Analysis of Space Frames under Vertical Earthquake Loads

Analyse de treillis spatiaux sous l'effet de charges sismiques verticales

Analyse von Raumfachwerken unter vertikaler Erdbebenbelastung

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SUMMARY
In this paper a rational and simple method is given for estimating forces in a space frame under vertical earthquake loads. A series of commonly used space frames of different types and spans were studied by computer analysis. Results of the investigation into the free vibration characteristics and earthquake response of space frames are reported, which form the basis of the proposed method.

RESUME
Une méthode simple et rationnelle permet le calcul des forces dans un treillis spatial sous l'effet de charges sismiques verticales. Une série de treillis de types et portées différents ont été étudiés à l'aide de l'ordinateur. Les résultats de l'étude des caractéristiques des oscillations propres et le comportement aux séismes des treillis sont présentés.

ZUSAMMENFASSUNG
Es wird in dieser Arbeit ein rationales und einfaches Verfahren zur Bestimmung der Stabkräfte des Raumfachwerkes unter vertikaler Erdbebenbelastung gegeben. Eine Reihe viel verwendeter ebener Raumfachwerke verschiedenen Typs und mit verschiedenen Spannweiten wurde mit Hilfe von Computerprogrammen untersucht und studiert. Die Ergebnisse der Untersuchungen über die Charakteristiken der Eigenschwingungen und die Antwortspektren der ebenen Raumfachwerke werden aufgezeigt. Sie bilden die Grundlage für das vorgeschlagene Verfahren.

1. INTRODUCTION

Space frame has found wide applications as the roof system of buildings, espe-
cially long-span roofs. The design of steel space frame to resist earthquake
loads is an urgent problem encountered to design engineers. Since the amount of
work involved in the dynamic analysis of space frame is extremely large, a simple
and rational method for estimating the seismic force in a space frame is required
for practical design purpose. Such a method should be based on the systematic
analysis of the free vibration characteristics and earthquake response of the
space frame. Some design codes specify that certain magnifying factor be in-
troduced to the static load to include the seismic effect, this seems to be
irrational and does not reflect the load-resisting behavior of the structure.

On the basis of paper [1], a series of commonly used space frame supported along
perimeters were studied by computer analysis and an appropriate expression for
the seismic force coefficient was found. The types of the space frame are:
(The names in the parenthesis indicate the terminology suggested in some English
literature)

Orthogonal Type

      (1) Two-way orthogonal lattice grids (Square on square)
      (2) Orthogonal square pyramid space grids (Square on square offset)
      (3) Orthogonal square pyramid space grids with openings (Square on larger
          square)
      (4) Square pyramid space grids of checkerboard pattern (Square on diagonal

Diagonal Type

      (5) Two-way diagonal lattice grids
      (6) Diagonal square pyramid space grids (Diagonal on square)
      (7) Square pyramid space grids with star elements

2. DYNAMIC BEHAVIOR OF SPACE FRAME

2.1 Free Vibration Characteristics

A space frame can be treated as a pin-con-
nected space truss system as shown in Fig. 1.
External loads are assumed to be point loads
acting at the joints of top chord or bottom
chord. The members resist only axial forces.
The free vibration of a space frame is thus
formulated as a equation of motion for a
freely vibrating undamped multi-degree-of-
freedom system. By solving the generalized
eigenvalue problems, the circular frequencies
and vibration mode vectors are obtained.

With each joint possessing three degrees of
freedom, a space frame of ordinary size will
have hundreds or thousands degrees of free-
dom and the amount of computing work will be
tremendous. In view of the fact that only
first dozens of vibration modes are needed in   Fig. 1 Space Truss System
analysis, a sub-space iteration method is
used, this results in a significant reduction of computational effort.

From paper [2], 22 space frames of different types and spans were taken for
dynamic analysis. The members of the space frame were selected by optimization,
so that the sectional area and static force were reasonably matching.

