

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 13 (1988)

**Artikel:** Fracture mechanics analysis of fatigue in plate girders

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**DOI:** <https://doi.org/10.5169/seals-13020>

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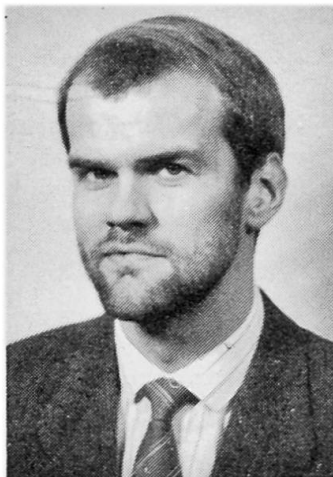
## Fracture Mechanics Analysis of Fatigue in Plate Girders

Analyse de durée de fatigue relative aux poutres en acier

Bruchmechanische Analyse der Ermüdung von Plattenträgern

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## SUMMARY

A theoretical study of the fatigue life of large scale plate girders subjected to variable amplitude loading is presented in this paper. Particular attention was paid to the remaining fatigue capacity after a surface crack had penetrated the plate material. Although the fatigue capacity subsequent to the penetration of the web plate is relative small it represents a valuable information for the planning of inspection and repair of the component.

## RÉSUMÉ

Ce compte rendu présente une étude théorique sur le comportement à la fatigue de poutres sous des charges variées. Une attention particulière est portée sur la capacité résiduelle de résistance à la fatigue après qu'une fissure en surface ait pénétré le matériau. Bien que cette période soit relativement courte, elle représente une information très importante pour la planification des contrôles et des réparations.

## ZUSAMMENFASSUNG

Dieser Bericht präsentiert eine theoretische Studie zur Ermittlung der Dauerschwingfestigkeit von Plattenträgern unter variierender Belastung. Besondere Beachtung wurde der zusätzlichen Lebensdauer geschenkt, die verbleibt, nachdem ein Oberflächenriß das Plattenmaterial durchdrungen hat. Wenn dieser Zeitraum auch relativ kurz ist, so stellt dies doch eine wertvolle Information zur Beurteilung von Inspektionsintervallen und Reparaturvorhaben dar.



## 1. INTRODUCTION

The overall purpose of this study was to investigate fatigue crack growth in large scale girders with special emphasis on the stage where a through crack approaches the girder flange. A general fracture mechanics approach was developed and particularly verified against test results for large-scale test girders of the type shown in Fig. 1.

## 2. FRACTURE MECHANICS ANALYSIS

### 2.1 Fracture mechanics approach to fatigue

With the existence of an initial flaw fracture mechanics methods can immediately be seen to offer a powerful and realistic means for analysing the fatigue strength of welded joints. It was shown by Irwin [1] that the elastic stresses in the region adjacent to a crack tip depend on the stress intensity factor  $K$ . The general form of  $K$  is

$$K = \sigma \sqrt{\pi a} \cdot F \quad (1)$$

where:  $\sigma$  = in-plane uniform stress applied remotely from the crack;  $a$  = half-crack length in the case of a through-crack or crack depth in the case of a surface crack;  $F$  = correction factor to account for the actual configuration of the body.

### 2.2 Fatigue crack propagation

An appropriate expression for the crack propagation rate is the increase of the crack length " $a$ " per load cycle, viz.  $da/dN$ . This rate again is dependent on the alternating stress intensity  $\Delta K$ .

An empirical relationship that adequately characterizes the crack growth rate in most types of welded steel materials has been suggested by Paris and Erdogan:

$$\frac{da}{dN} = C \cdot (\Delta K)^m \quad (2)$$

where  $C$  and  $m$  are generally taken as material constants.

### 2.3 Stress intensity factor

For a wide range of standard situations closed-form expressions for the stress intensity factor  $K$  exist. However, because of the complexities of most practical problems, exact solutions are not available. Two different approaches to estimate  $K$  have been applied in this work and will be presented in the following.

