

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 13 (1988)

**Artikel:** Monte Carlo study of strength of RC slender columns

**Autor:** Mirza, S.A.

**DOI:** <https://doi.org/10.5169/seals-13147>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 09.08.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**



## Monte Carlo Study of Strength of RC Slender Columns

Calcul de la résistance de colonnes élancées en béton  
armé à l'aide de la méthode de Monte Carlo

Berechnung der Tragfähigkeit schlanker Stahlbetonstützen mit der Monte Carlo Methode

**S.A. MIRZA**

Prof. of Civil Eng.  
Lakehead University  
Thunder Bay, ON Canada

### SUMMARY

The actual strength of a reinforced concrete member varies from the nominal strength which is based on specified strengths of constituent materials, geometric properties, and code design equations. The variability in the actual strength is caused by the variations in the strengths of concrete and steel, the cross section dimensions, the reinforcement placement, and the strength model itself among other factors. Computing the strength variability is an essential component in development of probability-based safety provisions for reinforced concrete design. This study was undertaken to investigate the variability of short-time ultimate strength of slender tied reinforced concrete columns of rectangular shape in cast-in-place construction. The columns studied were pin-ended with equal load eccentricities acting at both ends. The material strengths and geometric properties of the column were varied randomly and the resulting variations in the column ultimate strength were determined. The results of this study indicate that the slenderness ratio, the longitudinal steel ratio, and the end eccentricity ratio significantly influence the probability distribution properties of the column strength.

### 1. DESCRIPTION OF SIMULATED COLUMNS

Eighteen hypothetical rectangular tied columns subjected to single curvature bending with equal moments acting at both ends were used in this study. The graphical representation of the columns studied is shown in Fig. 1(c). Each column employed a different combination of longitudinal reinforcement ratio, specified concrete strength, and slenderness ratio ( $l/h$ ). All columns had the specified steel yield strength of 400 MPa, cross section size of 300 x 300 mm, concrete cover to tie reinforcement of 40 mm, and tie diameter of 10 mm. Each column was studied for end eccentricity ratios ( $e/h$ ) of 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5, and infinity (pure bending).

### 2. THEORETICAL STRENGTH MODEL

The theoretical resistance was calculated by using a strength model that generated the moment-curvature curves for different levels of axial load acting on a slender column. The maximum moment from the slender column moment-curvature curve for a given axial load level defined one point on the axial force-moment interaction diagram. This was repeated until sufficient points were obtained to define the entire interaction diagram for the slender column. The slender column resistances were then computed for specified end eccentricities through a curve-fitting subroutine applied to the generated points on the interaction diagram.

### 3. SIMULATION AND ANALYSIS OF DATA

Using the theoretical strength model and the probability distributions of the influencing variables described by Mirza et al. [2, 3, 4], the ultimate strengths ( $R$ ) were simulated 250 times by a Monte Carlo technique for each of the end eccentricity ratios for each of the slender columns studied. The simulated strengths were then divided by the corresponding ultimate strength ( $R_n$ ) predicted by the ACI Building Code [1] using the specified properties of the variables. This gave simulated samples of the ratios of the theoretical to ACI strengths ( $R/R_n$ ) referred to as strength ratios in this study. All computations for the ACI strength were carried out as they would be in a design office with the exception of the strength reduction factors  $\phi$ . The  $\phi$  factors for the cross section strength and those for the critical buckling strength of the column were taken equal to 1.0 in this study. The results of this study indicate that the slenderness ratio, the longitudinal steel ratio, and the end eccentricity ratio significantly influence the probability distribution properties of the column strength. As expected, the variability of concrete strength is a major contributing factor to the slender column strength variability in the region of low eccentricity ratios, whereas the variability in the steel strength makes a major contribution to the slender column strength variability when the end eccentricity ratios are high. The highlights of the results obtained are summarized in Figs. 1(a), (b), and (c) which plot the range of mean and one percentile strength ratios ( $R/R_n$ ) at different values of  $e/h$  for columns with slenderness ratio of 10, 20, and 30, respectively. Each of these figures represents six columns with identical slenderness ratios but different specified concrete strengths and longitudinal steel ratios. Note that each column was studied for twelve  $e/h$  ratios.

### REFERENCES

1. Building Code Requirements for Reinforced Concrete. ACI 318-83, American Concrete Institute, Detroit, Michigan, 1983.
2. MIRZA S.A., HATZINIKOLAS M. and MACGREGOR J. G., Statistical Descriptions of Strength of Concrete. ASCE Journal of the Structural Division, June 1979.
3. MIRZA S.A. and MACGREGOR J.G., Variability of Mechanical Properties of Reinforcing Bars. ASCE Journal of the Structural Division, May 1979.
4. MIRZA, S.A. and MACGREGOR J.G., Variations in Dimensions of Reinforced Concrete Members. ASCE Journal of the Structural Division, April 1979.

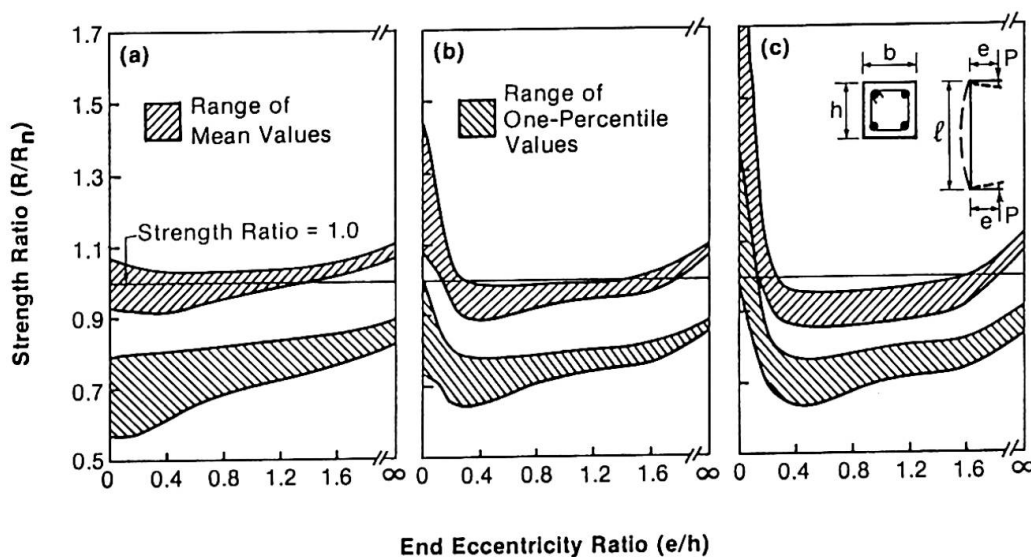


Fig. 1. Effect of end eccentricity ratio on the range of mean and one-percentile strength ratios: (a)  $l/h = 10$ ; (b)  $l/h = 20$ ; and (c)  $l/h = 30$ .