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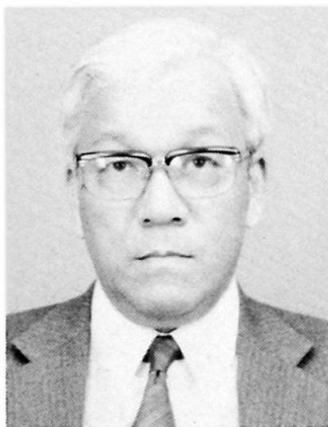
Optimum Design of Framed Structures using a Personal Computer

Projet optimal de charpentes à l'aide d'un ordinateur personnel

Optimaler Entwurf von Rahmentragwerken mit Hilfe eines "Personal-Computers"

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SUMMARY

Since the number of design variables and constraints increases for large-span bridges the numerical calculation for the optimum design becomes difficult. It is possible to carry out an optimum design of a complicated framed structure, using a personal computer, by the suboptimization method. The optimum design of steel highway bridges of statically indeterminate truss form has been carried out with sufficient accuracy.

RESUME

Pour la construction de ponts de grande portée, il est difficile d'en établir le projet optimal à cause de la multiplication des variables et des conditions de limite, et de la complication des calculs numériques. L'optimisation d'un projet de structure compliquée est cependant possible en utilisant la méthode de sub-optimisation, à l'aide d'un ordinateur personnel. Le projet optimal de ponts-routes en treillis hyperstatique démontre l'efficacité et la précision suffisante de cette méthode.

ZUSAMMENFASSUNG

Beim Entwurf von Brücken mit grossen Spannweiten erhöht sich die Anzahl der Variablen und Randbedingungen. Die numerische Erfassung des optimalen Entwurfes wird schwierig. Eine Optimalisierung mit Hilfe eines Personal-Computers ermöglicht auch für komplizierte Rahmentragwerke einen bestmöglichen Entwurf. Die Entwurfsmethode wird am Beispiel von Autobahnbrücken aus Stahl, bestehend aus statisch unbestimmten Fachwerken, dargestellt und zeigt eine Anwendbarkeit für die Ausarbeitung des tatsächlichen Entwurfs.



1. INTRODUCTION

Prof. Fenves emphasized the main point of the theme in his introductory report to be that the emergence of powerful personal computers, vastly expanded computer graphics, widely accessible distributed databases, microprocessor-controlled intelligent construction equipment, and knowledge-based expert systems will all drastically change structural engineering design and construction practices, and even the nature of the structures we design, build and operate.¹⁾ Based on this point, we are submitting the present paper. In this study, we are successful in generating an optimum design of a complicated structure using a personal computer.

In general, for large-span bridges, the number of design variables and constraints increases, and the numerical calculation for their optimum design has become difficult without using a large electronic computer. It is possible to carry out an optimum design of a complicated structure, using a personal computer, by suboptimization method. In a statically determinate structure, when only governed by stress, the correct optimum values of the design variables are obtained. In a statically indeterminate structure and a statically determinate structure governed by deflection, the approximate values of the design variables are obtained. In this case, it is possible to obtain comparatively more accurate values using a fully-stressed design method, and an optimality-criterion-based algorithm, respectively.

For the structural analysis a personal computer has a limited memory capacity. Therefore, in the analysis of a statically indeterminate structure, the method of forces or the method of deformations, is used depending upon the structural type. These method, which are used for the analyses of a wide variety of structures, are nearly as satisfactory for obtaining rigorous results as the finite-element-method. Additionally, the results of the optimum design will make it possible to carry out the automated design with an XY plotter.

A personal computer, Nihon Denki model PC-9800 with 640k byte memory, was used for this investigation. The FORTRAN Language with MS-DOS control was used for the program, where MS is a trademark of Microsoft Corporation and Dos means Disk Operating System.

2. METHOD OF OPTIMUM DESIGN USING SUBOPTIMIZATION

This method is mentioned in detail by the authors of this paper in ref. 2). A method of optimum design using suboptimization is outlined as follows: The design variables are divided into two groups, the variables \mathbf{x} of individual sections of elements and the variables \mathbf{y} of sections common to the whole structure.

