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## Ultimate State Aseismic Design of Reinforced Concrete Structures

Calcul à l'état limite de ruine des structures en béton armé sous l'effet de séismes

Grenztragfähigkeit von Stahlbeton unter Erdbebenbelastung

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Minoru Yamada, born 1930, received his doctor degree 1959 from the Kyoto University, Japan. His research findings on the shear explosion effect in short reinforced concrete columns was later verified in the Tokachi-Oki earthquake in 1968. He has been professor of structural engineering, Kobe University, Japan, since 1964.

### SUMMARY

The practical dimensioning formulae, not only for strength but also for deformation, for the ultimate state aseismic design of reinforced concrete structures are presented here not empirically but analytically, based upon the fundamental mechanical properties of materials. Using these formulae it is possible to estimate quantitatively the ultimate aseismic capacity of reinforced concrete structures.

### RESUME

L'article présente des formules pratiques de calcul à la rupture de structures en béton armé, sous l'effet de séismes. Ces formules, tenant compte de la résistance et des déformations, sont basées sur les propriétés mécaniques fondamentales des matériaux. L'application de ces formules permet de quantifier la sécurité des structures en béton armé, vis à vis de tremblements de terre.

### ZUSAMMENFASSUNG

Analytisch hergeleitete Bemessungsformeln für Stahlbetonbauten unter Erdbebenbelastung werden vorgeschlagen. Sie basieren auf den mechanischen Eigenschaften der beiden Komponenten Stahl und Beton und berücksichtigen auch die Verformungen. Mit diesen Bemessungsformeln wird es möglich, die Grenztragfähigkeit von Stahlbeton unter Erdbebenbelastung quantitativ abzuschätzen.



## 1. INTRODUCTION

The importances of the ultimate state design of reinforced concrete structures for earthquakes were already principally well recognized. However, the lack of practical dimensioning formulae [1] especially the lack of analytical evaluation formulae of plastic deformations or deterioration and fracture processes makes the practical application of ultimate state aseismic design impossible. It is proposed here from this point of view the dimensioning formulae of reinforced concrete aseismic elements not only ultimate resistances but also ultimate deformation based upon only characteristic mechanical values of materials, i.e. concrete and steel, not empirically but analytically.

## 2. ASEISMIC ELEMENTS OF REINFORCED CONCRETE STRUCTURES

### 2.1 Classification of Reinforced Concrete Aseismic Elements

Reinforced concrete structures are consisted of the following five aseismic elements:

- 1) Reinforced Concrete Short Columns (SC), with a shear span ratio ( $H/D$ ) shorter than 4, predominant of the influences of shear forces ( $V$ ) than bending moment ( $M$ ) and axial load ( $N$ ), show explosive shear fracture at a very small relative displacements without ductility.
- 2) Reinforced Concrete Medium Columns (MC), with a shear span ratio ( $H/D$ ) between 4 to 20, predominant of the influences of bending moment ( $M$ ) than shear force ( $V$ ) and axial load ( $N$ ), show bending yield at a fairly large relative displacement with some or sufficient ductility.
- 3) Reinforced Concrete Long Columns (LC), with a shear span ratio ( $H/D$ ) larger than 20, predominant of the influences of axial load ( $N$ ) than bending moment ( $M$ ) and shear force ( $V$ ).
- 4) Reinforced Concrete Shear Walls (SW), show large resistances but without ductility.
- 5) Reinforced Concrete Shear Walls with Openings (SWO), show medium resistance between shear wall without openings and rigid frames.

Under one way sway loading, like earthquake, these aseismic elements show quite different resisting and deformation behaviours of each other, such as shown in Fig. 1, under the fixed constraint of top and bottom ends.

### 2.2 Importances of the Differences of Deformability of Reinforced Concrete Elements

Because of the high rigidity of so-called "Scheibe-action" of floors, each aseismic elements are constrained to remove the same relative displacement between floors. Therefore the large differences of deformability of each aseismic elements hold no more the simple superposition principle of ultimate resistances of elements and cause the elasto-plastic deformation and fracture

