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Autor: Turk, Srdan

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Deformability of Composite Timber Beams

Déformabilité des poutres composées en bois

Deformabilität von zusammengesetzten Holzträger

SRDAN TURK
Professor D.Sc.
University of Edvard Kardelj
Ljubljana, Yugoslavia

SUMMARY
The report deals with the simple method for calculation of the bending and buckling behaviour of composite bars. The method gives, for the praxis, enough exact results considering any number of composite parts. The same principles can also be used for the calculation of the deformability of torsional beams and for the control of torsional stability (torsional buckling, lateral buckling).

RESUME
Cet article présente une méthode simple permettant de calculer la flexion et le flambement des poutres composées en bois. Les résultats obtenus sont suffisamment exacts pour une utilisation pratiques, quel que soit le nombre de composants. Les mêmes principes sont utilisables pour le calcul des phénomènes de torsion (déformabilité, flambement de torsion, déversement).

ZUSAMMENFASSUNG
Dargelegt wird eine einfache Methode für die Berechnung des Biege- und Knickverhaltens zusammengesetzter Holzstäbe. Die Methode bietet für die Praxis genügend genaue Resultate für eine beliebige Zahl zusammengesetzter Teile. Dieselben Prinzipien sind ebenfalls anwendbar für die Berechnung der Verformung von tordierten Trägern und für die Kontrolle der Torsionsstabilität (Drillknicken, Kippen).

1. THE BASIC PRINCIPLES FOR THE CALCULATION OF THE DEFORMABILITY OF COMPOSITE TIMBER BEAMS.

The starting point of our calculation is Fig.1, where in Fig.1/a deformations and stresses for a stiff-jointed beam are presented, in Fig.1/b an elastically jointed beam, and in Fig.1/c a composite beam without fasteners are presented. If r_i is the reduction-factor:

$$\sigma_v = \sigma_{io} + \sigma_{iv}\ ,\ \sigma_{io} = r_i \cdot \sigma_{ioa}\ ,\ \sigma_{iv} = \sigma_{iva} = \sigma_{ivb} = \sigma_{ivc} = \sigma_{vc}, R = \text{const} \quad ../1$$

Acc. to Fig.2 the stress $\sigma_y = \varepsilon_y \cdot E$ for the distance "y" from the neutral axis and thus $\sigma_{ioa} = C \cdot y_i$ and $\sigma_{iva} = C \cdot y_{iv}$, $C = E/R$ and:

$$\sigma_v = C \cdot (y_{iv} + r_i \cdot y_i)\ ,\ y_{iv} = y_v - y_i\ ,\quad C = E/R \quad ../2$$

The moment, taken over by the beam, if the beam has "n" parts and A is the complete area of the cross-section, and A_i the area of the part "i"

$$M = \int_A \sigma_v \cdot y_v \cdot dA_i = C \cdot \left\{ \sum_1^n \int_{A_i} y_{iv}^2 \cdot dA_i + \sum_1^n r_i \cdot y_i^2 \cdot A_i \right\} \quad ../3$$

$$M = C \cdot \left(\sum_1^n I_{ti} + \sum_1^n r_i \cdot I_{Ti}\right) = C \cdot I_e = E \cdot I_e / R,\ I_{ti} = \int_{A_i} y_{iv}^2 \cdot dA_i, I_{Ti} = y_i^2 \cdot A_i$$

where I_e is the effective moment of inertia, and if we denote

$$I_{ae} = \sum_1^n r_i \cdot I_{Ti}\ ,\ I_{am} = \sum_1^n I_{Ti}\ \ (\text{m...mathematical}),\ I_{ti} = \frac{b_i \cdot h_i^3}{12} \quad ../4$$

and

$$I_e = \sum_1^n I_{ti} + I_{ae}\ ,\ I_m = \sum_1^n I_{ti} + I_{am} (=\text{math. mom. of in.}) \quad ../5$$

Now r_i should be established. This can be simplified by introducing:

$$r_i = q \cdot k_i,\ 1{,}00 \geq q \geq 0{,}00\ ,\ 1{,}00 \geq k_i \geq 0{,}00;\ q = \text{const.} \quad ../6$$