Calculated results show some interesting features of the free vibration charac-
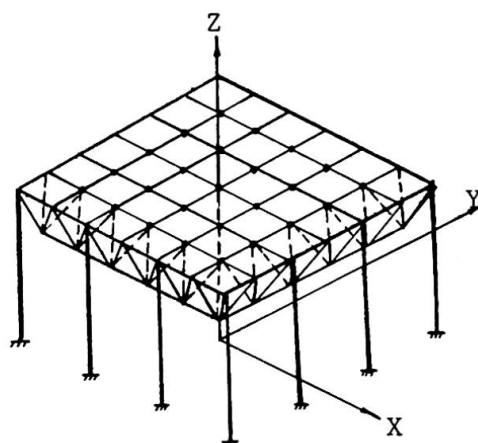
teristics of the space frame. The frequency spectrums of space frames are rather concentrated and fundamental periods for most space frames range from 0.4 – 0.6 sec. and fundamental period increases with the span. It was found that the vibration modes could be classified mainly as vertical modes and horizontal modes which appear alternately. The vertical modes of different types of space frame demonstrate essentially the same shape and the vertical frequencies for different space frames of equal span are very close to each other. Certain proportion could be established between the first three frequencies of the vertical mode, which are designated as f_{v1}, f_{v2} and f_{v3} respectively, i.e.

$$f_{v2} = 3 f_{v1} \text{ and } f_{v3} = (4 \sim 4.6) f_{v1}$$

2.2 Earthquake Response of Space Frame

In order to investigate the seismic behavior of space frames under vertical earthquake loads, use has been made of the following methods. First, a response spectrum method was used employing a proposed Chinese vertical spectrum [3] as shown in Fig. 2 . For comparison, a time history method was also used. The computation was implemented by Wilson-θ method employing vertical acceleration records of El Centro (1940) and Tianjin, China (1976). According to Chinese aseismic design code, the maximum vertical accelerations are 0.05g, 0.1g and 0.2g respectively corresponding to design intensities 7, 8 and 9. The peak values of the earthquake records were adjusted as the equivalents of the above design intensities.

Design Intensity	7	8	9
α_{vm}	0.115	0.23	0.46

Site Soil
Type I   Stable rock
Type II   Ordinary stable soil
Type III Saturated loose sand, silty sand, organic soft soil etc.

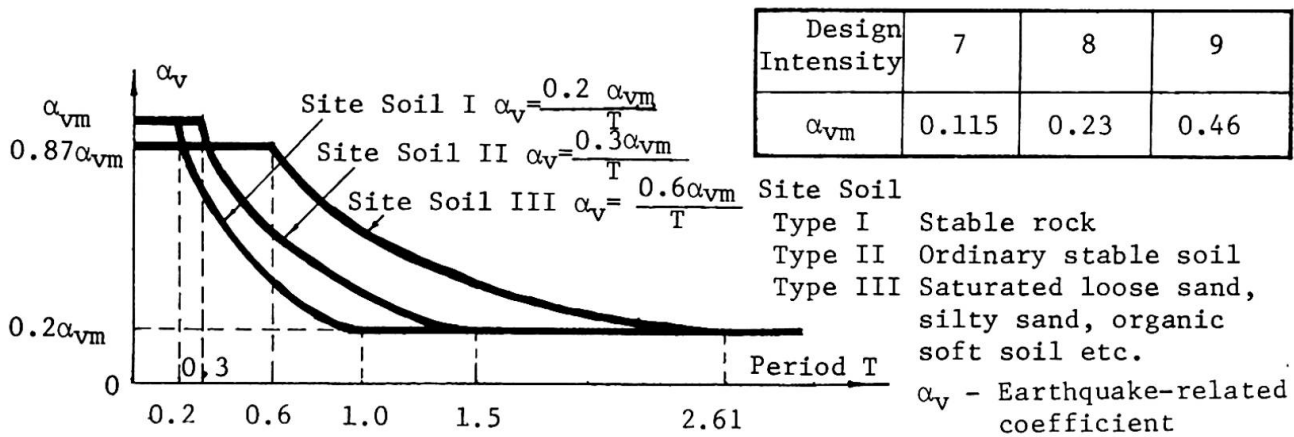α_v – Earthquake-related coefficient

Fig. 2 A Proposed Chinese Vertical Response Spectrum

It was found from the calculated results that forces in the space frame due to vertical earthquake are mainly contributed by the first three symmetrical vertical modes. Hence first ten symmetrical modes should be considered so as to include these vertical modes.