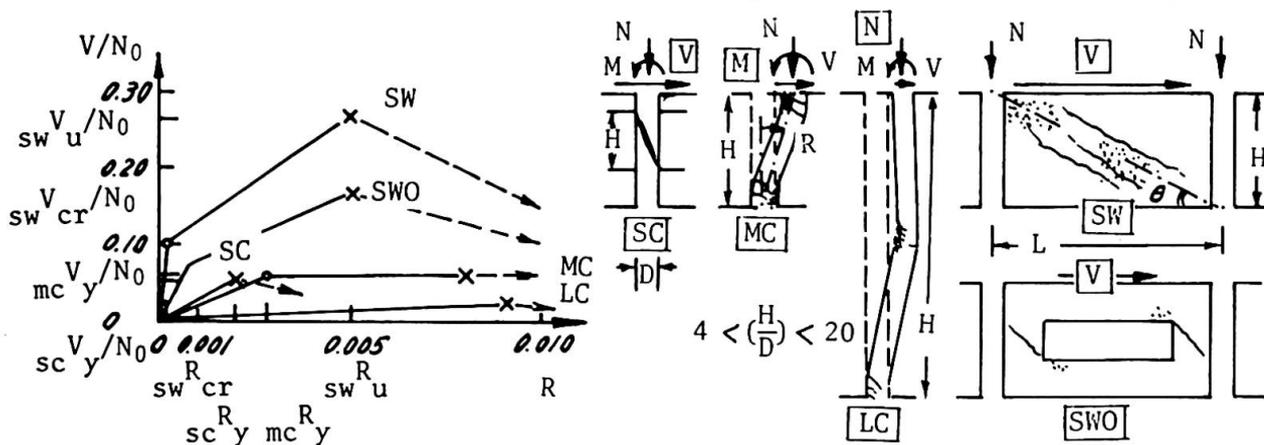


Fig.1 Resisting and deformation behaviours of aseismic elements

behaviours of whole structures very complex. The resistances of each elements are only able to superposed at the same values of displacements. Thus caused the whole resisting behaviour and fracture of reinforced concrete buildings [2] by the drastic reduction [1] of bearing capacity and ductility of short columns.

In this report a reinforced concrete cross section is abstracted into a poly-mass points-model such as shown in Fig.2(a) and the characteristic values of materials (concrete and steel) are illustrated in Fig.2(b) for analysis. (see Fig.2)

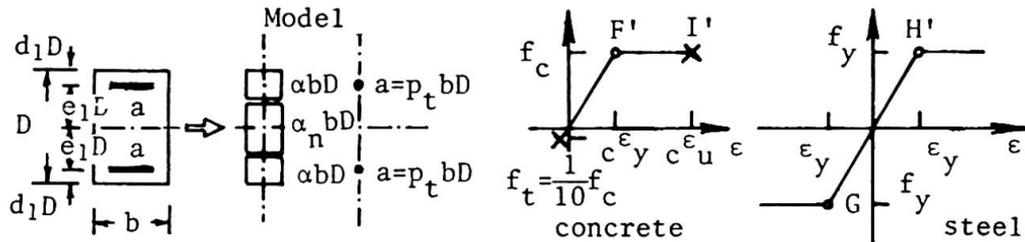


Fig.2 (a) Modeled cross section (b) Characteristic values of materials

### 3. REINFORCED CONCRETE SHORT COLUMNS

#### 3.1 One Way Sway Loading

Reinforced concrete short columns with a smaller shear span ratio (H/D) than 4, are predominated shear behaviour than bending and normal forces, and show violent shear explosion [2] at a shear resistance  $V_y^S$  with a very small relative sway displacement angle  $R_y^S$  or displacement  $\delta_y^S$  between stories as follows [3][4].

$$V_y^S = \tau_y A_{se} = \frac{7}{8}(1-d_1) f_c bD \sqrt{-0,10(X)^2 + 0,09(X) + 0,01} \quad (1),$$

$$R_y^S = c_y \gamma_y = \tau_y / G_c = f_c \sqrt{-0,10(X)^2 + 0,09(X) + 0,01} / G_c \quad (2),$$

$$\delta_y^S = R_y^S H \quad (3),$$

where

$$A_{se} = \frac{7}{8}(1-d_1) bD : \text{effective cross sectional area for shear,}$$

$$X = N/N_0 : \text{axial load level ratio,}$$

$$N_0 = f_c bD \{1 + 2(f_y/f_c) p_t\} : \text{ultimate strength of centrally loaded columns, (4),}$$

$$G_c = \tau_y / c_y \gamma_y = 0,9 \times 10^5 \text{ kg/cm}^2 : \text{shear modulus of concrete. (see Fig.3)}$$

