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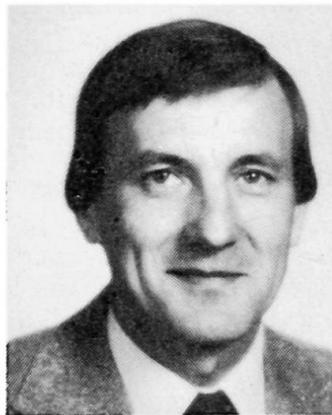
## Effects of Ice Formations in Cylindrical Water Tanks

Effet de la formation de glace dans des réservoirs à eau cylindriques

Auswirkungen der Eisbildung in zylindrischen Wassertanks

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### SUMMARY

A mathematical model for simulation of ice under stress is discussed and a method of determining the hoop stress induced in the wall of a cylindrical tank by an internal ice cap subjected to increasing temperature is developed. These predictions are validated by comparison with test data and it is concluded that hoop stresses capable of cracking the wall of a non-prestressed concrete tank may be induced by an expanding ice cap. A number of other defects observed during inspections of water tanks in Ontario, and attributed to the effects of a freezing environment, are discussed.

### RÉSUMÉ

Un modèle de simulation du comportement de la glace sous l'effet de contraintes est présenté. Une méthode de détermination des contraintes latérales, provoquées sur la paroi d'un réservoir cylindrique par un bouchon de glace soumis à une température croissante, est développée. Ces calculs sont comparés avec des résultats d'essais et font apparaître que les contraintes latérales sont capables de provoquer des fissures dans la paroi d'un réservoir en béton non précontraint, par l'augmentation du volume du bouchon de glace. Certains autres dégâts – résultant d'effets de la température – ont été observés lors de contrôles de réservoirs en Ontario.

### ZUSAMMENFASSUNG

Ein mathematisches Modell für Eis unter Spannung und eine Methode zur Bestimmung der Ringspannung in einem zylindrischen Behälter infolge Temperaturerhöhung der Eisdecke im Behälter werden vorgestellt. Die Berechnungen werden mit Versuchsdaten verglichen. Die auftretenden Ringzugkräfte können zur Rissbildung in schlaff bewehrten Stahlbetontanks führen. Verschiedene, bei Inspektionen von Wassertanks in Ontario, als Folge von Frostwirkung beobachtete Defekte, werden besprochen.



## 1. INTRODUCTION

A study [1] of 53 concrete cylindrical water tanks in the Province of Ontario, Canada, indicated performances ranging from two failed tanks through tanks exhibiting distress of varying degrees to tanks which were performing adequately. The major problems observed could be related to faults in construction methodology and to inadequate design criteria in view of the freezing environment during winter in Ontario. Ice formations consisting of a horizontal ice cap at the top water level and/or a vertical ice tube on the inside of the wall of the tank were observed during the inspections. Similar ice formations in water tanks in Finland have been described by Pitkanen [2].

Pressure exerted on the wall of a cylindrical water tank, as a result of warming and related expansion of an ice cap, is addressed in this paper. A mathematical model for simulation of ice under stress is discussed and a method of determining the hoop stress induced in the wall of a cylindrical tank by an ice cap subjected to increasing temperature is developed. These predictions are validated by comparison with test data from a model water tank. It is concluded that hoop stresses capable of cracking the wall of a non-prestressed concrete tank may be induced by an ice cap. A number of other defects observed during the inspections of the tanks in Ontario, and attributed to the effects of a freezing environment, are discussed. It is recommended that ice formations be prevented from forming in a tank by some appropriate means.

## 2. MECHANICAL PROPERTIES OF ICE

The mechanical properties of ice depend on many factors. The properties of different types of ice have been discussed by Michel [3] and Tuomioja et al [4]. Bergdahl [5] has proposed the following model, illustrated in Fig. 1, to describe the behavior of ice under pressure:

$$\dot{\epsilon} = \dot{\sigma}/E + BD\sigma^n \quad (1)$$

where  $\dot{\epsilon}$  is the strain rate,  $\dot{\sigma}$  is the stress rate,  $E$  is the modulus of elasticity,  $D$  is the self-diffusion coefficient and  $B$  and  $n$  are coefficients of viscous deformation. Suitable values of  $E$ ,  $D$ ,  $B$  and  $n$  have been suggested [5].

