II

Vierendeel Trusses in Timber and Frame Design

Poutres Vierendeel en bois et dimensionnement des cadres

Vierendeel-Träger aus Holz und Rahmenbemessung

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SUMMARY
Tests on vierendeel girders of timber, both with glued and nailed plywood gussets, showed dimensioning conditions for indetermined constructions.

RESUME
On a effectué des tests sur des poutres Vierendeel en bois; les goussets en contre-plaqué cloués ou collés. Les résultats de ces essais permettent de dégager certaines règles de dimensionnement pour les éléments porteurs hyperstatiques.

ZUSAMMENFASSUNG
Die experimentelle Untersuchung von Rahmenträgern aus Holz mit aufgeleimten oder aufgenagelten Knotenplatten aus Sperrholz ergab Bemessungskriterien für die Konstruktion statisch unbestimmter Tragwerke.

1. INTRODUCTION

In the scheme of research on statically indetermined constructions, tests were carried out in 1972 on vierendeel-trusses in timber with glued plywood gussets and others with nailed gussets, to obtain information about the stiffness, creep and strength, and redistribution of forces.
Motive to test the portalform of a vierendeel beam as type of construction was among others the verification of some aspects of building design, from which it e.g. appeared, in a worked out example of an office building in timber, that only minor differences exist in force distributions between buildings with stiff- and semi stiff connections.

2. SOME RESULTS OF THE INVESTIGATION ON VIERENDEEL-TRUSSES

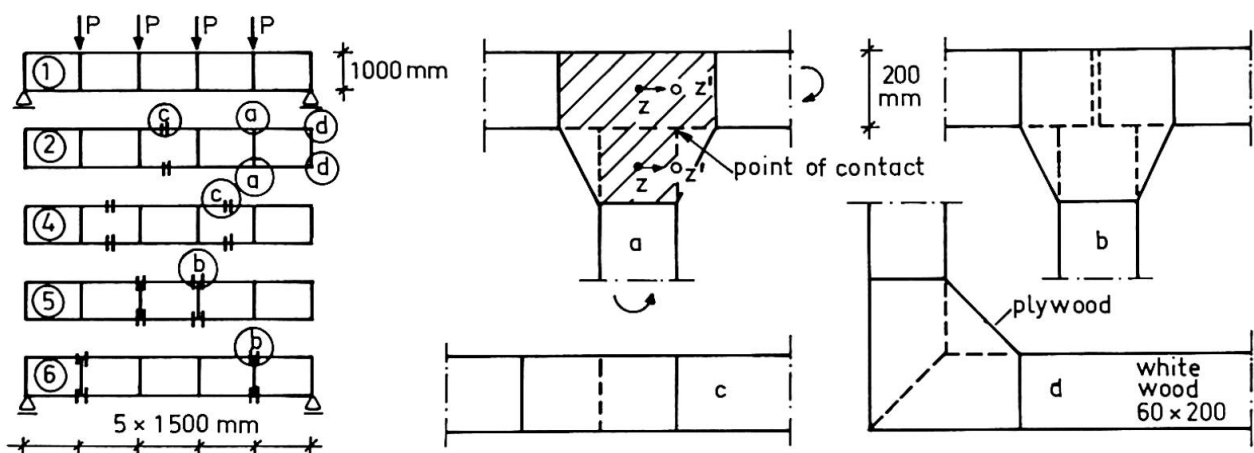In fig. 1 the scheme of the tested girders and shape of the joints is given.

Fig. 1 Places member joints: ‖, and plywood gussets.