Fracture condition of concrete under the combined stresses of  $\sigma$  and  $\tau$  are assumed to be:

$$\tau_y / f_c = \sqrt{-0,10(\sigma/f_c)^2 + 0,09(\sigma/f_c) + 0,01}.$$

Ductility of reinforced concrete short columns may only be expected with a very sufficient hoop reinforcement  $p_w$  more than 1% or more. (see Fig.3)

#### 3.2 Cyclic Sway Loading

Under cyclic sway loading, reinforced concrete short columns show a drastic reduction of resistances and deformation capacities. Therefore, it is unable to expect any resistances of such reinforced concrete short columns under cyclic sway loadings.

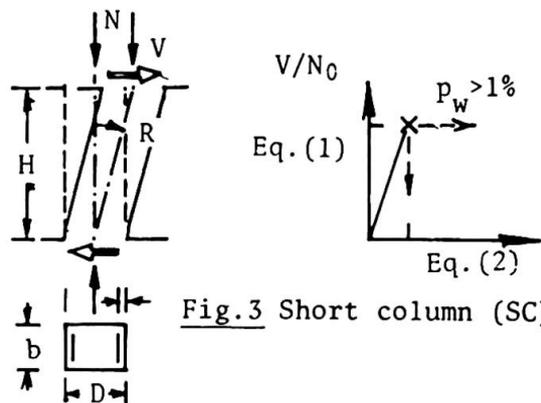


Fig.3 Short column (SC)



4. REINFORCED CONCRETE MEDIUM COLUMNS

4.1 One Way Sway Loading

Reinforced concrete medium length columns with a shear span ratio ( $H/D$ ) between 4 to 20 are predominated by the influences of bending moment  $M$  than shear  $V$  and axial force  $N$ .

Story yield shear force  $V_y^B$  and story yield sway displacement  $\delta_y^B$  under bending yield of both (top and bottom) fixed ends of medium length columns are:

$$V_y^B = 2M_y/H \tag{5},$$

$$\delta_y^B = (H^2/6)\phi \tag{6},$$

where

(a) for lower axial load level:  $0 \leq X \leq \frac{\alpha}{1+2(f_y/f_c)p_t}$

$$M_y = f_c bD^2 \{N/f_c bD + 2(f_y/f_c)p_t\} e_1 \tag{7},$$

$$\phi_y = - \frac{\epsilon_y}{2e_1 D} \left\{ \frac{N/f_c bD + (f_y/f_c)p_t}{\alpha + (f_y/f_c)p_t} + 1 \right\} \tag{8}.$$

(b) for higher axial load level:  $\frac{\alpha}{1+2(f_y/f_c)p_t} \leq X \leq \frac{\alpha + (3/4)\alpha_n}{1+2(f_y/f_c)p_t}$

$$M_y = f_c bD^2 \left\{ 2 - \frac{N/f_c bD}{\alpha + (3/4)\alpha_n} \right\} \{ \alpha + (f_y/f_c)p_t \} e_1 \tag{9},$$

$$\phi_y = - \frac{\epsilon_y}{2e_1 D} \left\{ \frac{N/f_c bD}{\alpha + (3/4)\alpha_n} - 2 \right\} \tag{10},$$

where

$$2\alpha + \alpha_n = 1 \text{ (cf. Fig. 2) (see Fig. 4(a))} \tag{11}.$$

4.2 Cyclic Sway Loading

For only the cases of constant curvature amplitude and tensile yield ( $0 < \frac{N}{f_c bD} \leq \alpha$ ) Under the assumption of the formation of cyclic plastic hinge zones with a hinge length of  $\lambda D$  ( $\lambda=1$ ) at the both (top and bottom) fixed ends of columns, then story yield shear force  $V_y^B$ , story slip shear force  $V_s^B$ , and story yield sway displacement  $\delta_y^B$ , story sway displacement amplitude  $\delta_a$  under bending yield are:

$$V_y^B = \frac{2M_y}{H}, \quad V_s^B = \frac{2M_s}{H} \tag{12},$$

$$\delta_y^B = \frac{H^2}{6}\phi_y, \quad \delta_a = \delta_y + \lambda D (\phi_a - \phi_y) H \tag{13},$$

where

$$M_y = f_c bD^2 \left\{ \frac{N}{f_c bD} + 2(f_y/f_c)p_t \right\} e_1 \tag{14},$$

$$M_s = f_c bD^2 \left\{ 2(f_y/f_c)p_t - \frac{N}{f_c bD} \right\} e_1 \tag{15},$$

$$\phi_y = \frac{2e_1 D}{\epsilon_y} = 2 \tag{16}.$$

see Fig. 4(b).