#### 2.3.1 Empirical solution

The first and most convenient approach is the use of an empirical equation for the stress intensity factor for a surface crack, presented by Newman and Raju [2]. This equation reads

$$K = (\sigma_t + H \cdot \sigma_b) \cdot \sqrt{\pi a/Q} \cdot F(a/t, a/c, c/b, \theta) \quad (3)$$

where:  $\sigma_t$ ,  $\sigma_b$  = remote tension and bending stresses respectively;  $H$  = function, dependent on  $a/t$ ,  $a/c$ ,  $\theta$ ;  $Q$  = shape factor for elliptical crack;  $F$  = boundary-correction factor;  $t$  = plate thickness;  $c$  = half length of surface crack;  $b$  = half-width of cracked plate;  $\theta$  = parametric angle of the ellipse.

#### 2.3.2 Hybrid method

The stress distribution and hence the value of the stress intensity factor at the tip of a crack in a structure or component will be influenced by edges, surfaces etc., called "boundaries" in the following. The present method may be

used to estimate the value of the stress intensity factor for a crack in a structure containing several boundaries by separating a complex configuration into a number of simpler auxiliary configurations each containing the crack and a single boundary. For this specific boundary solutions are available (e.g. [3] and [4]) for two- or three-dimensional crack problems in finite sheets and bodies. Many of these solutions define the stress intensity factor as

$$K = F \cdot \sigma \cdot \sqrt{\pi a} \quad (4)$$

with  $F$  being a correction factor for this configuration. This simplified, cost-effective technique is described more comprehensively in [5]. The total correction factor for the detail under consideration is the product of the single correction factors for the auxiliary configurations.

### 3. STRESS ANALYSES

It is obvious that the knowledge of the state of stress in the vicinity of the potential crack site is a prerequisite to any fatigue life calculation. At the Marine Structures Laboratory of the Norwegian Institute of Technology in Trondheim large-scale structural models have been tested under fatigue loadings. The type of specimen that is analyzed in the present work is a welded, built-up plate girder with transverse stiffeners on one side (cfr. Fig. 1). Weld-induced misalignments were present in the entire structure. Stress analyses that were conducted by means of the Finite Element Method provided the stresses in the structure in the length and width dimension as well as in the thickness direction at the end of one specific stiffener.

### 4. FATIGUE CRACK GROWTH CALCULATIONS

#### 4.1 Computation of fatigue life

The fatigue crack growth calculations were based on a computer program "LIFE" [5]. A modified version of "LIFE" allows for the fatigue calculation of the actual structural detail, the welded plate girder.

In an I-girder the extension of a crack due to cyclic loading takes place in four stages:

- From an initial defect to the penetration of the girder web. (Stage 1)
- Along the stiffener-to-web weld to the tension flange. (Stage 2)
- Through the flange until this is penetrated. (Stage 3)
- As a through crack towards the ends of the flange until the total remaining intact cross section of the flange fails due to yielding. (Stage 4)

#### 4.1.1 Surface crack in the web (Stage 1)

The early crack growth was calculated by means of Eq. 3. In the actual case of a plate girder the through-thickness stress distributions were far from linear. Hence, Newman-Raju's equation was corrected with regard to stress gradients.

The number of load cycles from an initial crack depth " $a_i$ " to a certain depth " $a$ " is according to Eq. 2

$$N = \frac{1}{C} \cdot \int_{a_i}^a \frac{da}{(\Delta K)^m} \quad (5)$$

The Stage 1 life spans calculated lay well within the  $\mu$  and  $\mu+2$  scatterband in the S-N approach.

#### 4.1.2 Through crack in the web (Stage 2)

After the crack had penetrated the web plate thickness, it propagated both



upwards and downwards along the stiffener weld. The further crack growth was determined by means of the hybrid method mentioned earlier. Here the global membrane stress distribution governed the growth. The magnitude of stresses and consequently the stress intensity factors were different at the two tips of the crack. The correction factors that account for the actual configuration of the body allowed for the effect of non-uniform stresses and the presence of the girder flange ahead of one of the crack tips.

In this second stage of crack growth the growth rate was different at the two crack tips. Hence the crack extension was controlled alternating at the two tips with the crack length "a" increasing with small increments.

#### 4.1.3 Crack within the flange

The further considerations apply for the lower crack tip only. It was this tip that was decisive for the fatigue life of the detail.