Besides trusses with continuous horizontal members, trusses with joints in these girders on different places, low loaded (c) or high loaded (b) at the elastic stage, were tested. The loading was short-term and long-term followed by recovery periods and at last, the ultimate load test was done. The strength of the nailed connections could be explained from the mechanism of failure of the nails and the volume effects of the crushing strength of the wood. Tests on separate joints showed a highly non-linear force-deformation-diagram with a great hysteresis and even a linear approximation up to working load is dependent on the load level and load history. To account for this, a rheological model was set up, that could describe the deformations and the hysteresis by only a few parameters, leading to the loading expression of the moment M on the nail pattern of the gussets of:

$$M \approx C\,F_u\,\frac{I_p}{R}\sqrt{\frac{\psi}{\psi_m}} \geq 1.3\,F_u\,\frac{I_p}{R}\sqrt{\frac{\psi}{\psi_m}} = M_u\sqrt{\frac{\psi}{\psi_m}} \quad (= M_u \text{ as } \psi > \psi_m) \qquad (1)$$

F_u = ultimate transverse nail load; I_p = inertiamoment of nail pattern in sq.mm

ψ = rotation of connection $\qquad\qquad$ ψ_m = max. value of ψ before full plasticity

R = greatest nail distance to the centre of rotation in mm

C = 4/3 à 3/2 depending on the form of nail pattern. Using always C = 1.3, it is allowed to optimize I_p/R and disregard very excentric placed nails.

For a connection only loaded by ultimate tension P_u, C = 1, so the moment deformations at $M_{u/\omega}$ are much greater than the tensile deformations at

$P_{\mu/\omega}$ and the equivalent linear translational stiffness factor is much greater than the rotational one. (It is not possible to give one stiffness factor).
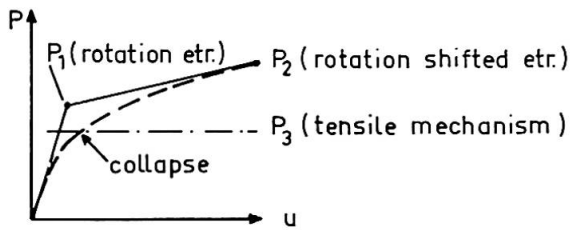

Fig. 2 scheme P-u

By the great plastic rotation of the joints, the stud contacted the horizontal member and the centre of rotation of the nail pattern shifted accordingly to this point of contact. So hardening occurred and a higher ultimate moment because of this eccentricity. (fig. 1-a).

Besides a joint rotation mechanism, a tensile force mechanism is possible as minimum mechanism (fig. 4).

The ultimate values of load P on the trusses can be calculated in 3 different manners. Based on the mechanism of rotation around the nailgroup centre a value P_1 is found representing the beginning of flow (fig. 2). A value P_2 can be calculated with the shifted rotation centre (fig. 1-a) at greater displacements and finally a value P_3 (fig. 2) is found if a tensile force mechanism is the limiting factor. In the tests was seen that P_3 (without hardening) actually occurred (fig. 2; 4).

For the mechanism of joint rotation it can be deduced from plasticity the the diminishing of the maximum moment by an axial force H is:

$$M_{max} \approx M_u \left[1 - \left(\frac{H_{actual}}{H_{ultim.}} \right)^2 \right] \quad (2)$$

In table 1 the so calculated ultimate truss loads P are given

In beam 2 with continuous horizontal girders an incomplete mechanism appeared. (fig. 4). In this case the ultimate moment in the continuous members was necessary for a mechanism, but it appeared that there was practically no plastic

Table 1. Beams with nailed joints

ultimate force P		calculated in kN		
		joint rotation		
beam nr. (fig. 1)	measurement P in kN	P_1 min. beginning	P_2 max. shifted centre	P_3 tensive force mechan.
3	11,0	11,3	12,4	11,0
4	11,6	11,2	12,0	13,2
5	11,8	9,9	12,4	11,7
6	14,9	10,9	14,4	21,0
2	13,7	13,5 equilibrium method		
		___ = occurred min. mechanism		

deformation of these members. Because of knots and the many nailholes a brittle fracture with a volume-effect was possible in the horizontal tensile member. Regarding the trusses with glued gussets a mechanism was possible in the high loaded, glued, joints near to the studs (fig. 1 beam 5; 6). The rotational capacity was not sufficient for a redistribution to a mechanism with the, in the elastic stage low loaded, joints in the girders in between the studs. (see fig. 1, beam 3; 4.). They acted as if these girders were continuous. For continuous horizontal members, or members with low loaded joints in the elastic stage, a partial mechanism appeared in the glued joints near the outer studs before the wood reached its maximum moment, what was possible by the great elastic deformation of the members.

