad \mathbf{u} = \mathbf{u}^o + \eta(\mathbf{d}_1 - \mathbf{D}_1) + \varsigma(\mathbf{d}_2 - \mathbf{D}_2) \tag{6}$$

where $\mathbf{u}^o$ is the displacement vector of the reference line. The finite element presentation of Eq. (6)

$$\mathbf{u} = \sum N_i(\mathbf{u}_i^o + \eta(\mathbf{d}_1 - \mathbf{D}_1)_i + \varsigma(\mathbf{d}_2 - \mathbf{D}_2)_i) \tag{7}$$

is inserted into an incremental form of the Green-Lagrangian strain tensor

$$E_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) \tag{8}$$

containing normal strain and shear strains. The stiffness matrices $\mathbf{K}_1$ and $\mathbf{K}_g$ and the vector of internal forces $\mathbf{F}$ for an element are evaluated by numerical integration. In the applications a two-noded element with a one point Gaussian rule [5] is used.

3. CONSTITUTIVE MODEL

The strain rate $\mathbf{D}$ is decomposed into elastic, plastic, creep and thermal parts

$$\mathbf{D} = \mathbf{D}_e + \mathbf{D}_p + \mathbf{D}_c + \mathbf{D}_T \tag{9}$$

The elastic part $\mathbf{D}_e$ is obtained from Hooke's law

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\epsilon}_e \tag{10}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{C}$ an elastic constitutive tensor and $\boldsymbol{\epsilon}_e$ the elastic strain tensor. A rate form of Eq. (10) is

$$\overset{*}{\boldsymbol{\sigma}} = \mathbf{C}\mathbf{D}_e + \frac{\partial \mathbf{C}}{\partial T}\dot{T}\boldsymbol{\epsilon}_e \tag{11}$$

where $\overset{*}{\boldsymbol{\sigma}}$ is the Jaumann rate of $\boldsymbol{\sigma}$. The J_2-flow theory is used to evaluate the plastic strain rate

$$\mathbf{D}_p = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{12}$$

where $f = \sqrt{3J_2}$ and J_2 is the second invariant of the deviatoric stress. The thermal strain rate is

$$\mathbf{D}_T = \alpha \dot{T} \mathbf{I} \tag{13}$$

where α is the coefficient of thermal expansion. In short duration loadings $\mathbf{D}_c$ is assumed to vanish. The yield condition is expressed by the formula

$$F = f - \sigma_y = 0 \tag{14}$$

where the yield stress σ_y is dependent on a hardening parameter κ and temperature T. According to the consistency condition during plastic flow

$$dF = \frac{\partial F}{\partial \sigma} d\sigma + \frac{\partial F}{\partial \kappa} d\kappa + \frac{\partial F}{\partial T} dT = 0 \tag{15}$$

By inserting the flow rule Eq. (12) into the equation

$$\overset{*}{\sigma} = \mathbf{C}(\mathbf{D} - \mathbf{D}_p - \mathbf{D}_T) + \frac{\partial \mathbf{C}}{\partial T} \dot{T} \epsilon_e \tag{16}$$

and by using the consistency condition Eq. (15) $\dot{\lambda}$ is obtained [6]. Eq. (16) gives then

$$\overset{*}{\sigma} = (\mathbf{C} - \frac{1}{h}\mathbf{b}\mathbf{b}^T)(\mathbf{D} - \mathbf{D}_T) + \frac{1}{h}\mathbf{b}(\frac{\partial \sigma_y}{\partial T}\dot{T} - \mathbf{a}^T \frac{\partial \mathbf{C}}{\partial T}\dot{T}\epsilon_e) + \frac{\partial \mathbf{C}}{\partial T}\dot{T}\epsilon_e \tag{17}$$

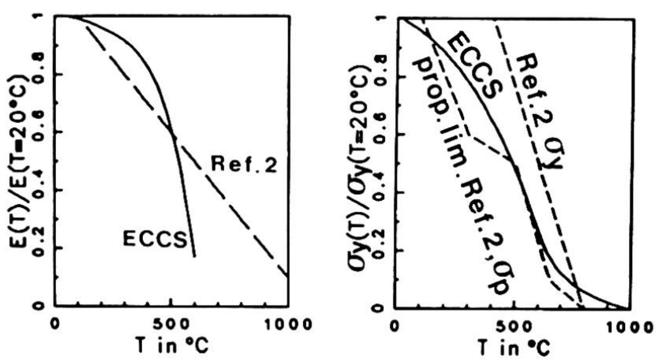

Fig. 2 Modulus of elasticity E and yield stress σ_y vs. temperature

where $\mathbf{a} = \partial f / \partial \sigma$, $\mathbf{b} = \mathbf{Ca}$, $h = \mathbf{a}^T\mathbf{b} + E_p$ and superscript T means transpose. The plastic hardening modulus E_p is obtained from the tangent modulus E_t and the modulus of elasticity E by the formulas

$$E_p = \frac{EE_t}{E - E_t}, \qquad E_t = \frac{d\sigma_y}{d\bar{\epsilon}_p} \tag{18}$$

