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### Monte Carlo Study of Strength of RC Slender Columns

Calcul de la résistance de colonnes élancées en béton armé à l'aide de la méthode de Monte Carlo

Berechnung der Tragfähigkeit schlanker Stahlbetonstützen mit der Monte Carlo Methode

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#### SUMMARY

The actual strength of a reinforced concrete member varies from the nominal strength which is based on specified strengths of constituent materials, geometric properties, and code design equations. The variability in the actual strength is caused by the variations in the strengths of concrete and steel, the cross section dimensions, the reinforcement placement, and the strength model itself among other factors. Computing the strength variability is an essential component in development of probability-based safety provisions for reinforced concrete design. This study was undertaken to investigate the variability of short-time ultimate strength of slender tied reinforced concrete columns of rectangular shape in cast-in-place construction. The columns studied were pin-ended with equal load eccentricities acting at both ends. The material strengths and geometric properties of the column were varied randomly and the resulting variations in the column ultimate strength were determined. The results of this study indicate that the slenderness ratio, the longitudinal steel ratio, and the end eccentricity ratio significantly influence the probability distribution properties of the column strength.

#### 1. DESCRIPTION OF SIMULATED COLUMNS

Eighteen hypothetical rectangular tied columns subjected to single curvature bending with equal moments acting at both ends were used in this study. The graphical representation of the columns studied is shown in Fig. 1(c). Each column employed a different combination of longitudinal reinforcement ratio, specified concrete strength, and slenderness ratio ( $\ell/h$ ). All columns had the specified steel yield strength of 400 MPa, cross section size of 300 x 300 mm, concrete cover to tie reinforcement of 40 mm, and tie diameter of 10 mm. Each column was studied for end eccentricity ratios ( $e/h$ ) of 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5, and infinity (pure bending).

#### 2. THEORETICAL STRENGTH MODEL

The theoretical resistance was calculated by using a strength model that generated the moment-curvature curves for different levels of axial load acting on a slender column. The maximum moment from the slender column moment-curvature curve for a given axial load level defined one point on the axial force-moment interaction diagram. This was repeated until sufficient points were obtained to define the entire interaction diagram for the slender column. The slender column resistances were then computed for specified end eccentricities through a curve-fitting subroutine applied to the generated points on the interaction diagram.

### 3. SIMULATION AND ANALYSIS OF DATA

Using the theoretical strength model and the probability distributions of the influencing variables described by Mirza et al. [2, 3, 4], the ultimate strengths ( $R$ ) were simulated 250 times by a Monte Carlo technique for each of the end eccentricity ratios for each of the slender columns studied. The simulated strengths were then divided by the corresponding ultimate strength ( $R_n$ ) predicted by the ACI Building Code [1] using the specified properties of the variables. This gave simulated samples of the ratios of the theoretical to ACI strengths ( $R/R_n$ ) referred to as strength ratios in this study. All computations for the ACI strength were carried out as they would be in a design office with the exception of the strength reduction factors  $\phi$ . The  $\phi$  factors for the cross section strength and those for the critical buckling strength of the column were taken equal to 1.0 in this study. The results of this study indicate that the slenderness ratio, the longitudinal steel ratio, and the end eccentricity ratio significantly influence the probability distribution properties of the column strength. As expected, the variability of concrete strength is a major contributing factor to the slender column strength variability in the region of low eccentricity ratios, whereas the variability in the steel strength makes a major contribution to the slender column strength variability when the end eccentricity ratios are high. The highlights of the results obtained are summarized in Figs. 1(a), (b), and (c) which plot the range of mean and one percentile strength ratios ( $R/R_n$ ) at different values of  $e/h$  for columns with slenderness ratio of 10, 20, and 30, respectively. Each of these figures represents six columns with identical slenderness ratios but different specified concrete strengths and longitudinal steel ratios. Note that each column was studied for twelve  $e/h$  ratios.

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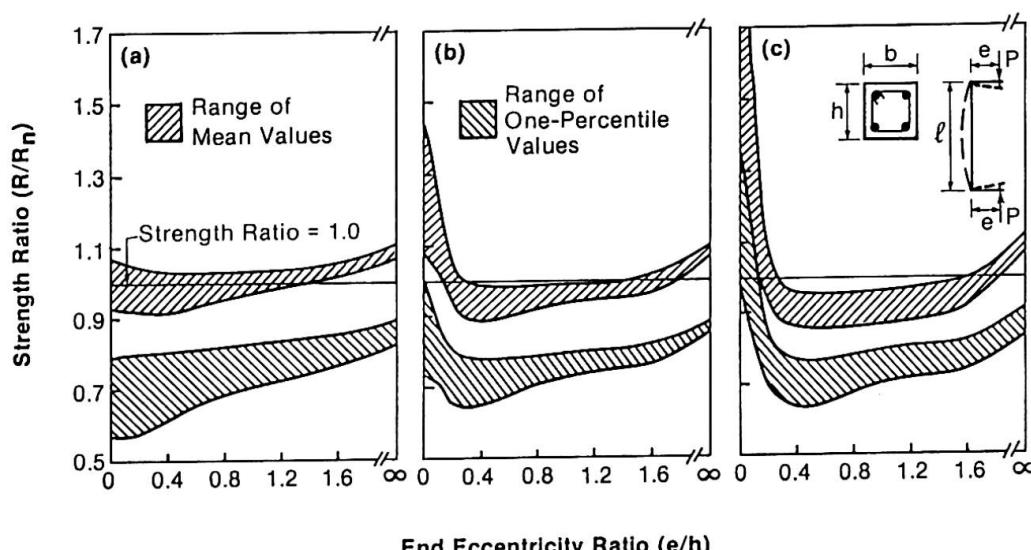


Fig. 1. Effect of end eccentricity ratio on the range of mean and one-percentile strength ratios: (a)  $\ell/h = 10$ ; (b)  $\ell/h = 20$ ; and (c)  $\ell/h = 30$ .