In general, the constraints are given by:

$$\begin{aligned} g_{iI}(\mathbf{x}_I, \mathbf{x}_{II}, \dots, \mathbf{x}_N, \mathbf{y}) &\leq 0 \quad (i_I = 1, \dots, m_I), \\ g_{iII}(\mathbf{x}_I, \mathbf{x}_{II}, \dots, \mathbf{x}_N, \mathbf{y}) &\leq 0 \quad (i_{II} = 1, \dots, m_{II}), \quad \dots \dots \dots (1) \\ &\dots \dots \dots \end{aligned}$$

$$g_{iN}(\mathbf{x}_I, \mathbf{x}_{II}, \dots, \mathbf{x}_N, \mathbf{y}) \leq 0 \quad (i_N = 1, \dots, m_N),$$

The expressions of functions of only \mathbf{y} are given by

$$g_i(\mathbf{y}) \leq 0 \quad (i = 1, \dots, m) \quad \dots \dots \dots (2)$$

where, the variables \mathbf{x} belonging to I, II, ... and N are called \mathbf{x}_I , \mathbf{x}_{II} , ... and \mathbf{x}_N , respectively, and the constraints belonging to I, II, ... and N are given by g_{iI} , g_{iII} , ... and g_{iN} , and N analytical member elements of a structure are by I, II, ... and N. The expression of an objective function is given by

$$z = f(\mathbf{x}_I, \mathbf{x}_{II}, \dots, \mathbf{x}_N, \mathbf{y}) \quad \text{minimize.} \quad \dots \dots \dots (3)$$

Then for the objective function and constraints, the groups of the variables \mathbf{x}_I , \mathbf{x}_{II} , ... and \mathbf{x}_N belonging to the element I, II, ... and N, respectively, are

independent of each other. In the case of a statically determinate structure, the change of its sectional dimensions does not have an effect on the distribution of the internal stresses. Then, for this case, too, as to the objective function and constraints, the groups of the variables x_I, x_{II}, \dots and x_N are independent of each other. Therefore, the constraints are given by

$$\begin{aligned} g_{iI}(x_I, y) &\leq 0 \quad (i_I = 1, \dots, m_I), \\ g_{iII}(x_{II}, y) &\leq 0 \quad (i_{II} = 1, \dots, m_{II}), \\ &\dots \dots \dots \end{aligned} \quad \dots \dots \dots (4)$$

$$\begin{aligned} g_{iN}(x_N, y) &\leq 0 \quad (i_N = 1, \dots, m_N), \\ g(y) &\leq 0. \end{aligned} \quad \dots \dots \dots (5)$$

and the objective function z is expressed by

$$z = z_I + z_{II} + \dots + z_N = f_I(x_I, y) + f_{II}(x_{II}, y) + \dots + f_N(x_N, y) \quad \text{minimize,} \dots \dots (6)$$

where

$$\begin{aligned} Z_I &= f_I(x_I, y) \\ Z_{II} &= f_{II}(x_{II}, y), \\ &\dots \dots \dots \\ Z_N &= f_N(x_N, y). \end{aligned} \quad \dots \dots \dots (6)a$$

Show the objective functions of the element I, II, ... and N, respectively.

In case of a statically indeterminate structure, Eqs.(4) and (6)a can be approximately applicable. Then, the following relations are obtained the optimum values of x for any constant value of y , for

$$\begin{aligned} x_I &= h_I(y), \\ x_{II} &= h_{II}(y), \\ &\dots \dots \dots \\ x_N &= h_N(y). \end{aligned} \quad \dots \dots \dots (7)$$

Then, by substituting Eqs.(7) into Eq.(6) the following expression is obtained:

$$z = f_I\{h_I(y), y\} + f_{II}\{h_{II}(y), y\} + \dots + f_N\{h_N(y), y\} \quad \text{minimize} \dots \dots (8)$$

The constraints will be reduced to only eq.(5) and $y < 0 \dots (9)$. It is then necessary to obtain the optimum solutions of the variables y by minimizing Eq.(8) under the constraints of Eqs.(5) and (9).

3. EXAMPLE OF OPTIMUM DESIGN

3.1 Example I : Two-panel two-span continuous truss

As an example I of an optimum design using a personal computer by the proposed method, the optimum design of a two-panel two-span continuous truss, as shown in Fig.1, is carried out. In this figure, ①, ②, ③ and ④ show the number of members, and (1), (2), (3) and (4) show the classification of the member sections. In this case, loads of $P=6000$ tons are applied to the panel point of the upper chord.