Fisher [6] examined a fatigue crack failure in a steel bridge where the cracked detail was very similar to the actual plate girder. He based his calculations on the assumption that the crack extended into the flange as a semi-circular crack, the centre being the point of crack initiation. Both the welds and the embraced air split between the plates were disregarded. This idealization was adapted in the present investigation since Fisher's results were found to be in good agreement with the actual field behaviour of those details. Several correction factors account for the different boundaries. Once the crack had penetrated the flange the further crack growth in a plain steel plate (the flange plate) could be considered to occur under uniform tension load.

#### 4.2 Comparison between theoretical and experimental results

The validity of the fracture mechanics model was checked by comparing the computed fatigue lives with a-N-results from the laboratory tests. The subjects of this comparison were two quite similar plate girders with web plate thicknesses  $t=20$  mm and  $t=40$  mm.

A regression analysis concerning the crack shape development ( $a/2c$ ) had been performed. This analysis was based on crack surface recordings from the laboratory tests. The resulting regression lines were different for the two web plate thicknesses.

As for Stage 1 of crack growth, the crack being a surface crack, the a-N calculations agreed well with the experimental data when initial crack depths in the range from 0.1 to 0.2 mm were assumed.

For Stage 2, the approach of the lower crack front to the girder flange is demonstrated in Fig. 2. As the crack front in the real case generally does not proceed as a straight line two curves for the maximum and minimum distance from the flange are given. In the case of Fig. 2 the crack did not reach the flange in the course of the tests. Therefore the curves from the test results cease at a distance from the flange larger than zero.

Laboratory test results for the final two stages were not available since those cracks which had extended into the flange had been repair welded.

The calculated relative fatigue lives for the respective stages turned out to be the same for both of the web plate thicknesses:

Stage 1:	92 %
Stage 2:	6 %
Stage 3:	2 %

#### 4.3 Sensitivity studies

In the following the sensitivity of the fatigue life to changes in the most important quantities will be demonstrated.

The difference in Stage 1 fatigue life for the two plate thicknesses corresponds approximately to a factor 2.0, the thinner plate having the longer life.

Increasing the assumed initial crack depth within a reasonable range by a factor of 3.0 causes a decrease in fatigue life by 26 % for the 20 mm case and 20 % for the 40 mm case.

The dependence of the shape of the crack front ( $a/2c$ ) on the crack depth had been expressed in terms of results from several regression analyses. The conclusions with respect to these circumstances are twofold. Firstly, a crack growth law obtained from a thin plate analysis should not be applied to larger thicknesses, and secondly the assumption of a crack growth law from a large plate regression analysis may cause an error, but this error is in the conservative direction.

With a stress gradient as in the plate girder the main crack will not initiate very far from the end of the stiffener since the nominal stress along the stiffener is highest at this place. It is obvious that, the shorter the stiffener, the lower the stress field at its end and the longer the fatigue life in Stage 1 and, not least in Stage 2. A thorough evaluation of the two phenomena buckling and fatigue may lead to an optimum solution of this problem. A possible gain in endurance of several 10 % for a shorter stiffener demonstrates that this can be as advantageous as e.g. time consuming post-weld improvement methods.

In order to find the significance of the weld toe radius this parameter has been altered within the range from 0.67 to 2.0 mm. The difference between the lowest and the highest of the calculated endurances is 10 %, what is not very much seen in relation to other influencing parameters. An expensive weld surface treatment to increase the toe radius, however, would also reduce the size of a present initial surface defect, or even remove this totally. Thus, a twofold improvement of the fatigue capacity might be achieved by grinding, shot peening etc.

In design the statistical nature of fatigue behaviour must be taken care of by applying a crack growth parameter  $C$  that provides 97.7 % probability of survival, what signifies that the  $C$ -value from the upper bound 95 % confidence limit has to be applied. In the course of the calculations it turned out that the endurance estimated on the basis of the 95 % confidence limit is only about 35 % of the endurance based on the mean  $C$  value.

## 5. CONCLUSIONS

A fracture mechanics approach has been developed and used in the analysis of plate girders which have been tested in the Marine Structures Laboratory of the Norwegian Institute of Technology. The trend in crack growth is well predicted. However, the endurance in the first stage from an initial defect to a through-thickness crack is uncertain because of the uncertain size of the initial defect. The fracture mechanics approach is particularly useful in tracing the later stages in crack growth which are not covered by S-N data and which are useful in planning inspection and repair [7].