3. TIMBER FRAME DESIGN

The method of minimum mechanism of collapse can be used if only nailed joints are involved in the inelastic rotation. Because of the great rotational capacity every complete mechanism, where elastic deformations can be neglected, is possible. However, if the construction is not yet statical determinated for the minimum joint-mechanism, the ultimate moment in the wood is necessary in that

part to form a mechanism, and control of this partial mechanism, with brittle wood "hinges", is necessary. In this case, and also for glued connections, a design method has to be used, accounting for elastic and small plastic deformations. Such compatibility method, based on the virtual work equations, is recommended by the C.E.B. for concrete and can be extended for members with joints. Starting point is to choose sufficient hinges (restraint-releases) in the whole construction to make it statically determinate; so one hinge less than necessary for a mechanism. At the design stage, long-term effects on force distribution can be disregarded because of the increase of the possible deformations with creep. For nailed joints the short-term enforced displacements too, can be omitted because of the great deformation capacity. As a first approximation, the influence of the shearing force can be neglected, accounting this deformation by an adjustment of the modulus of elasticity. Also the influence of the normal force is small for frames; even for members with joints, as was seen in the tests.from the complete joint tensile mechanism, neglecting all joint rotations, that was possible. However, adjustments for collumn-instability and strength of the connections, (eq. 2) are necessary. For these reasons the virtual work equations of the n hinges become: ($k = 1 \ldots n$).

$$\int \frac{M_k \cdot M_o}{EI} \, ds + X_k \int \frac{M_k^2 \, ds}{EI} + \sum_i^{i \neq k} X_i \int \frac{M_k \cdot M_i \, ds}{EI} + \sum_m \frac{M_k \, M_o}{C_m} + X_k \sum_m \frac{M_k^2}{c_m} +$$

$$+ \sum_i^{i \neq k} X_i \sum_m \frac{M_k \, M_i}{C_m} = - \theta_k - \sum_m M_k \, \theta_m \qquad (3)$$

X_k = statical unknown at hinge k; M_o = bending moment on the member due to the external load, and M_k, due to $X_k = 1$; EI = bending stiffness of members and C_m of the joints, in, or intermediate between the release sections; θ_k = the in elastic rotation at hinge k if the hinge is a joint and $\theta_k = 0$ if the hinge is in the wood of the member; $\sum M_k \, \theta_m$ = the inelastic contribution of the joints between the critical sections.

Choosing all joints as release sections, there are no intermediate joints and inelastic contributions, so the equations (3) become: ($k = 1 \ldots n$)

$$\int \frac{M_k \, M_o \, ds}{EI} + \sum_i X_i \int \frac{M_k \, M_i \, ds}{EI} + \frac{X_k}{C_k} = - \theta_k \qquad (4)$$