The values of vertical seismic forces in the members, whether top chords, bottom chords or webs, are higher near the central region of the space frames and decrease gradually towards the perimeters. A seismic force coefficient k_i is introduced as the ratio between the forces in ith member due to vertical earthquake and static load,

$$k_i = F_i^e \Big/ \left| F_i^s \right| \quad (i = 1,2, \dots n) \tag{1}$$

where n is the total number of the members.

k_i values are then plotted at the midpoint of each member on the plan of space frames for different types and spans. An example is shown in Fig. 3 for a Type 1 space frame. By designating the k_i values on the symmetrical axes to scale, it can be shown that an approximately linear relation exists. Thus the
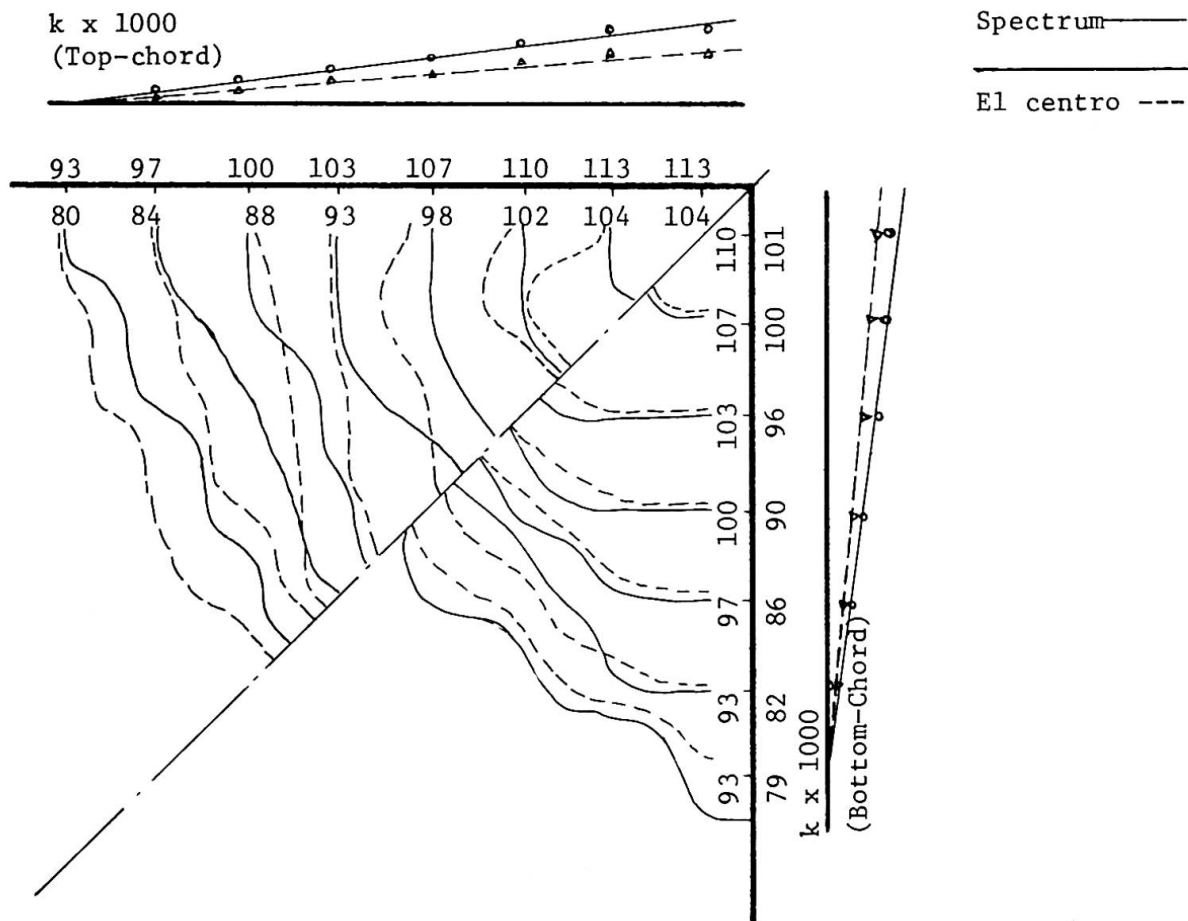
k x 1000
(Top-chord)

