II

Research on Double-Tapered Glulam Beams

Recherches sur des poutres à hauteur variable en lamellé-collé

Untersuchungen über das Tragverhalten von Satteldachträgern aus Brettschichtholz

RICHARD M. GUTKOWSKI
Prof. of Civil Engineering
Colorado State University
Ft. Collins, CO, USA

GEORGE R. DEWEY
Structural Engineer
J. F. Sato and Associates
Denver, CO, USA

SUMMARY
Reliability based design and limit states design have the advantage of unifying codes and establishing consistent safety margins. Improved knowledge of ultimate strength performance and service load displacements, based on refined analysis procedures and experimental verifications, are important prerequisites to such code formats. This paper reports a recent theoretical and experimental study of the service and ultimate behavior of double-tapered glulam beams. Important implications for future design codes are briefly described.

RESUME
Les méthodes de dimensionnement basées sur les états limites permettent une meilleure vue d'ensemble des différentes normes et la détermination des facteurs de sécurité. On peut améliorer les connaissances sur la résistance ultime et les déformations sous charges de service par des méthodes de calcul sophistiquées et par des essais en laboratoire; ceci est une condition essentielle pour pouvoir appliquer la théorie aux états limites. On décrit, dans cet exposé, les résultats d'une étude théorique et expérimentale effectuée sur des poutres en lamellé-collé à hauteur variable. On a étudié le comportement sous les charges de service ainsi que la résistance ultime. On décrit brièvement quelques points importants dont il faudrait tenir compte lors de l'élaboration de futures normes.

ZUSAMMENFASSUNG
Auf Grenzzuständen beruhende Bemessungsmethoden erleichtern die Vereinheitlichung der Normen und ermöglichen die Festlegung der Sicherheitsmargen. Verbesserte Kenntnisse über Grenztragwiderstand und über das Verformungsverhalten unter Nutzlasten, die sowohl durch verfeinerte Berechnungsverfahren als auch durch Versuche erreicht werden, sind hiefür wichtige Voraussetzungen. Dieser Beitrag berichtet über eine neuere theoretische und experimentelle Untersuchung über das Verformungs- und Tragverhalten von geraden Satteldachträgern aus Brettschichtholz. Daraus ergeben sich wichtige Folgerungen für zukünftige Normen.

1. INTRODUCTION

Glued-laminated timber (glulam) beams are popular structural members in build-
ings, bridges and other structures. As such a sound understanding of the
physical behavior of any such member is essential. It is important that glulam
members be accurately modeled in accordance with their inhomogeneous, ortho-
tropic material properties. Rapid computer analysis methods are now commonplace
and permit the analyst to eliminate many of the former analytical assumptions.
The finite element method of analysis is a well established method of analysis
and has been shown [1,2,3,4] to produce improved solutions to many problems in-
volving wood. Analytical and experimental work reported in this paper indicates
similar success for this method in the analysis of double-tapered glulam beams.

2. SCOPE OF THE STUDY

The analysis of a double-tapered shape (Fig. 1) is a complex problem. Unusual
stresses are induced in the member by virtue of the varying cross-section. Con-
centration of stresses at the apex, possible deep beam behavior, and potential
radial ($\perp$ to grain) stresses further complicate the analysis. Analytical
studies and a full-scale testing program comprise the major components of this
paper and are part of a more extensive investigation to be reported later. In
the experimental study, four full-size double-tapered members are investigated
at working and ultimate loads. Results are compared with related past research
and theoretical predictions obtained by an established orthotropic finite ele-
ment technique [5] applied to the model in Fig. 2.
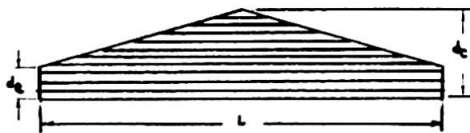
Fig. 1 Typical Beam
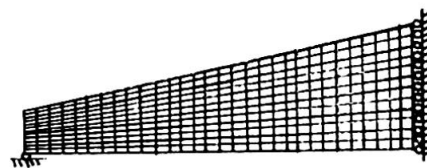
Fig. 2 Finite Element Mesh