3.1.1 Design variables

Among design variables the grade of material S the upper flange plate and web plate thicknesses T_u, T_w , respectively, of the classification of the sections (1) and (2), the web plate thickness T_w of classification (3), and the flange and web plate thickness T_u, T_w , respectively, and the flange width B_u of the classification (4), are related to x , and the truss height H , the truss chord width BF and the truss upper chord height BW are related to y . Concerning S , the steel of 41 kg/mm^2 in tensile strength is expressed by 4, the steel of 50 kg/mm^2 is expressed by 5 and 58 kg/mm^2 by 6.

3.1.2 Constraints

In the case of the suboptimization, the constraints contain the limit of stress, the limits of values of the design variables at the limit of ratio of plate width to thickness below 80, the limit of the slenderness ratio of $l/r \leq l/r_x$ where r_x and r_y are the radii of gyration about the x and y axes



respectively, and $1/r \leq 200$ at the section (1); the limit of ratio of plate width to thickness for prevention from local buckling, the limit of slenderness ratio of $1/r \leq 1/r_x$, and $1/r \leq 120$ at the section (2); the limit of limit of slenderness ratio of $1/r \leq 200$ at the section (3); the limit of ratio of plate width to thickness for prevention of local buckling and $1/r \leq 120$. In case of overall optimization, the constraints are only the upper and lower limits of the variables H , BF and BW .

The allowable stresses are a function of the discrete variable S , but it is considered to be a continuous as shown in the following equations:

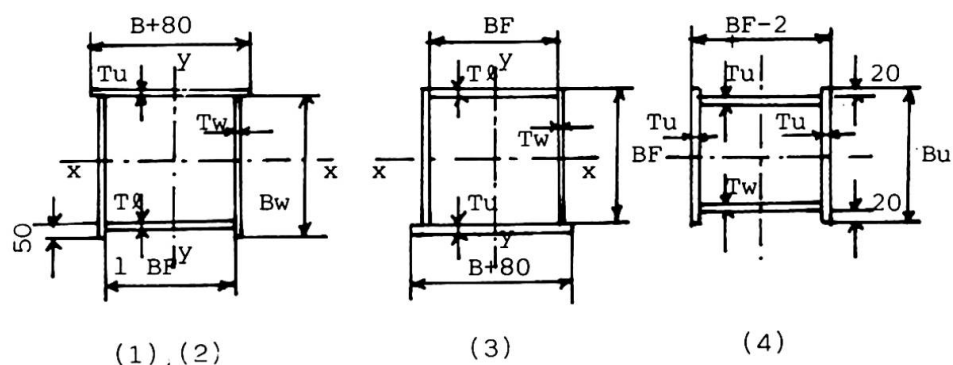


Fig.1 Notations of 2-span continuous truss (unit:mm)

$$\begin{aligned} FG1 &= 4S^2 - 41S + 120, \\ FG2 &= -13S + 145, \\ \sigma_{ua2} &= -2.2S^2 - 15.2S + 34, \\ \sigma_{ua3} &= 100S^2 - 2600S + 15500, \end{aligned} \quad \dots\dots\dots(10)$$

and

$$\sigma_{ta} = 100S^2 - 400S + 1400.$$

The compressive allowable stress without considering local buckling, σ_{cag} , is obtained from the following equations:

$$\begin{aligned} l/r \leq FG1: \sigma_{cag} &= \sigma_{ta}, \\ FG1 < l/r < FG2: \sigma_{cag} &= \sigma_{ta} - \sigma_{ua2}(l/r - FG1), \\ FG2 < l/r: \sigma_{cag} &= 12000000 / \{ \sigma_{ua3} + (l/r)^2 \} \end{aligned} \quad \dots\dots\dots(11)$$

The compressive allowable stress against local buckling, σ_{cal} , is obtained from the following equations:

$$\gamma_d = 0.35S^2 - 8.75S + 69.0 \quad \dots\dots\dots(12)$$

$$C_d < \gamma_d: \sigma_{cal} = \sigma_{ta} \quad \dots\dots\dots(13)$$

$$C_d \geq \gamma_d: \sigma_{cal} = 2200000(1/C)^2$$

where

$$C_d = B/T$$

B : the width of a plate,

T : the thickness of a plate.