An ice cap in a concrete or steel tank subjected to an increase in temperature will be subjected to biaxial in-plane compressive stress since the coefficient of thermal expansion of ice is 4 to 5 times that of concrete or steel. The relationship between stress and strain, given in Eqn. (1), is nonlinear and the effect of combined stresses may be considered using a procedure [6] which defines the creep strain rate in the form

$$\dot{\epsilon}_{cr}^* = G\sigma^{*n} \quad (2)$$

where  $G$  is a constant and  $\sigma^*$  is the octahedral stress defined by

$$\sigma^* = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (3)$$

where  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  are principal stresses. Thus, from Eqn. (1), the creep strain rate may be expressed as

$$\dot{\epsilon}_{cr} = BD\sigma^{*n} \quad (4)$$

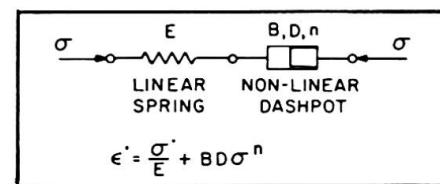


Fig. 1 Model for ice

The in-plane stress condition at any level in an expanding ice cap within a cylindrical tank is analogous to a disc subjected to a uniform radial pressure,  $p$ , around its circumference. For this case, the in-plane principal stresses have a magnitude equal to  $p$  throughout the disc and consequently, from Eqn. (3) with  $\sigma_1 = \sigma_2 = p$  and  $\sigma_3 = 0$ , the octahedral stress in the ice cap is given by  $\sigma^* = p$ .

A creep strain rate of the form given in Eqn. (2) is adopted for the analysis of creep under combined stress in the ABAQUS computer program [7]. Figure 2 shows the variation, with time and related temperature, of the pressure in the ice as predicted by ABAQUS for a test reported by Lindgren [8]. The good agreement observed between the test data and the prediction of ABAQUS demonstrates the validity of Eqns. (1) and (2).

### 3. ANALYSIS OF CYLINDER WITH ICE CAP

Figure 3(a) shows an ice cap contained within a long cylinder. Assuming that a linear pressure distribution develops (creep of the ice neglected) when the ice cap is subjected to an increase in temperature, which varies linearly over its thickness (Fig. 3(b)), the problem reduces to that of a long cylinder subjected to a band of linearly varying pressure around its circumference (Fig. 3(c)). This condition may be analyzed using an approach suggested by Timoshenko and Woinowsky-Krieger [9]. It can be shown [10, 11] that, within the loaded area at a location defined by the parameters  $b$  and  $c$  in Fig. 3, the hoop stress,  $\sigma_h$ , in the wall of the cylinder is given by

$$\sigma_h = Kpd/2t \quad (5)$$

where the parameter,  $K$ , is given by

$$K = - \{c(e^{-\beta c} \cos \beta c + e^{-\beta b} \cos \beta b - 2)/2\ell + [e^{-\beta c}(\sin \beta c - \cos \beta c - 2\beta c \cos \beta c) - e^{-\beta b}(\sin \beta b - \cos \beta b - 2\beta b \cos \beta b)]/4\beta\ell\}, \quad (6)$$

$p$  is the maximum pressure,  $\ell$  is the thickness of the ice cap,  $d$  is the diameter of the cylinder,  $t$  is the thickness of the cylinder wall and  $\beta^4 = 12(1 - \nu_T^2)/d^2 t^2$  where  $\nu_T$  is the Poisson's ratio of the cylinder material. Similar expressions can be developed [10] for hoop stresses in the portions of the cylinder wall outside the loaded area. Flexural stresses in the wall of the cylinder can be determined in a similar manner [10].

The pressure,  $p$ , at the top of the ice cap may be found by considering compatibility of deformations at the location of maximum deformation in the cylinder wall [10] and is given by

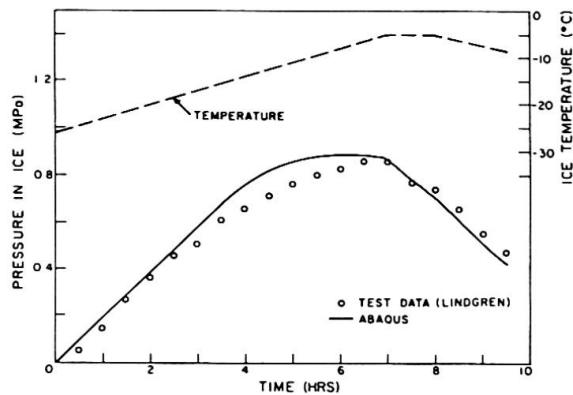


Fig. 2 Simulation of Lindgren test [8]