A bi-linear approximation of the force-displacements diagram of a joint is apparently accurate enough, so in eq. (4) it is possible to replace X by X_u, the ultimate moment in the hinges and if there are only connector hinges, all values of the left part of equation (4) are known and so θ_k is immediately known. If there are wood hinges too, where $\theta = 0$, a set of simultaneous equations in X_i of the unknown wood-hinges remain, and in general not all the values of X_i of these hinges reach the ultimate value. X_i can be eliminated or resolved from the equations (starting at an end-hinge) and used in the connector-hinge-equations, where $\theta \neq 0$. So all terms are known then and all θ's can be determined and controlled to have the correct sign and value. If not, adjustments to the first trial X_u- or X_i-values must be made, putting $\theta = 0$ where the sign was wrong. Fig. 3 gives an illustration of this method applied from symmetry to a quarter of the vierendeel-beam.
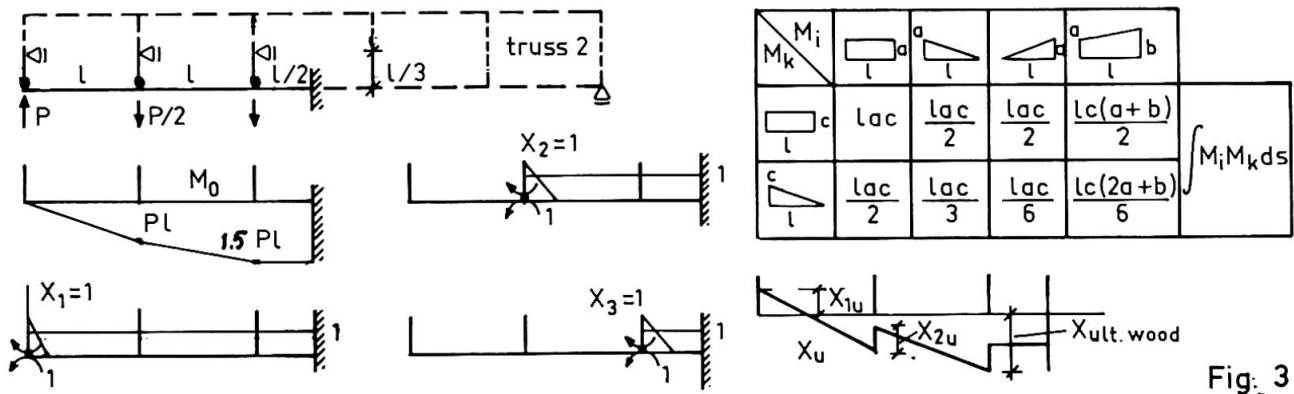
Fig. 3

In this case however, it is easy to look to the equilibrium of the horizontal girders assuming all mechanical joints flowing and controling if the deformation of the continuous girders is enough to justify that assumption (table 1 beam 2).

4. SOME CONCLUSIONS

- A pure mechanism (elasticity neglected) is possible for nailed joints if only these joints are involved in the mechanism and design can be based on this mechanism.
- When the ultimate moment in the wood is necessary to form a mechanism, an incomplete mechanism is possible, because of the small plasticity of the wood, and a compatibility method has to be used according to eq. (4).
- The same applies for glued joints if the force distribution of the mechanism is not to far away from the elastic distribution (elasticity can not be ignored). So, because of the minor redistribution, it seems better for timber with glued connections to use the simple approximation of linearity up to the ultimate stage, and the linear calculation of the rolling shear of the glued gussets, as a reasonable lower bound of the strength.
- The strength of the nail-pattern in the gussets for a moment is greater than 1.3 x the linear calculated value (eq. 1).
- The deflections of the vierendeel beam at working stage can be found by the method for built up beams, or by putting $\theta = 0$ everywhere in eq. (4).
- From the recovery periods in the tests, it was seen that visco-plasticity was probably small. So a visco-elastic model is possible to determine the creep factor. This factor was about 1.8 for the nailed beam and about 1.6 for the glued one.