2. THE CALCULATION OF THE FACTORS OF REDUCTION AND OF THE APPROPRIATE NUMBER OF FASTENERS.

The Eq.3 can be composed so that the moment M is divided into the local moments $M_i = C \cdot I_{ti}$ and into the associated moment $M_a = C \cdot I_{ae}$, i.e. $M = \sum_1^n M_i + M_a$. If for the moment M_a first the calculation of his

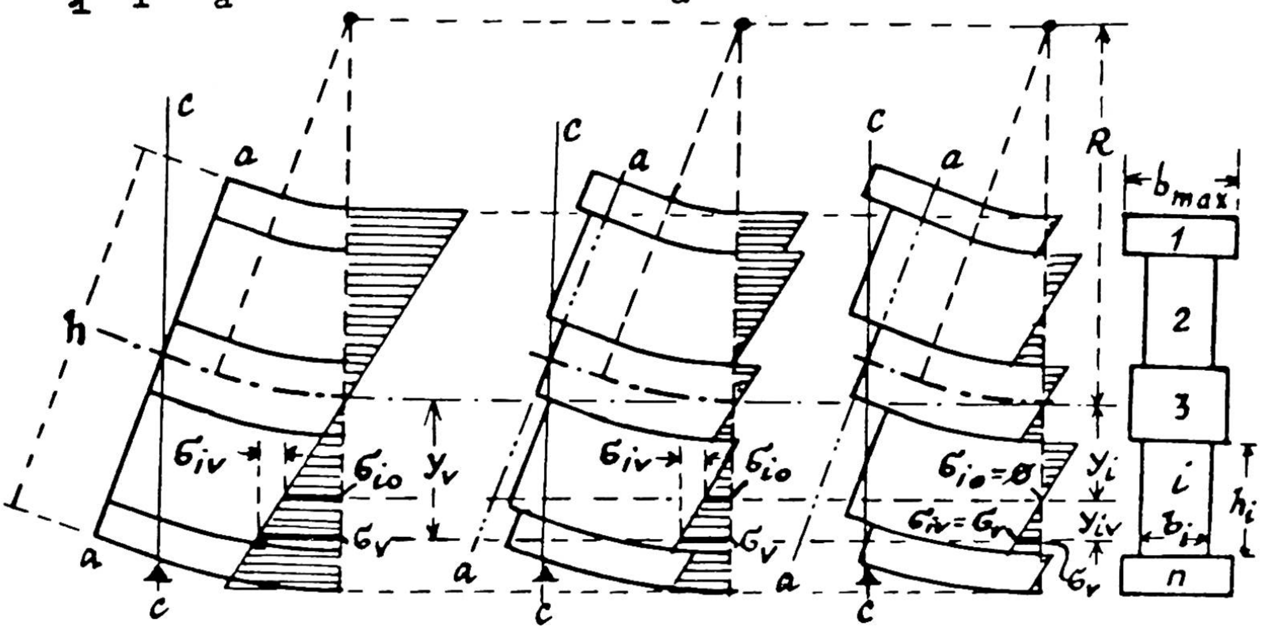

Fig.1/a Fig.1/b Fig.1/c

deformation is considered with the consideration of the effective moment of inertia I_{ae}, and then of the mathematical moment of inertia I_{am}, here including the deformations with regard to the transversal forces, it follows:

$$f = M_a.L^2/K.E.I_{ae} = M_a.L^2/K.E.I_{am} + a.M_a/A_o.G_d \qquad ../7$$

Here is E the modulus of elasticity for timber, L is the span(Fig. 3/a,b), K is the factor dependent on the shape of the moment line, for the praxis, $K \doteq 10$ is sufficient; "a" is for a rectangular cross section a=1,2, and A_o is the area of the cross-section outlined rectangle; G_d is the reduced shear modulus(deformability of fasteners!). From Eq.7 the value for I_{ae}:

$$I_{ae} = I_{am}/(1+m), \qquad m = a.I_{am}.K.E/L^2.A_o.G_d \qquad ../8$$

Supposing that all k_i are equal to 1,oo, i.e. $r_i = q$ = const., acc. to Eq.4 : $I_{ae} = q.I_{am}$ and finally(acc. to Eq. 8):

$$q = 1/(1+m), \quad \text{for } k_i = \text{const.} = 1{,}oo, \quad r_i = q \qquad ../9$$