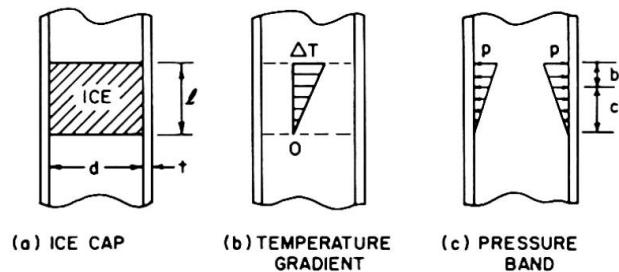


Fig. 3 Cylinder with ice cap



$$p = \frac{c}{\ell} \Delta T E_I (\alpha_I - \alpha_T) / [K \frac{n}{2} + (1 - \nu_I) \frac{c}{\ell}] \quad (7)$$

where the location of maximum deformation, is defined by

$$\frac{c}{\ell} = 0.667 + 0.01\beta\ell \quad (8)$$

and  $\alpha_I$  is coefficient of thermal expansion of ice,  $\alpha_T$  is coefficient of thermal expansion of cylinder material,  $E_I$  is modulus of elasticity of ice,  $\Delta T$  is temperature increment at the top of the ice cap and  $n = E_I d / E_T t$  where  $E_T$  is the modulus of elasticity of the cylinder material.

A typical variation of hoop stress in the wall of a tank, as predicted by the above theory, is shown in Fig. 4, which also shows for comparison corresponding variations obtained using ABAQUS [7] to model the ice cap in the tank but neglecting creep in the ice. It can be seen that the hoop stress variations obtained by both approaches are similar. An idealized variation of the hoop stress proposed for design purposes is also indicated in Fig. 4. The locations of peak hoop stress and zero hoop stress in the wall of a tank are indicated. The zero stress locations are a function of  $\beta$  and  $\ell$ . The magnitude of the peak hoop stress may be determined using Eqns. (5), (6) and (7). Charts to aid these computations, as well as those for flexural stresses, have been developed by Kong and Campbell [11].

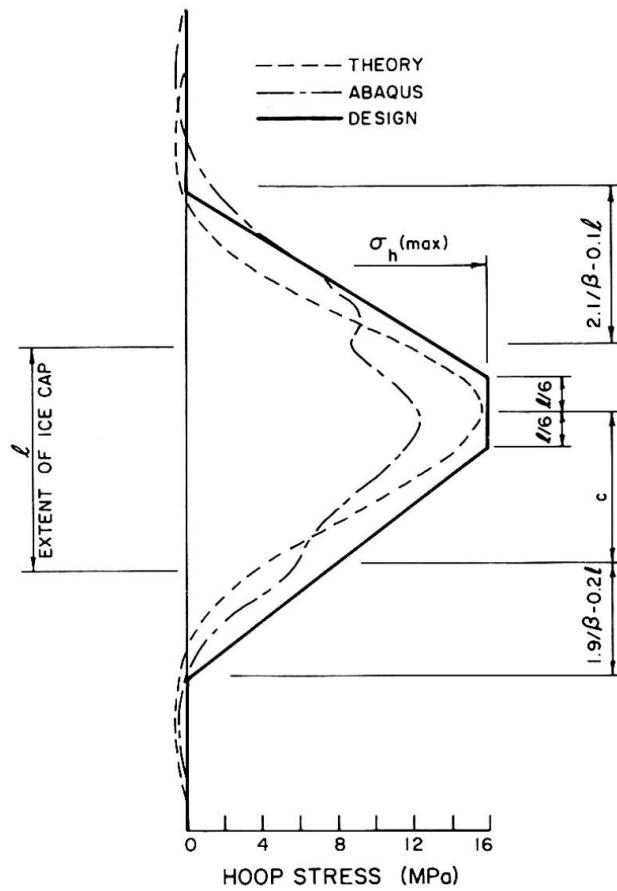


Fig. 4 Variation of hoop stress

#### 4. EFFECT OF CREEP OF ICE

Creep of the ice will reduce the pressure exerted by an ice cap subjected to a rise in temperature over a finite time. The maximum static values of pressure and related hoop stress, determined as described previously, can be adjusted for creep using charts similar to that in Fig. 5. These charts have been developed from a study [11] of restrained ice discs under different values of initial temperature, heating rate and parameter  $\eta$ , which is a measure of the degree of confinement. The peak pressure developed in the ice has been expressed as a percentage of the

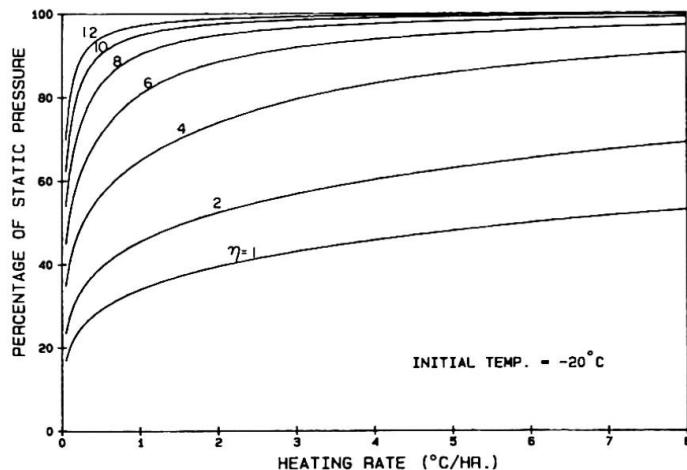


Fig. 5 Effect of creep of ice

corresponding maximum static pressure which would occur if creep were neglected. Figure 5 shows that, for a particular initial temperature, the effect of creep diminishes with increasing values of  $\eta$ , and heating rate. These reduced values of static pressure may be used in conjunction with Fig. 4 to determine the variations of hoop stress in the wall of a cylindrical tank.