Then, the compressive allowable stress σ_{ca} is obtained from the following equation:

$$\sigma_{ca} = \sigma_{cag} \times \sigma_{cal} / \sigma_{ta} \quad \dots\dots\dots(14)$$

The ratio, γ , of plate width to thickness for prevention of buckling of the compression members is expressed as a function of S as follows:

$$\gamma = -6S + 64.0 \quad \dots\dots\dots(15)$$

A limit of the ratio of plate width to thickness is set up for the tension members to be 80.

3.1.3 Objective function

The objective function Z consists of material and fabrication costs, and is expressed by

$$Z = \sum_{k=1}^K \tilde{H}_{kl} (SMH) + \sum_{i,j} H_{ij} (SMH) + \sum_j \rho V_j \cdot C \cdot (CM) = Z_1 (SMH) + Z_2 (SMH) + Z_3 (CM) \\ = (CM) \times (Z_1 \mu + Z_2 \mu + Z_3) \quad \dots\dots\dots(16)$$

where

- ρ : the unit weight of steel material,
- C : the coefficient for unit cost of the steel material,
- (CM) : the unit cost of the steel material,
- (SMH) : the unit cost for one man hour,
- H_{ij} : the man hour of the i -th manufacturing operation of the j -th element which is a function of the design variables.
- \tilde{H}_{kl} : the man hour of the k -th manufacturing operation of the l -th element which is a fixed value,
- $\mu = (SMH)/(CM)$.

C is considered a function of T (thickness of plate) and S , and is expressed in the form of an equation with C_1 and C_2 , where C_1 is a function of S , and according to the "Prime Costs of Steel Highway Bridges in Japan in 1975",³⁾ is expressed as follows:

$$C_1 = 0.125S^2 - 0.955S + 2820, \quad \dots\dots\dots(17)$$

and C_2 which is a function of T is expressed as follows:

$$C_2 = 0.0348T^2 - 0.0845T + 1.2091. \quad \dots\dots\dots(18)$$

Then, the following expression is obtained:

$$C = C_1 \times C_2 \quad \dots\dots\dots(19)$$

Z_1, Z_2 and Z_3 are divided by CM to be dimensionless.

Then, it can be considered that only μ is related to the costs. H_{ij} is obtained from the "Prime Costs of Steel Highway Bridges in Japan in 1972"⁴⁾ which were determined by the method of squares from actual examples at bridge fabricating shops in Japan, and can be expressed as follows:

$$H_{1j} = 3W_t \cdot HA(S)/T, \\ H_{2j} = 0.055N_b \cdot HA(S), \\ H_{3j} = 3.5 \times \{1.0 + (0.01B - 0.5)\} \times HS(S), \\ \quad \quad \quad (if B is below 50cm, B is equal to 50cm), \quad \dots\dots\dots(20) \\ H_{4j} = 0.58L_1 \cdot HA(S), \\ H_{5j} = 0.58L_2 \cdot HA(S), \\ H_{6j} = 0.28A_\gamma \cdot HA(S),$$

where the suffix i of H_{ij} shows marking-off, hole boring, cutting of member edges, shop butt welding, shop fillet welding and shop painting, and W_t =the weight of members, N_b =the number of holes, B =the plate width, L_1 =the total welded length of butt welds equivalent to 6mm fillet size, L_2 =the same length as L_1 of fillet welds, A_γ =the surface area, $HA(S)$ =the coefficient for man hours depending on S , and can be expressed by

$$HA(S) = 0.085S^2 - 0.725S + 2.54 \quad \dots\dots\dots(21)$$

L_1 and L_2 assumed as a functions of T are calculated by the following equations.

$$L_1 = \bar{L}_1 \times \eta_1(T), \\ L_2 = \bar{L}_2 \times \eta_2(T), \\ \eta_1(T) = 1.2T^2 + 3.8T + 1.3, \\ \eta_2(T) = 0.0476T^2 + 0.1952T + 0.7572.$$

where, \bar{L}_1 =the total actual welded length of butt welds, \bar{L}_2 =the total actual welded length of butt welds.