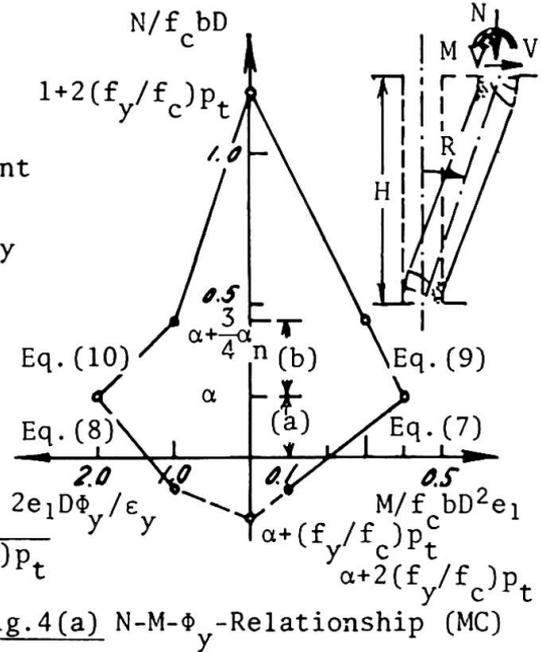


Fig. 4(a) N-M- $\phi$ -Relationship (MC)

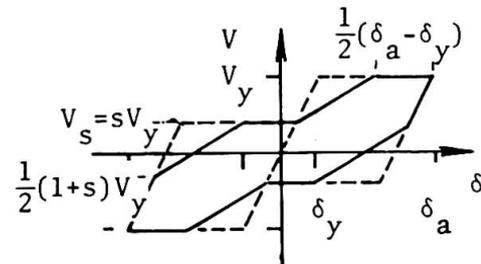


Fig. 4(b) Hysteresis loop of medium column

### 4.3 Ductility Factor

Plastic hinges of reinforced concrete medium columns are formed only by the tensile yielding of longitudinal reinforcement at G and the rotation capacity of them are limited by the reach of concrete strains to the ultimate value  $\epsilon_u = 0,004$  (ultimate compressive strain of concrete at I').

The ductility factor  $\mu$  of reinforced concrete medium columns under one way sway are:

$$\mu = \frac{\phi_u}{\phi_y} = \frac{1}{n_1} \left( \frac{c \epsilon_u}{\epsilon_y} - \frac{1}{2} \right) \frac{2}{3} (1 - d_1 - n_1) \geq 1 \tag{17},$$

see Figs.5,6.

From Eq.(17) the plastic hinges are formed only under the lower axial load level than the intersection point of G and I' so,

$$n_1 = 0,53 \tag{18}.$$

The ductility factor  $\mu$  of reinforced concrete medium columns under cyclic sway are computed under the assumption of the reduction  $\gamma$  of concrete resistances by the repetition number of loadings  $N$  such as shown in Fig.7, and the fatigue fracture occurs by reach of the compressive strain of concrete at the compressive longitudinal reinforcement to the compressive ultimate strain  $\epsilon_u$ , then the relationship between curvature amplitude  $\phi_a$  and number of cycles until fracture becomes

$$\phi_a = \frac{1}{D} \frac{c \epsilon_u}{1 - \frac{1}{8} \log_{10} N_B \frac{N}{f_c b D} - d_1} \tag{19}.$$

For the axial load level the relationship is illustrated in Fig.8, in which the the values at  $N_B = 10^0 (= 1)$  approximately corresponding to the values at  $N/f_c b D = 0,2$  in Fig.5.