The yield stress σ_y is obtained from a tension test as a function of the logarithmic inelastic strain

$$\bar{\epsilon}_p = \int \sqrt{\tfrac{2}{3}\mathbf{D}_p \cdot \mathbf{D}_p} dt \tag{19}$$

where t is a load parameter. Using the relationship between the strain rate $\mathbf{D}$ and the rate of Green-Lagrange strain $\dot{\mathbf{E}}$ an appropriate constitutive equation for the incremental Lagrangian formulation is obtained in the form

$$\dot{\mathbf{S}} = \mathbf{C}_L \dot{\mathbf{E}} \tag{20}$$

where $\mathbf{S}$ is the 2nd Piola Kirchhoff stress. The temperature dependency of E, σ_y and α is in accordance with Ref. [1] or alternatively with Ref. [2], depicted in Fig. 2. The material model is evaluated at discrete integration points on the cross section. The transverse shear stresses are taken into account.

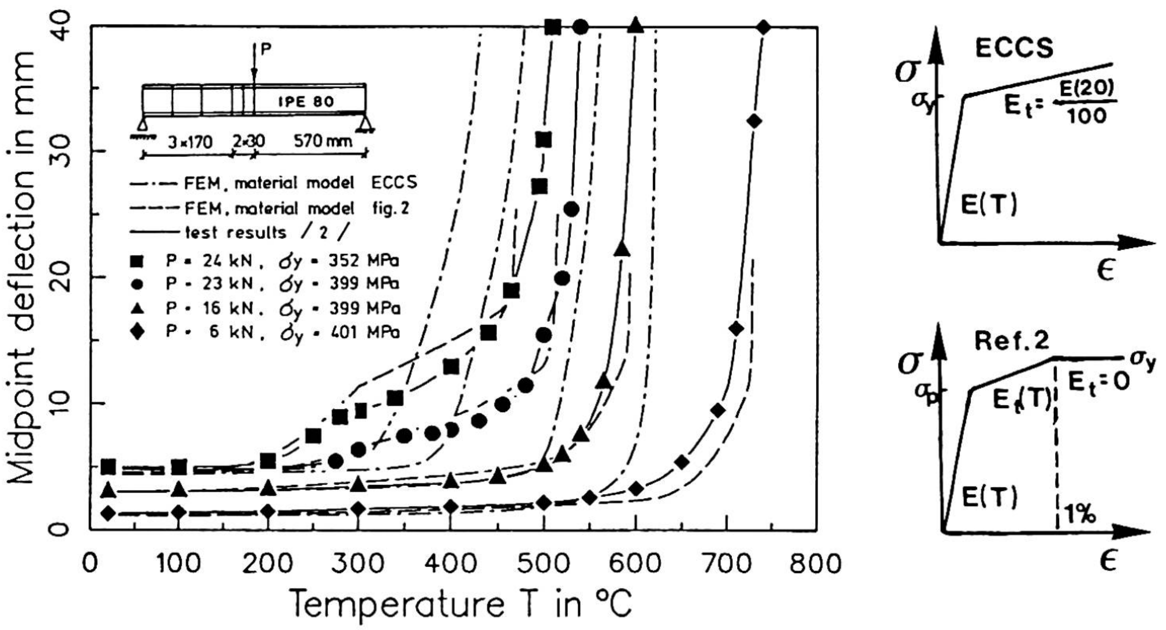

Fig. 3 Central deflection vs. temperature of a beam with various load magnitudes

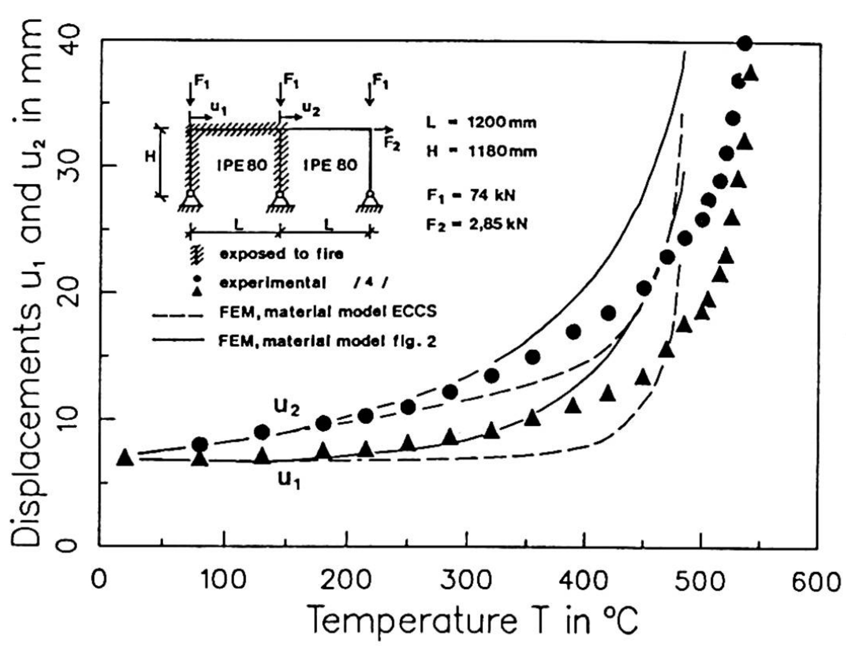

Fig. 4 Horizontal deflections of a two-bay frame

4. APPLICATIONS

Applications are chosen mainly to enable comparisons with experimental data. The FEM program developed is applicable for three-dimensional beams and frames, but, due to the lack of test results on three-dimensional cases, plane structures are analyzed. An IPE 80 beam with a point load at midspan is considered first, merely to verify the material model. The problem definition and the results corresponding to the experimental measurements in [2] are given in Fig. 3. In most cases the ECCS material parameters [1], which are mainly not defined in temperatures over 600^oC, result in conservative estimates for displacements, also under this temperature. The results corresponding to the material parameters of Ref. [2] agree reasonably well with experimental data.