The reduced shear modulus G_d is shown in Fig.4. In Fig.4/a we have the real example with G=G and the displacement x_f, in Fig.4/b we have the fictitious example $G \rightarrow G_d$, and the same displacement x_n, but now without the dislocation of the elements "i" and "j", which is acc. to Fig.4/a equal to z_{pij} ($\tau_{ji} = \tau_{ij}.s_{ij}/s_{ji}$!):

$$x_f = \tau_{ij}.d_{ij}/G + \tau_{ji}.d_{ji}/G + z_{pij} = x_n = \tau_{ij}.d_{ij}/G_d + \tau_{ji}.d_{ji}/G_d \qquad ../10$$

and:

$$G_d = G/(1+z_{pij}.G/H_{ij}.\tau_{ij}.s_{ij}), H_{ij} = d_{ij}/s_{ij} + d_{ji}/s_{ji} \qquad ../11$$

The dislocation z_{pij} at the shear force P_{ij} in one fastener, which have the allowable loading N_{sij} and the dislocation with regard to this loading z_{nij}, is then: $z_{pij} = z_{nij}.P_{ij}/N_{sij}$. If there are n_{sij} fasteners on the length of 100 cm, we get $z_{pij} = z_{nij}.\tau_{ij}.s_{ij}.100/N_{sij}.n_{sij}$. By introducing this in Eq.11, and regarding $Q_{ij} = N_{sij}.H_{ij}/100.z_{nij}$ we get:

$$G_d = G/(1+G/Q_{ij}.n_{sij}) = G_{do}/(1+G_{do}/G), G_{do} = Q_{ij}.n_{sij} \qquad ../12a$$

To have the same G_d along the whole depth of the beam, G_{do} = const.:

$$Q_{ij}.n_{sij} = Q_{ts}.n_{sts}, \text{ i.e. } n_{sij} = n_{sts}.Q_{ts}/Q_{ij} \qquad ../12b$$

Here is t-s any jointed plane, where we can take any number of fasteners(n_{sts} on 100 cm).

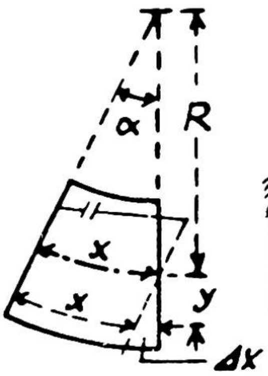

Fig.2

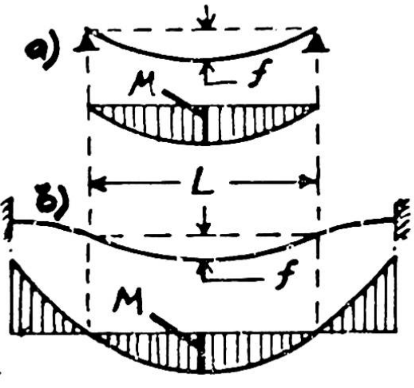

Fig.3/a,b

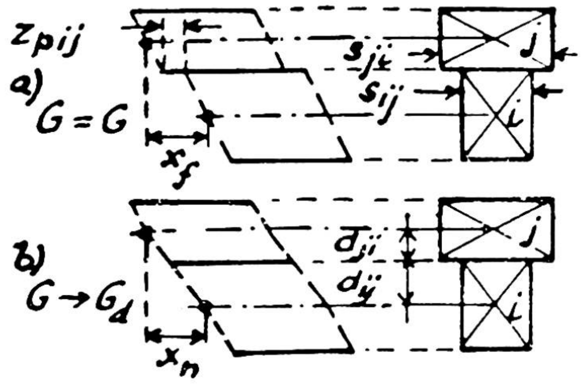

Fig.4/a,b

If we want less fasteners n'_{sij} on boundary areas, we take into account $r_i \to r_i' = q.k_i$, $k_i < 1{,}00$ any values, for i=t, i=s recommendable $k_t = k_s = 1{,}00$. In this case the shear force in the joint i-j diminishes from $P=\tau_{ij}.s_{ij}.\Delta L$ to $P'=\tau'_{ij}.s_{ij}.\Delta L$, i.e. the factor of reduction of fasteners $t_{ij}=\tau'_{ij}/\tau_{ij}$, so (r... boundary [edge] element):

$$t_{ij}=\sum_{j}^{r} k_p.A_p.y_p \Big/ \sum_{j}^{r} A_p.y_p\ ,\quad p=j,\ j+1,\ j+2,\dots\ r \qquad ../13$$

