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Dynamic Analysis of Layered Beams by Exact Finite Element Analysis

Analyse dynamique de poutres à lamelles par la méthode des éléments finis

Dynamische Analyse von geschichteten Stäben anhand der exakten Finite-Elemente-Methode

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1.THEORY

The general dynamic-stiffness matrix and the calculation procedure of exact shape-functions and natural frequencies of elastic layered beams for transverse undamped free vibrations have been presented in the previous papers [1], [2] and [3]. The method is exact for beams with two faces and one core or three faces and two cores if the cross-section of the beam is symmetrical. If there are more faces then this method can be used approximatively [3]. Neglecting damping, rotational inertia and shear deformations of the faces, the differential equation for the lateral vibration is [3]

$$EI_oEI_sv^{(6)} - (EIk + NEI_s)v^{(4)} + (Nk + cEI_s)v^{(2)} - ckv$$
$$+\mu EI_s\ddot{v}^{(2)} - \mu k\ddot{v} = EI_sp^{(2)} - kp, \quad (1)$$

where EI_o is the sum of bending stiffnesses of the faces, EI_s is the Steiner-term, $EI = EI_o + EI_s$, k is the shear stiffness of the beam, N is the axial force, c is the modulus of Winkler-type foundation, μ is the mass/unit lenght of the beam, $p = p(x,t)$ is the lateral loading intensity of the beam, $v = v(x,t)$ is the lateral displacement of the beam, $v' = \partial v/\partial x$, $\dot{v} = \partial v/\partial t$, where x is the beam axis and t is time.

The equation is solved by superposing the exact shape-functions $\phi_i(x)$ (see Fig. 1.) of undamped free vibrations (the functions $\phi_i(x)$ satisfied the governing differential equation and the corresponding boundary conditions)

$$v(x,t) = \sum_{i=1}^{\infty} \phi_i(x)Y_i(t). \quad (2)$$

The homogeneous boundary conditions can be presented in the form [3]

$$/_0^L(Q\phi - M\varphi - M_o\gamma) = 0, \quad (3)$$

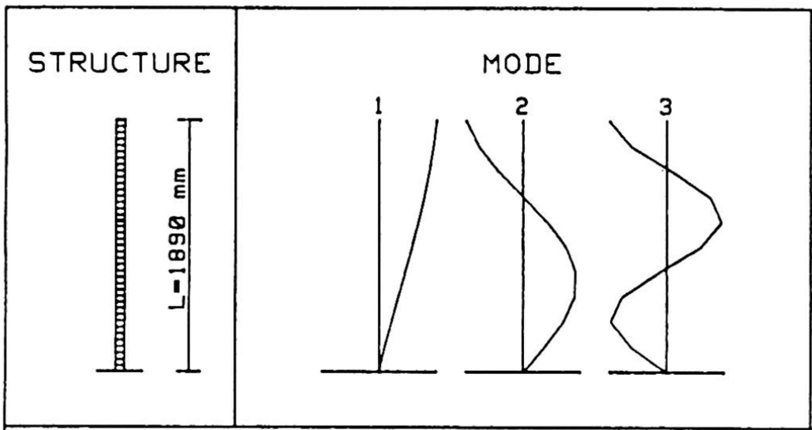
where Q is the total shear force of the beam, M is the total bending moment, M_o is the sum of the bending moments of faces, γ is the rotation angle due to slip of the core and $\varphi = \phi' - \gamma$. The equations for these variables can be found in the reference [3]. Substituting the trial (2) into the equation (1), multiplying the equation with a test function ψ_j and integrating over the lenght L of the beam leads us to the equations

$$\sum_{i=1}^{\infty} M_{ij}[\ddot{Y}_i(t) + \omega_i^2 Y(t)] = P_j(t), \quad (4)$$

$$M_{ij} = \int_0^L (\mu EI_s\phi_i^{(2)} - \mu k\phi_i)\psi_j dx, \tag{5}$$

$$P_j(t) = \int_0^L (-kp + EI_s p^{(2)})\psi_j dx. \tag{6}$$

The functions $\ddot{Y}_i(t) + \omega_i^2 Y_i(t)$ can be solved from the equation (4) as soon as the test functions ψ_j are chosen and then finally the functions $Y_i(t)$ can be calculated from Duhamel's integral. The functions ϕ_j are used as the test functions and then all the integrations in the equations above can be performed analytically.



FREQUENCY [Hz]

MODE	THIN FACES	THICK FACES	EXPER. [4]
1	12.75	12.87	12.80
2	39.53	40.15	42.00
3	71.44	72.66	-

N=0 and c=0: EIs = 1.418E+01 kNm^2 μ = 5.620E-01 kg/m
EIo = 3.646E-03 kNm^2 k = 6.944E+00 kN

Fig. 1. Exact shape functions for a shear wall model

2.EXAMPLES

The numerical examples concerning the vibrations of wooden leight weight units and shear walls are presented in the Poster. Some test results have been calculated by using the approximative finite elements (ABAQUS). Multi-point constrains are used in these calculations, so that the results can be compared with the results calculated by using the theory of layered beams. The more accurate modelling of the structures considered is also performed so that the error due to the basic assumptions of the theory of layered beams can be seen.

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