3. BACKGROUND

Much research has been aimed at accurately predicting radial tension stresses.
Wilson [6] studied the radial tension problem in 1939. He employed the simple
radial stress formula

$$f_r = \frac{3}{2} \frac{M}{Rbd} \qquad (1)$$

in which R = radius of curvature; M = applied bending moment; b = width of beam,
and d = depth of beam. The formula is strictly correct only for a curved member
of constant cross section subject to pure bending and is extremely conservative
for pitched and curved beams of variable cross section. Norris [7] verified
Wilson's formula by adopting a plane stress solution obtained by Carrier [8] in
his study of curved wood beams (of constant cross section and subjected to pure
bending).

In a succession of studies Foschi [9,10,11,12] and Fox [13] worked independently
and in collaboration [14] to determine stress distributions in the central
portion of double-tapered pitched and curved timber beams subjected to combined

bending, shear and axial forces. Elasticity solutions (experimentally verified) were used and a significant outcome of their work was the prediction of maximum radial stress according to

$$f_r = K_r \frac{6M}{bd^2} \qquad (2)$$

in which K_r is a radial stress factor depending on the pitch of the upper edge and d_c is the midspan depth. Values predicted by Eq. 2 are for pure bending and proved to be (overly) conservative for other loadings. Equation 2 has been incorporated in some design codes [15,16].

Thut [2] developed and employed an orthotropic trapezoidal finite element model to determine the stress distributions in the central portion of pitched-tapered and curved glulam beams. He recommended stress factors for maximum radial tension and compression stresses at the apex section due to pure bending. Later studies by Gopu [3,4] were aimed at developing a rational design procedure for double-tapered pitched and curved beams. His experimental study consisted of testing six full-size unreinforced and two reinforced Douglas-fir beams. A key finding was that failure in members of practical configuration (including re-inforced members) occurs in a radial tension mode. He also conducted an extensive parameter study using a modified version of Thut's orthotropic finite element model. Double-tapered beams with straight soffits were included in the range of the parameter study. Gopu refined Eq. 2 to provide more accurate prediction of the maximum radial stress for uniformly loaded members and developed a similar formula for determining the maximum flexural stress.

Previous research on straight-bottomed tapered beams has been limited. In an early study Maki and Kuenzi [17] derived closed-form theoretical equations for the principal stresses. An equation for flexural stress was derived using elementary Bernoulli-Euler theory of bending for an isotropic material. Expressions for maximum shear and radial stresses, at a section, were developed as functions of the flexural stress, the pitch of the taper and material properties. Tests were conducted on small specimens of aluminum and Sitka Spruce. However, none were tested to failure. It was further recommended that the interaction of the principal stresses, as proposed by Norris [18] be used to predict the ultimate strength of straight-bottomed tapered beams. The findings of Maki and Kuenzi are the basis of design methods presently required in one code [15].

4. EXPERIMENTAL WORK

4.1 Specimens

Four specimens (Table 1) were tested at working and ultimate load in a simple beam configuration. Moisture content at time of testing ranged from 5 to 7% for

Table 1

Specimens	d_e (in)	d_c (in)	width (in)	span (ft)	MOE (psi)	E_R (psi)
DF42DT	12	42	5 1/8	30	2,030,000	56,000
SP42DT	12	39	5	30	1,460,000	78,000
DF27DT	12	27	5 1/8	30	2,040,000	44,000
SP27DT	12	27	5	30	1,530,000	60,000

DF = Douglas-fir (1 1/2 in. lams), SP = southern pine (1 in. lams), DT = double-tapered.

Douglas-fir and 6 to 8% for southern pine. Listed MOE values are the average value for all laminations. In the finite element analysis these values were used except for the row of elements adjacent to the straight bottom. For the bottom row elements, a weighted MOE was used based upon the proportion of the element area occupied by tension laminations and lesser grade material. E_R values were measured from compression tests on undamaged pieces of the tested specimens. All other elastic parameters were calculated by a known procedure [19] which uses the longitudinal modulus as a predictor.