The second correction arises because of the increase of deformations of fasteners and is acc. to Fig.5 equal c_{ij}:

$$c_{ij} = (y_j-y_i).(1-q)/\{y_j.(1-k_j.q)-y_i.(1-k_i.q)\} \qquad ../14$$

and:

$$n'_{sij} = n_{sij}.t_{ij}.c_{ij}\ \ (n_{sij}\ \text{for}\ k_i=1{,}00,\ \text{Eq.12/b}) \qquad ../15$$

In Eq.7, the values "a" should be presented also for other cross-sections. To avoid the known complicated equation, we take (Fig.6):

$$a \doteq 1{,}2.b_{max}.\sum H_{ij} \Big/ \sum (d_{ij}+d_{ji}) \qquad ../16$$

3. APPLICATIONS OF THE GIVEN METHOD IN BENDING, BUCKLING AND TORSION.

In bending, the effective moment of inertia I_e (from Eq.5) is taken for the calculation of deformations. To calculate the stresses we put from Eq.3 the value $C = M/I_e$ in the Eq.2 and we get:

$$\sigma_v = (M/I_e).(y_{iv}+r_i.y_i) \qquad ../17$$

A special example is a lattice beam, flanged additionally. Here the influence of diagonals and verticals is exchanged by the web of the width b''_o (which is not considered in the sums acc. to Eqs.3,4,5).

In buckling, the total slenderness is calculated acc. to equation:

$$\lambda_{tot} = \sqrt{\lambda_e^2 + \lambda_1^2}\ ,\ \lambda_e = L_i/i_e,\ i_e=\sqrt{I_e/A} \qquad ../18$$

where L_i is for slenderness competent length, and λ_1 the local slenderness of the most inconvenient part of the beam.

In torsion, the method can be used in the calculation of the composite box-cross-sections, acc. to Fig.7. Supposing the equal rotation-angle "u" of every single part and of the whole, we get:

$$u = T.L/C_d\ ,\ C_d = \sum C_{di} + C_{da}\ ,\ C_{di}=G.Y_i,\ C_{da}=G_a.Y_a \qquad ../19$$

where T is the moment of torsion, L is the length, where the angle "u" is measured, C_d is the torsional stiffness, G is the timber shear modulus, G_a is the reduced shear modulus, Y_i is the local tors. moment of inertia, Y_a the associated tors. mom. of inertia:

$$G_a = G/(1+100.z_{nij}.G.D/N_{sij}.n_{sij}.U_{ij}),\ Y_a \doteq 4(H-D)^2.(B-D)^2.D/\sum U_{ij} \qquad ../20$$

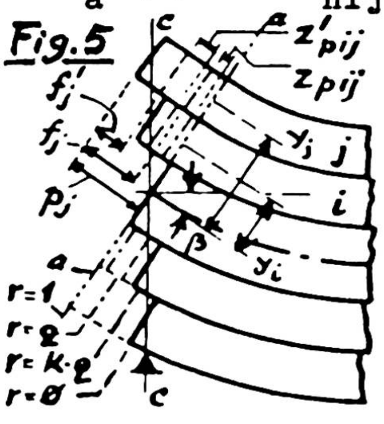

Fig.5

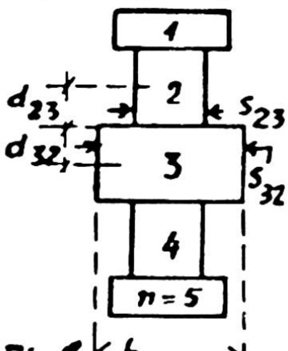

Fig.6

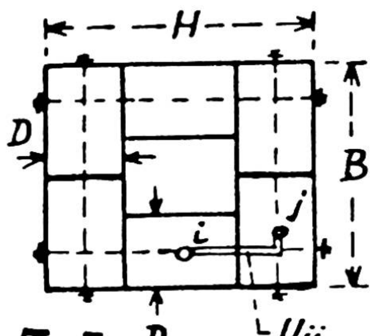

Fig.7