N_b is calculated by the following equation:

$$N_b = A_g \times \sigma_a / \rho_s \quad \dots\dots\dots(24)$$

where

- A_g : the sectional area of a member,
- σ_a : the allowable stress,
- ρ_s : the yield strength of a high tensile bolt depending on S , which can



be calculated by the following equation:

$$\rho_s = -335S^2 + 3955S - 6580 \quad \dots\dots\dots(25)$$

\tilde{H}_{k1} is calculated by the following expression:

$$\tilde{H}_{k1} = 7.0 \times A_{\ell 2} + 56 + 35/NP, \quad \dots\dots\dots(26)$$

where

NP: the number of panels.

$$A_{\ell 2} = 2.5 + 0.25A, \quad \dots\dots\dots(27)$$

and

$$A_{\ell 1} = 0.5\ell - 15.0(\text{m}), \quad \dots\dots\dots(27)$$

$$A_{\ell 2} \leq 0, \quad \dots\dots\dots(27)$$

where

ℓ : the length of a member (m).

Then, the optimum values of the variables x are obtained with a fixed value of y for each element, by solving the objective function (16) under the above-mentioned constraints with the SLP method.

3.1.4 Overall optimization

Among the design variables, BF, BW and H are related to y . Since Eq.(7) which is obtained by suboptimization, satisfies all of the constraints except Eq.(5), it would be enough if only the constraints could satisfy Eq.(5), in overall optimization. The above-mentioned SUMT will be applied to the method of the overall optimization.

As S is a discrete variable, the integral value is obtained by the branch and prune method.

3.1.5 Results of calculation and discussions

The optimum values of the variables, penalty function and objective function are shown in Table 1, from the results of optimum design by the proposed method.

K	IS	Bf (cm)	Bw (cm)	H (cm)	F (1000yen)	Z (1000yen)	Rk
1	0	90.0	85.0	600.0	10457.0	7228.2	50.0
1	1	90.9	88.8	620.6	9816.5	7116.7	50.0
1	2	95.8	86.8	703.5	9404.0	6942.3	50.0
2	0	95.8	86.8	703.5	9597.3	6942.3	1.0
2	1	95.2	89.2	725.7	6961.4	6905.8	1.0
2	2	93.9	88.2	821.3	6899.4	6842.8	1.0
2	3	95.2	88.2	827.6	6896.4	6838.9	1.0
3	0	95.2	88.2	827.6	6901.7	6838.9	0.02
3	1	95.2	88.4	828.5	6839.9	6838.7	0.02
3	2	95.3	88.4	835.7	6839.6	6838.3	0.02
3	3	95.2	88.4	836.2	6839.6	6838.3	0.02

Table 1 State of convergence for 2-panel 2-span continuous truss

The results of a two-panel two-span continuous truss calculated by this method are summarized in Table 2. In this table, H_0 shows the initial value of H_0 and the values of the design variables and the penalty and objective functions are shown depending on the values of H_0 . Case 1, Case 2 and Case 3 in table 2 show the different optimum values for the initial values of $H_0=500(\text{cm})$, $H_0=600(\text{cm})$ and $H_0=700(\text{cm})$, respectively. According to the results of the optimum design, the values of Case 1, Case 2 and Case 3 are almost equal.

global minimum value. In this optimum design, SMH=4000 (yen/hour), CM=80000

(yen/ton) and $\mu=0.05$ are employed.

Case	H ₀ (cm)	Bf (cm)	Bw (cm)	H (cm)	S ₁	S ₂	S ₃	S ₄	T _{u1} (cm)	T _{u2} (cm)
1	500.0	95.2	88.4	832.3	4.0	6.0	5.0	6.0	1.19	3.17
2	600.0	95.2	88.4	836.2	4.0	6.0	5.0	6.0	1.19	3.17
3	700.0	95.3	88.3	836.6	4.0	6.0	5.0	6.0	1.19	3.10
Case	T _{u4} (cm)	T _{w1} (cm)	T _{w2} (cm)	T _{w3} (cm)	T _{w4} (cm)	B _{u4} (cm)	F (1000yen)	Z (1000yen)		
1	3.86	1.04	2.76	1.66	2.95	124.1	6839.7	6838.4		
2	3.86	1.04	2.81	1.25	2.95	124.0	6839.6	6838.3		
3	3.88	1.04	2.69	1.53	2.95	124.7	6839.4	6838.3		