As a second example a two-bay test frame of Ref. [4] is analyzed. The problem definition and the calculated horizontal deflections vs. temperature by using the material parameters of ECCS [1] or Rubert and Schaumann [2] are given in Fig. 4.

Fig. 5 shows the calculated horizontal deflections in the first floor of a multistorey frame (ECCS calibrating frame I in Ref. [3]) and the deformed configurations when exposed to a local fire. The results correspond to ECCS material parameters. Results of a modified frame in which the lower end of the central column is hinged are shown by dashed lines.

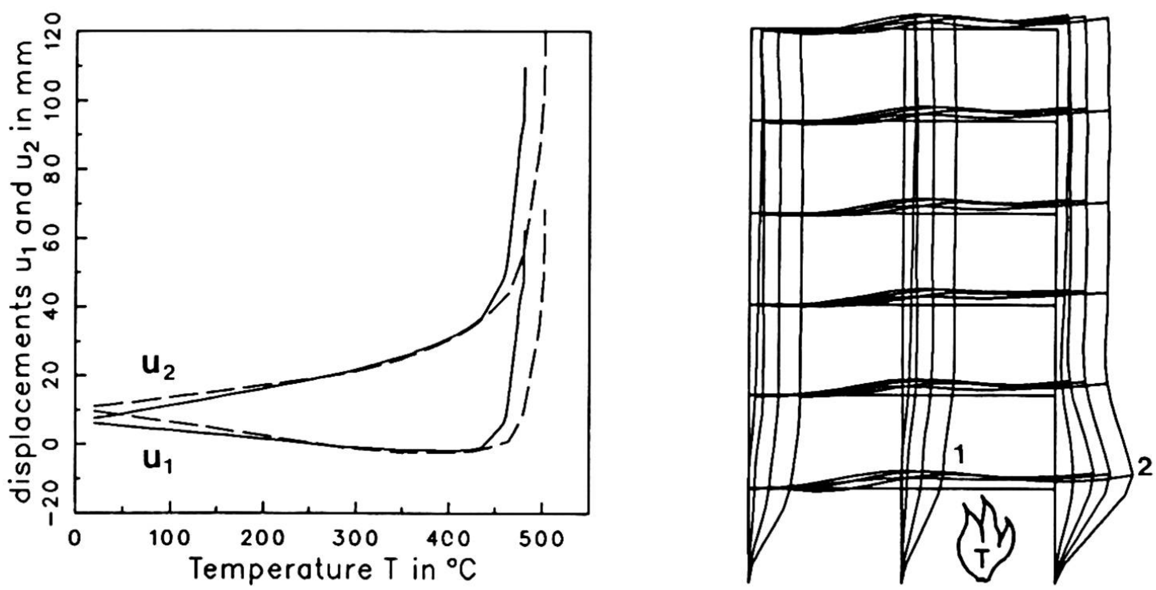

Fig. 5 Horizontal deflections of points 1 and 2 and deformed shapes magnified by a factor of 30 at temperatures 320, 460, 470 and 480°C

REFERENCES

1. European Recommendations for the Fire Safety of Steel Structures. ECCS-Technical Committee 3. Elsevier, 1983.

2. RUBERT A. & SCHAUMANN P., Temperaturabhängige Werkstoffeigenschaften von Baustahl bei Brandbeanspruchung. Stahlbau, 54, 1985, No. 3, 81-86.

3. VOGEL U., Calibrating Frames. Stahlbau, 54, 1985, No.10, 295-301.

4. RUBERT A. & SCHAUMANN P., Tragverhalten stählerner Rahmensysteme bei Brandbeanspruchung. Stahlbau, 54, 1985, No.9, 280-287.

5. ZIENKIEWICZ, O.C., The Finite Element Method. 3 rd edn., Mc Graw-Hill, London 1977, 787 p.

6. HILL R., On the Classical Constitutive Relations for Elastic-Plastic Solids. Recent Progress in Applied Mechanics. Almqvist and Wiksell, Stockholm, 1967, 241-249.