The relationships between the ductility factor  $\mu$  and the number of cycles to fracture  $N_B$  are (see Fig.9),

$$\mu = 1 + \{ \lambda D (\phi_a - \phi_y) H \} / \delta_y \tag{20}.$$

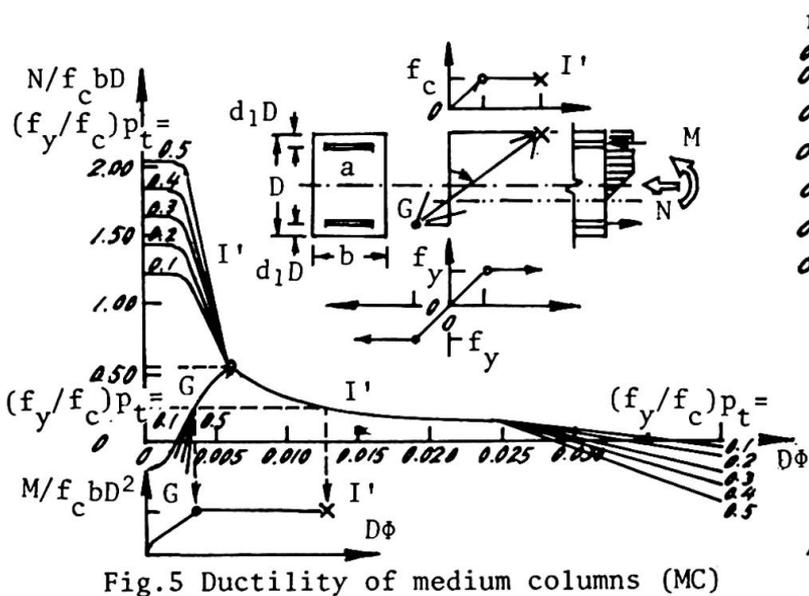


Fig.5 Ductility of medium columns (MC)

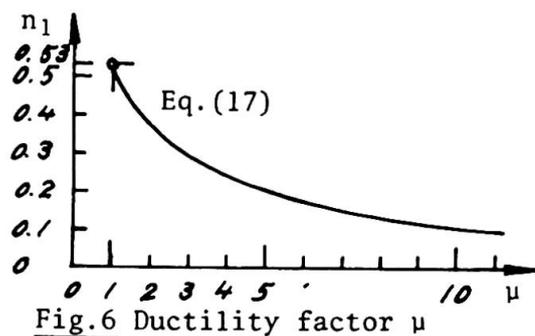


Fig.6 Ductility factor  $\mu$

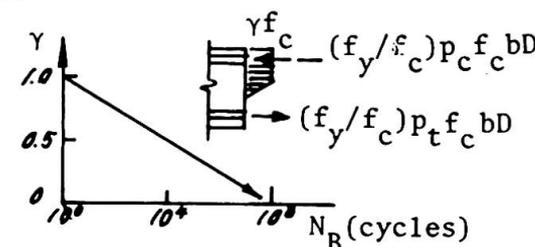


Fig.7 Deterioration of concrete strength

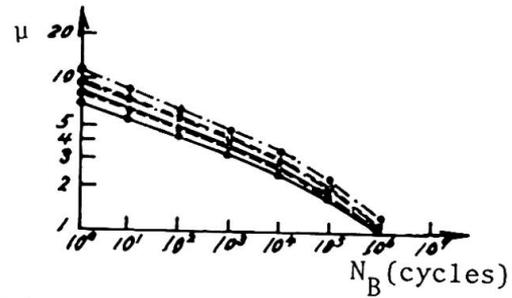
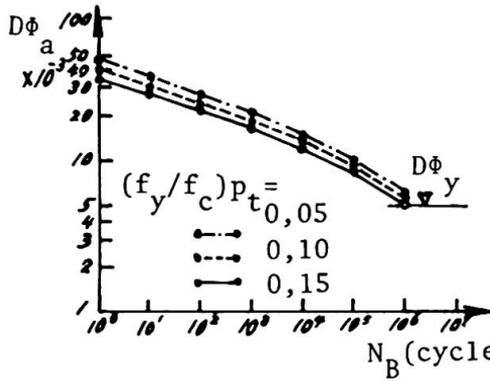


Fig.8 Curvature amplitude-number of cycles to fracture

Fig.9 Ductility factor-number of cycles to fracture

5. REINFORCED CONCRETE LONG COLUMNS

5.1 One Way Sway Loading

One way sway loading of reinforced concrete long columns show unstable states. The ultimate N-M-Interaction curves indicate the influences of the values of shear span ratios (H/D) (see Fig.10). So there exist no ductility in long columns with larger shear span ratios (H/D) > 20.