REFERENCE

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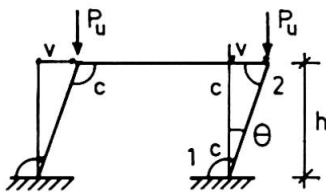
APPENDIX   PROPOSAL FOR TIMBER FRAME DESIGN

elementary mechanism
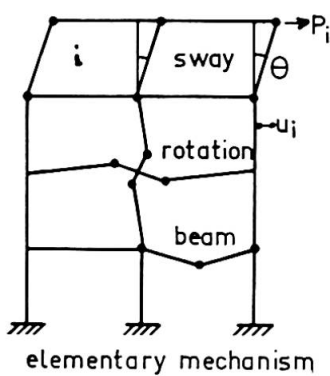
$$\sum M_{uj} \cdot \theta_j = \sum P_i u_i \qquad M_{uj} \text{ and }$$

$$\sum P_u \cdot v = \sum M_j = \sum C_j \theta_j \rightarrow$$

$$P_u \cdot v = 2C \frac{v}{h} \rightarrow P_u = \frac{2C}{h}$$

1. Mechanical joint mechanism is possible.
   The true (minimum) collapse mechanism has one degree of freedom, so is an elementary mechanism or combinations of these resulting in hinge eliminations. For timber the wood hinges must disappear, so the beam mechanism with the wood hinge in the middle is not possible.
   In general also rotational mechanisms contain one or more wood hinges for stiffness reasons. However, combinations of rotation- and sway-mechanisms (with wood-hinge elimination if relevant) result in elementary sway-mechanisms in the joints. So only joint-sway mechanisms have to be considered. Equilibrium of mechanism i requires that: M_{uj} and θ_j are plastic moments and relative rotations of joint j respectively, and W_i and δ_i are loads and their corresponding displacements in the mechanism i. In M_{uj} is the influence of the normal force eq. 2. For buckling at yield, from the method of "split rigidities", the critical load Pcr is:

$$\frac{1}{P_{cr}} = \frac{1}{P_u} + \frac{1}{P_b} \quad \dots \quad (5)$$

P_u = buckling load of the rigid member system
   $= \sum C/n$

P_b = Euler buckling load of the elastic system with rigid joints. P_{cr} is immediately known by splitting rigidities in column, and joint with girders together.

2. Partial mechanism with "wood-hinges".

If there are not enough joints for a joint-mechanism in a part of the construction comptability has to be controled according to eq. 4:

$$\int \frac{M_k M_o}{EI} ds + \sum_i M_{u,i} \int \frac{M_k M_i}{EI} ds = -\theta_{t,k} \quad \text{with} \quad \frac{M_{u,k}}{C_k} < \theta_{t,k} (< \theta_{ult,k}) \text{ or total}$$
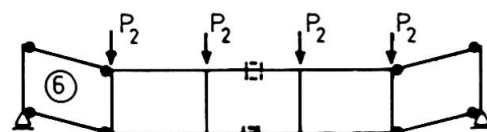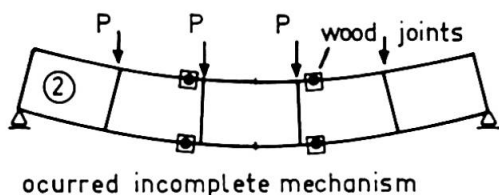
joint rotation must be greater than the elastic part at yield and $\theta_{t,k} = 0$ for woodhinges. As above, instabilitey (at yield) can be easily checked by the method of split rigidities.

occurred tensile force mechanism
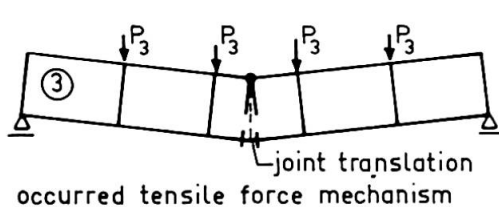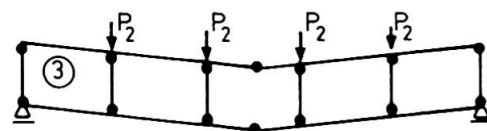
possible joint rotation mechanisms
(beam 3 and 6) and begining of flow (P₁)

ocurred incomplete mechanism

to be checked incomplete mechanism []
(force distribution not totally known)

Fig. 4 Same mechanisms of failure.