A maximum tensile hoop stress of 6.5 MPa has been computed [11] for a concrete tank having a mean diameter of 7140 mm, a wall thickness of 400 mm and containing an ice cap 1000 mm thick. The initial temperatures in the ice cap were  $-20^{\circ}\text{C}$  and  $0^{\circ}\text{C}$  at the top and bottom surfaces, respectively, and the heating rate was  $2^{\circ}\text{C}$  per hour at the top surface.

## 5. TESTING OF MODEL TANK

Tests [10], similar to those reported by Bergdahl [5], were carried out on a model steel tank to examine the ability of ABAQUS with Eqn. (1) incorporated to model the behavior of an ice cap in a water tank. The tank had an internal diameter of 886 mm, a wall thickness of 14 mm and a height of 1310 mm. Biaxial electrical resistance strain gauges (aligned circumferentially and axially) were attached to the wall of the tank. Thermocouples were placed at various levels within the tank, as well as on the wall. The tank was wrapped with 400 mm of fibreglass insulation over its entire height, and 100 mm of insulation was used underneath the tank.

The tank was filled with tap water to the desired level and the temperature in the cold room facility, in which the tank was located, was lowered gradually to around  $-25^{\circ}\text{C}$ . Ice formed initially on the top surface of the water and the ice cap gradually grew in thickness downwards. After the ice cap reached the desired thickness, the temperature in the room was raised gradually to  $0^{\circ}\text{C}$ . The test was concluded when melting of the ice cap occurred adjacent to the wall of the tank.

Three tests were carried out. The thicknesses of the ice cap were 240, 300 and 308 mm and the heating periods were 14, 16 and 11 hours, respectively.

Typical test data, showing the variation of the hoop stress in the tank wall with time at a depth of 46 mm below the top surface of the ice cap, are given in Fig. 6. Good agreement is obtained with a simulation of the test using ABAQUS.

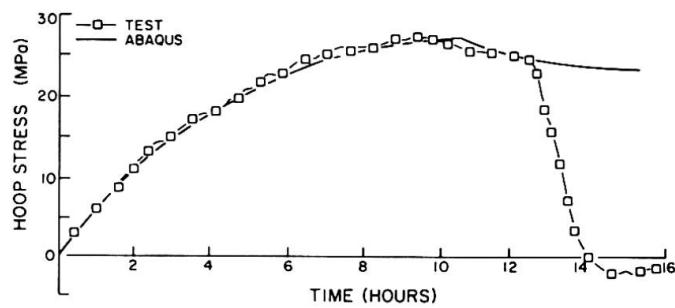


Fig. 6 Hoop stress vs. time

Type	Description
1	Wall delamination
2	Vertical cracks in wall
3	Wall/floor joint deterioration
4	Vertical voids in shotcrete
5	Jack-rod spalls
6	Shotcrete cover coat delamination
7	Waterproof coating failure
8	Cover coat cracking
9	Cold joints and horizontal cracks
10	Corrosion of prestressing steel

Table 1 Types of defects in tanks



## 6. DEFECTS OBSERVED IN CONCRETE TANKS

Ten of the main defects observed during surveys of concrete water tanks in Ontario are listed in Table 1. Most of the defects may be attributed to the influence of a freezing environment and possible mechanisms causing these defects are discussed in Ref. [1]. Delamination of the wall was prevalent and has been attributed to radial tensile stresses in the wall. Such stresses may be induced by either freezing and thawing with related dilation of saturated concrete in the outer portion of the wall exposed to solar radiation, or by trapping a lens of unfrozen water between an inward moving freezing front and the impermeable frozen zone in the interior of the wall section. Vertical cracking in the wall was attributed to pressure and induced hoop stress from internal ice formations which have been addressed in this paper.

## 7. CONCLUSIONS

A mathematical model for ice has been validated by data from tests on a model steel water tank containing an ice cap and has been used to develop a method for prediction of the hoop stress in the wall of a tank containing an ice cap. Hoop stresses large enough to crack the wall of a non-prestressed reinforced concrete tank may be induced by internal ice formations. It is recommended that internal ice formations be eliminated by suitable means such as providing insulation to the tank.

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