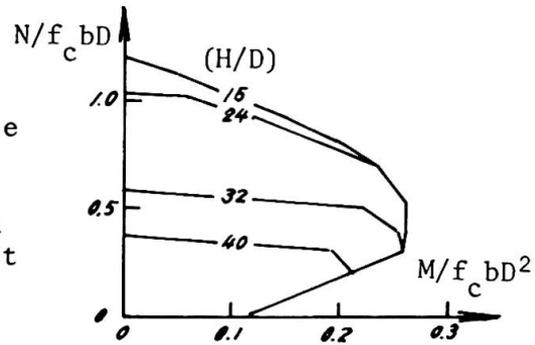


Fig.10 Ultimate N-M-Interaction curve of long columns (LC)

6. CRITICAL SHEAR SPAN RATIOS

There exist two critical shear span ratios of reinforced concrete columns, i.e. between short columns (SC) with shear explosion and medium column (MC) with bending yield (H/D)<sup>I</sup><sub>cr</sub>; and between medium column (MC) with bending yield and long column (LC) with buckling (H/D)<sup>II</sup><sub>cr</sub>.

6.1 Critical Shear Span Ratio (H/D)<sup>I</sup><sub>cr</sub>

The critical shear span ratio (H/D)<sup>I</sup><sub>cr</sub> is characterized by the intersecting condition of bending yield of (MC) and shear explosion of (SC) [2][4] as follows:(Fig.11)

(a) for lower axial load level:  $0 \leq X \leq \frac{\alpha}{1+2(f_y/f_c)p_t}$

$$(H/D)_{cr}^I = \frac{2\{X+2(1+X)(f_y/f_c)p_t\}\{(1/2)-d_1\}}{(7/8)(1-d_1)\sqrt{-0,10X^2+0,09X+0,01}} \quad (21),$$

(b) for medium axial load level:

$$\frac{\alpha}{1+2(f_y/f_c)p_t} \leq X \leq \frac{\alpha+(3/4)\alpha_n}{1+2(f_y/f_c)p_t}$$

$$(H/D)_{cr}^I = \frac{2\{\alpha+(f_y/f_c)p_t\}\{(1/2)-d_1\}}{(7/8)(1-d_1)\sqrt{-0,10X^2+0,09X+0,01}} \quad (22),$$

then

$$N = XN = 0,53f_c bD \quad (23).$$

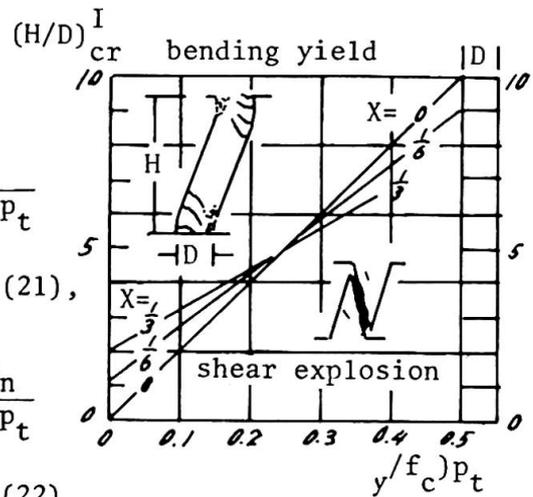


Fig.11 Critical shear span ratio (H/D)<sup>I</sup><sub>cr</sub>

6.2 Critical Shear Span Ratio (H/D)<sup>II</sup><sub>cr</sub>

The critical shear span ratio (H/D)<sup>II</sup><sub>cr</sub> is characterized by the buckling of long columns. The analytical results of ultimate N-M-Interaction curves in Fig.10 show unstable states occurs under longer columns (H/D)<sup>I</sup><sub>cr</sub> > 20. For double curvature critical shear span ratio (H/D)<sup>II</sup><sub>cr</sub> varies with the axial load levels X too.

7. REINFORCED CONCRETE INFILLED SHEAR WALLS

The resisting mechanism of reinforced concrete infilled shear walls against horizontal shear load like earthquake excitation are abstracted into a compression bracing field by concrete [5] such as shown in Fig.12.