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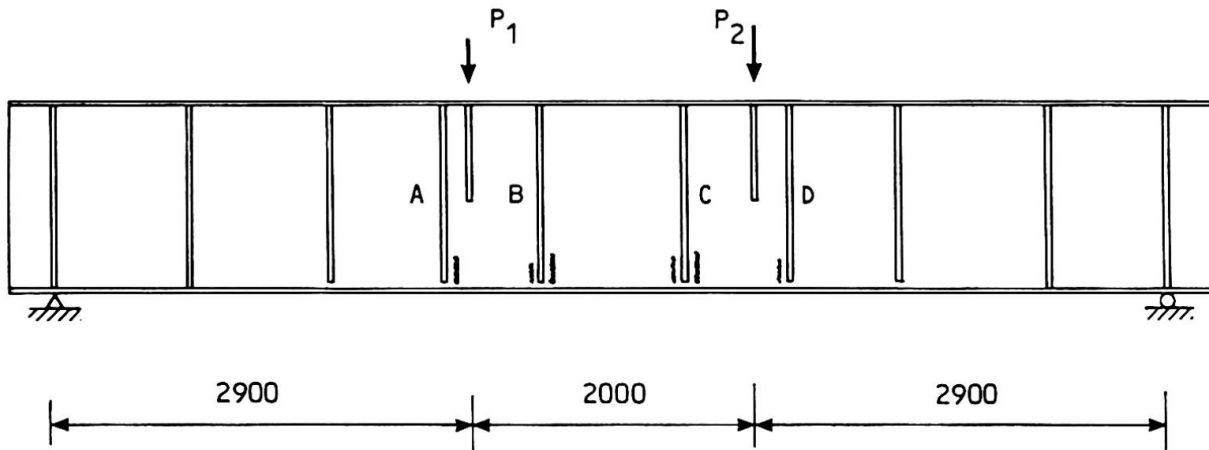


Fig. 1 Plate girder; fatigue cracking at lower ends of stiffeners A-D.

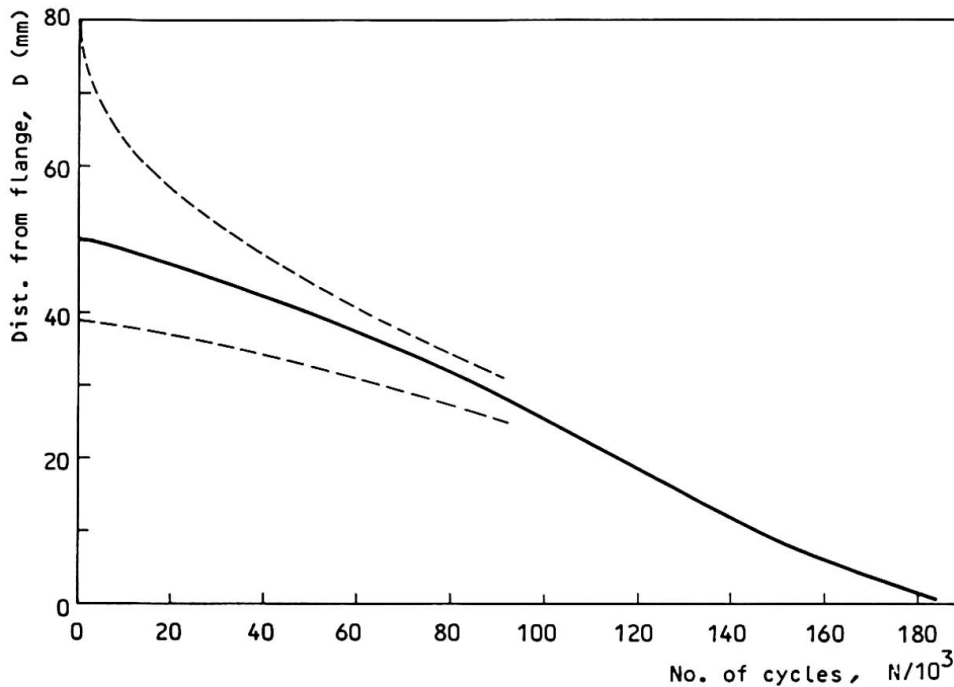


Fig. 2 Crack growth in Stage 2  
 Solid line: Calculated values  
 Dashed lines: Measured values for  $D_{max}$  and  $D_{min}$ .