Spectrum ———

El centro ---

Fig. 3   Distribution of k Values in a Type 1 Space Frame
         (60 m x 60 m, Site Soil II, Design Intensity 8)

contour of the k_i values could be depicted
in the shape of a cone as in Fig. 4 , where
the top of the cone is the center of the
space frame. For a rectangular plan, the
base of the cone is a ellipse instead of a
circle. The ordinate of the point on the
surface of the cone can be taken as the k_i
value for the member located at the same
point. The peak value is k_{max} at center and
minimum value is k_{min} along the perimeters,
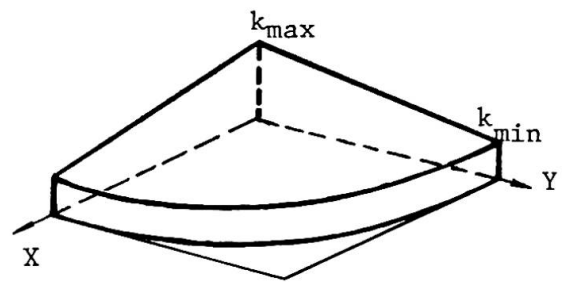which are related by a reduction factor β,
i.e.

$$k_{min} = \beta \, k_{max} \qquad (2)$$

Fig. 4   Cone-shaped Distribution
         of Seismic Force Coeffi-
         cient

3. METHOD OF ANALYSIS

In the process of space frame design, the force in each member and displacement
at each node due to static loads are usually found first, therefore it is con-
venient to determine the force due to earthquake load on the basis of known
static forces. Thus from eqation (1)

$$F_i^{\,e} = k_i \left| F_i^{\,s} \right| \quad (i = 1, 2, 3 \ldots n) \qquad (1a)$$

As shown above, the distribution of the seismic force coefficient k over the plan
of a space frame is in the shape of a cone. Once the peak value k_{max} and the

reduction factor β are found, the value k_i for any member can be determined without difficulty.

After analyzing the distribution of k value of different types of space frame, it can be seen that the k values for top chord members are slightly higher than those for bottom chord members. In most cases, k values for web members near the central region of the space frame are relatively high, this is due to the fact that the sectional areas of the web members are determined by construction-al requirements and are not fully-stressed as the chord members do. Therefore in determining the k_{max} and β values it is justifiable to take the top chord members to represent the whole space frame.

3.1 Peak Value of Seismic Force Coefficient

The factors influencing the seismic force in a space frame are numerous, such as the magnitude of earthquake load, the type of space frame, the grid module and depth of the space frame as well as the aspect ratio of the plan layout.
Yet the only factor that can most comprehensively reflect the above parameters is the fundamental frequency of the space frame, which can express the dynamic behavior of the structure satisfactorily. Hence it is necessary to find out the relation between the peak value of seismic force coefficient k_{max} and the fundamental frequency f.

A series of space frames under various site soil conditions were analyzed and the relations between k_{max} and f are shown in Fig. 5 and 6. It was found that the seimic force of a space frame depends to a large extent on whether the layout of the top chord members is orthogonal or diagonal. The k_{max} values of the diagonal type are much higher than those of orthogonal type. It also can be seen that for different site soil, an approximately linear relation exists between k_{max} and f, and k_{max} approaches to a definite value as f increases. The above relation can be concluded as a broken line shown in Fig. 7 where the slope of the inclined section was found by regression analysis of the calculated results. It is suggested to determine k_{max} from Fig. 7 where a and b can be taken from Table 1.