Table 2 Comparison of optimum values for 2-panel 2-span continuous truss

3.2 Example II: Ten-panel 2-span continuous truss

Secondly, as another example of an optimum design using the proposed method, the optimum design of a steel highway bridge of ten-panel 2-span continuous truss type with a live load of TL20, as shown in Fig. 2, is carried out. For

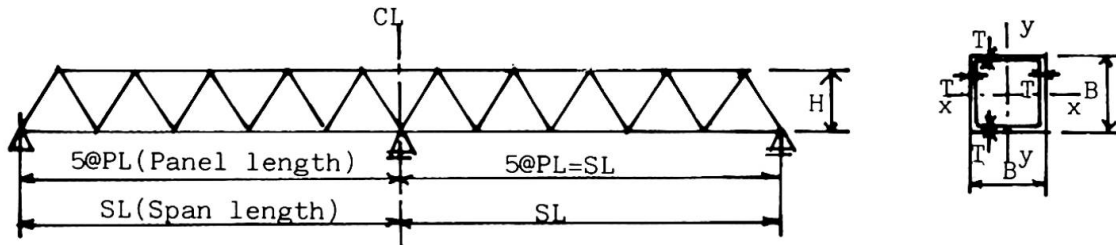


Fig.2 10-panel 2-Span continuous truss and notation

Member					Upper chord					End post	Lower		
Case	L (cm)	H (cm)	B (cm)	S ₁ -S ₂₀	T ₁ (cm)	T ₂ (cm)	T ₃ (cm)	T ₄ (cm)	T ₅ (cm)	T ₆ (cm)	T ₇ (cm)	T ₈ (cm)	
1	2@25=50	328.4	17.2	4.0	3.22	4.62	3.02	0.90	2.64	1.97	0.90	1.62	
2	2@30=60	384.1	23.8	4.0	1.97	2.41	1.58	0.91	2.62	1.29	0.90	1.23	
3	2@35=70	424.4	28.1	4.0	1.99	2.66	1.91	0.90	2.31	1.24	0.90	1.20	
chord			Diagonal										
Case	T ₉ (cm)	T ₁₀ (cm)	T ₁₁ (cm)	T ₁₂ (cm)	T ₁₃ (cm)	T ₁₄ (cm)	T ₁₅ (cm)	T ₁₆ (cm)	T ₁₇ (cm)	T ₁₈ (cm)	T ₁₉ (cm)	T ₂₀ (cm)	Z (¥1000)
1	1.57	0.90	3.91	0.90	0.90	0.90	1.17	0.90	0.90	0.90	1.03	1.93	33518.4
2	1.09	0.90	3.32	0.90	0.90	0.90	1.02	0.90	0.90	0.90	0.90	1.43	36202.8
3	1.16	0.90	2.32	0.90	0.90	0.90	0.99	0.90	0.90	0.90	0.90	1.33	38989.2

Table 3 Optimum values

examples of the optimum design of the bridge, 2@25(m), 2@30(m) and 2@35(m) in its effective span length and 10(m) in its effective roadway width are treated.



The variables, constraints and objective function of example II are almost the same as those of example I, S and T among the design variables are related to x , and B and H are related to y . The constraints contain the limit of stress, the limit of deflection, the upper and lower limits of values of the design variables, the limit of ratio of plate width to thickness for prevention from local buckling and $l/r < 120$. The objective function is the same as the two-panel two-span continuous truss. Also, $SMH=4000(\text{yen/hour})$, $CM=80.000(\text{yen/ton})$ and $\mu=0.05$ are applied to this example. The results of the optimum design are given in Table 3. Case 1, Case 2 and Case 3 show the case of the span length of 2@25(m), 2@30(m) and 2@35(m), respectively, and the effective width of 10(m).

4. CONCLUSION

As the present study demonstrates for a statically indeterminate structure and a statically determinate structure with restriction of displacement, it is a little insufficient from the accurately and community to any structural type, but it is sufficient as a useful method for an actual design of bridges. In addition, the result of the optimum design will make it possible to carry out an automated design with an XY plotter.

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