7.1 Initial Cracking Load and Cracking Sway Displacement

Initial cracking shear force  $V_{cr}$  and relative story sway angle  $R_{cr}$  are under the assumption of uniform distribution of shearing stresses in web panel of wall with a thickness of  $t$ :

$$V_{cr} = \tau_{cr} Lt = \frac{f_c}{10} Lt (=0,1f_c Lt) \tag{24},$$

$$R_{cr} = \frac{\tau_{cr}}{G_c} = \frac{f_c}{10} \frac{1}{G_c} = \frac{2(1+\nu)}{E_c} \frac{f_c}{10} (= 0,000001166 f_c) \tag{25}.$$

7.2 Ultimate Resistance  $V_u$  and Fracture Sway Displacement  $R_u$

Ultimate resistance and fracture sway displacement are under the assumption of the formation of concrete bracing:

$$V_u = f_c B_e t \cos\theta = \frac{2}{3} f_c Lt \sin\theta \cos\theta \tag{26},$$

$$R_u = \frac{c^e_u}{\sin\theta \cos\theta} = \frac{0,002}{\sin\theta \cos\theta} \tag{27},$$

see Fig.12.

7.3 Cyclic Sway Loading

Under cyclic sway loading infilled shear walls show their resistances only at the verginal part so the resistances and displacements under cyclic sway loading are abstracted into a fragmental pair resistances until ultimate resistances [7][8] such as shown in Fig.13.

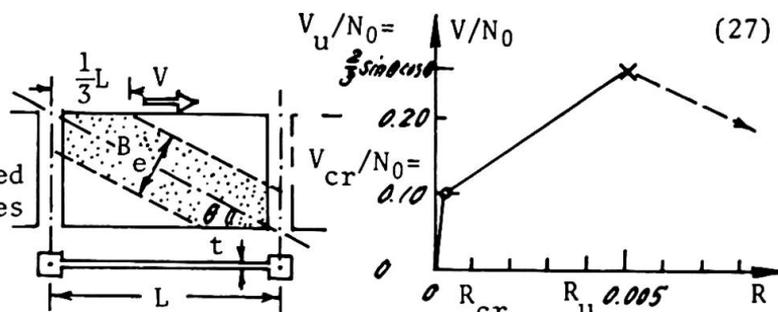


Fig.12 Shear walls (SW)

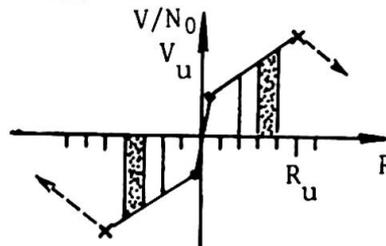


Fig.13 Hysteresis loop of shear wall

8. REINFORCED CONCRETE INFILLED SHEAR WALLS WITH OPENINGS

The resisting mechanism of reinforced concrete infilled shear walls with openings against horizontal shear load are very complicated by the differences of fracture processes according to the different sizes and positions of openings  $H_i$  in walls [6]. Their load-deformation relationships are situated between shear walls without openings and surrounding rigid frames such as illustrated in Fig.1. Analytical evaluation of resistances and deformations for such shear walls with openings are only possible in the cases of symmetric single openings in walls with opening width ratio  $l_1=L_0/L_i$ , and depth ratio  $h_1=H_0/H_i$  [6] in Fig.14.

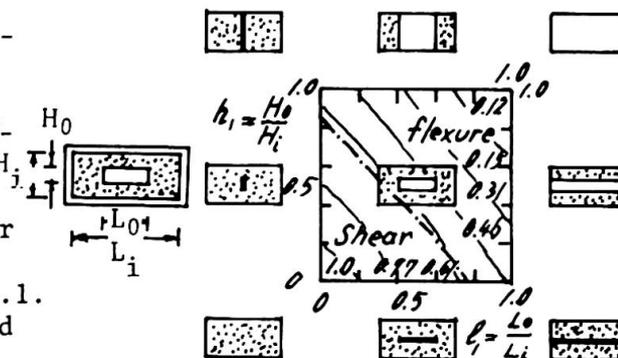


Fig.14 Ultimate resistances of shear walls with openings [6]



## 9. CONCLUDING REMARKS

Resisting elements of reinforced concrete structures against horizontal load like earthquake excitation are classified into short columns (SC), medium columns (MC) long columns (LC), shear walls without openings (SW) and shear walls with openings (SWO). The analytical formulae of their ultimate resistances as well as ultimate deformations, and critical shear span ratios are presented here based only upon the fundamental mechanical characteristic values of elemental materials (concrete and steel).

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