For commonly used space frame in practice, the k_{max} values are usually taken from the inclined section of the broken line in Fig. 7 for type I and II site soils and from horizontal section for type III site soil. Thus the seismic force coefficient is always higher for unfavorable soil condition.
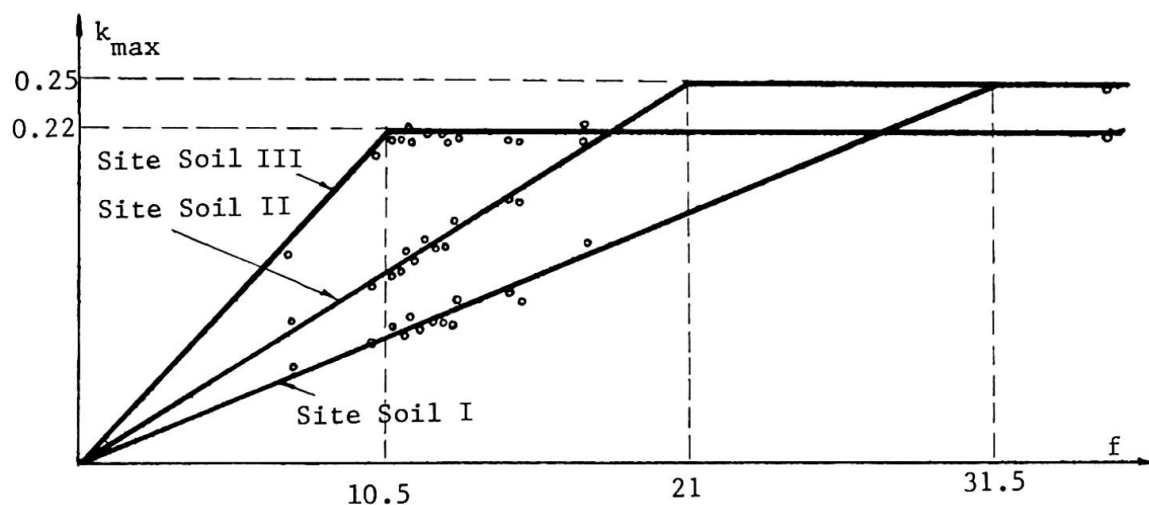

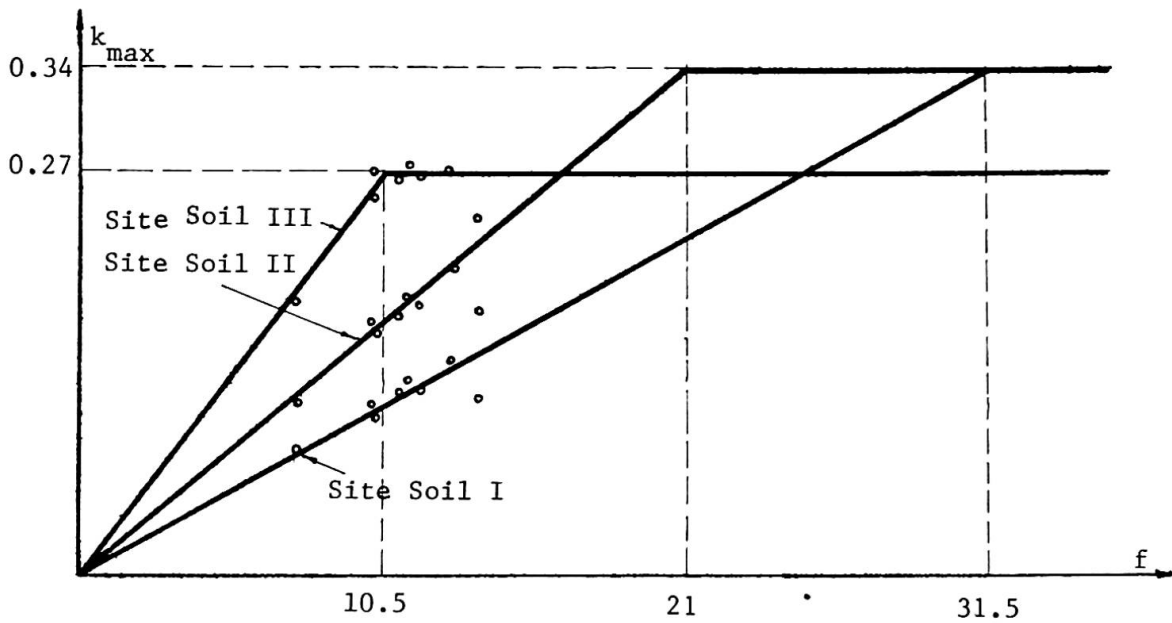Fig. 5 k_{max} - f Relationship for Orthogonal Type Space Frame

Fig. 6 k_{max} - f Relationship for Diagonal Type Space Frame

Type of Site Soil	a		b
	Orthogonal Type	Diagonal Type	
I			31.5
II	0.25	0.34	21
III	0.22	0.27	10.5

Table 1 Values of a and b in Fig. 7
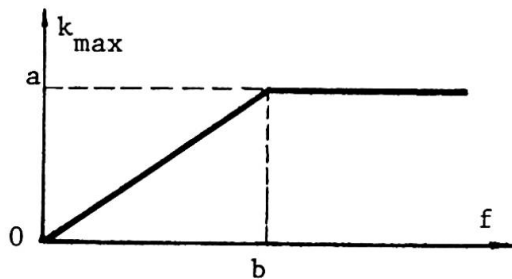
Space Frame		β
Orthogonal Type	Square	0.81
	Rectangular	0.87
Diagonal Type	Square	0.56
	Rectangular	0.80

Fig. 7 Suggested k_{max} - f Diagram   Table 2 Reduction Factor β

3.2 Reduction Factor β

From the calculaed results, it can be shown that the reduction factor β is only influenced by the type and plan layout of the space frame, and irrelevant to the variation of span or type of the site soil. β value for rectangular plan is usually higher than that for square plan. It is suggested to use the β values as set forth in Table 2 which are obtained from the average values of the computed values for top chord members of various types of space frame.

3.3 Seismic Force Coefficient k_i

After determining the values k_{max} and β as mentioned above, the seismic force coefficient of any member of a space frame can be found from the following formula:

$$k_i = K\ C\ k_{max} \left[\ 1 - \frac{L_i}{L}\ (1 - \beta\ )\ \right] \qquad (3)$$

where K – Coefficient considering seismic design intensity*

C – Structure-related coefficient depending on the type of structure of different constructional materials, also covering the effects of damping, ductility etc.**

L_i– Distance from the center 0 of the space frame to the midpoint of ith member

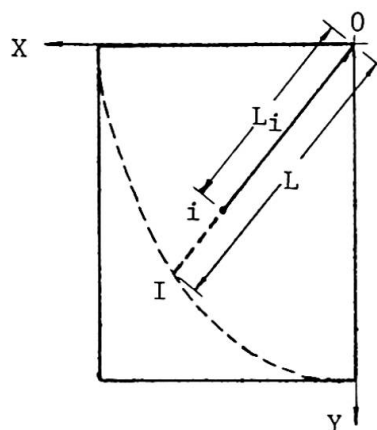L – Length 0I in Fig. 8.I is a point on the inscribed ellipse (or circle) of the plan

Fig. 8 Plan of a Space Frame

3.4 Method for Calculating Fundamental Frequency f

In order to determine k_{max} it is necessary to get the fundamental frequency f of the space frame first, which can be obtained from dynamic analysis or testing result. In practical engineering design, energy method could be used to calculate the fundamental frequency, using the curved surface of the vertical displacements under static loads as an approximation of the first vibration mode. Thus, the formula for calculating fundamental frequency can be written as:

$$f = \sqrt{g\ \frac{\sum\limits_{k=1}^{N} W_k\ Z_k}{\sum\limits_{k=1}^{N} W_k\ Z_k^{\ 2}}} \qquad (4)$$

where W_k is the static load at kth node point, Z_k is the vertical displacement under static load at kth node point and N is the total number of node points. As can be seen from Table 3, this method is efficient for obtaining relatively accurate values of fundamental frequency of different types of space frame.

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* According to Chinese aseismic design code, for design intensities 7, 8 and 9 the K values are taken as 0.5, 1 and 2 respectively.

**C value is essentially emperical, it is suggested to be 0.35 for steel space frame.

Space Frame		Fundamental Frequency	
Type	Size in m	Accurate Analysis	Approximate Value fr.eq.(4)
1	24 x 36	16.643	16.966
2	48 x 48	12.324	12.547
3	48 x 48	10.920	11.109
4	36 x 36	16.410	16.671
5	50.4 x 61.6	10.267	10.544
6	60 x 60	10.396	10.627
7	48 x 48	11.791	12.024

Table 3   Comparison of Accurate and Approximate Values of f

4. CONCLUSION

By means of the computer analysis of a series of space frames of different types
and spans, the free vibration characteristics and the pattern of the internal
forces in the space frame due to vertical earthquake are basically clarified.
A practical method of design is given. For any space frame, once the fundamen-
tal frequency is known, the peak value k_{max} of seismic force coefficient and
the reduction factor β are easily determined. Then the seismic force in any
member of the space frame can be calculated by formula (1a), which is simply
the product of the seismic force coefficient and the force due to static load.
The infuluences due to the type of space frames and their site soil conditions
as well as the variation in the dynamic behavior of the space frame are all
included in the proposed method so that a more rational estimation of seismic
forces can be obtained as compared to current practice.

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