4.2 Conduct of Tests

Each beam was subjected to a pair of symetrically placed loads located 4 ft to each side of the apex. Loads were applied to a spreader beam by a single hydraulic actuator. Stabilizer frames were used to prevent lateral motion. In the working load tests both longitudinal ($||$ to grain) and radial ($\perp$ to grain) strains were recorded by specially devised strain transducers [3,4]. Deflection at various locations were measured by direct current displacement transducers. In the ultimate load tests no strain data was taken and only midspan load-deflection data was measured.

4.3 Results

For repetitive working load tests, load deflection data was essentially identical for all cycles and was linear elastic. The "total" (actuator) loads applied were 10000 lbs. and 6000 lbs. for the nominal 42 inch and 27 inch deep specimens, respectively. No measurable creep or visible distress occurred. Results of the ultimate load tests are as follows:

DF42DT - Radial tension failure at mid-depth of the centerline at a total load of 52000 lbs. Maximum deflection was 2.5 in.

SP42DT - Bottom finger joint (96 in. from one end) failed at a total load of 46000 lbs. Continued loading to a 54000 lbs total produced a flexural failure at the same location. Maximum deflection was 3.3 in.

DF27DT - Radial tension failure at midspan 18" above the bottom surface at a total load of 24800 lbs. Maximum deflection was 2.34 in.

SP27DT - Flexural failure initiated at a finger joint (48 in from centerline) at a total load of 33000 lbs. Maximum deflection was 3.50 in.

5. COMPARISON WITH THEORIES

5.1 Stresses

Longitudinal and radial strain data had to be measured at different locations through the depth of each selected cross-section. A least-squares fit to each strain distribution was employed to produce values at common points before conversion to stresses were made. A third degree polynomial was found to give good fits to the resulting stress distributions. Midspan experimental flexural stress distributions are compared with the finite element predictions in Fig. 3 and 4. Results for the other specimens were very similar.
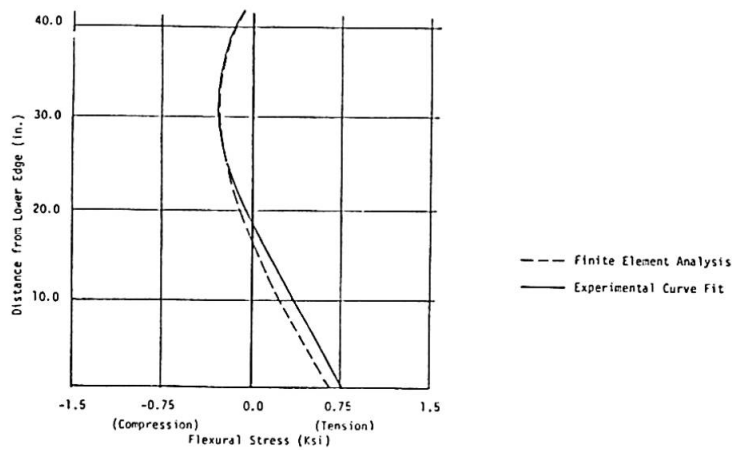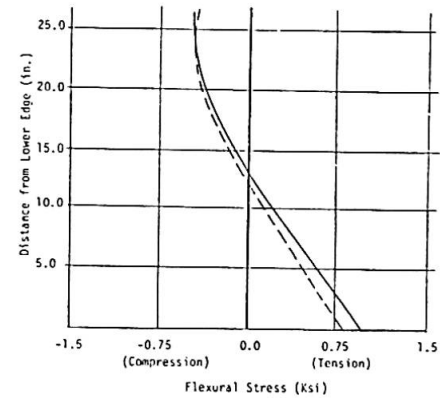
Fig. 3   DF42DT ⨎ Flexural Stress

Fig. 4   SP27DT ⨎ Flexural Stress

Radial stress comparisons are shown in Figs. 5 and 6. Discrepancies evident in some cases are partially attributed both to the sensitivity of the theoretical values to E_R, which exhibited high variability, and the high scatter in MOE values of the individual laminations.
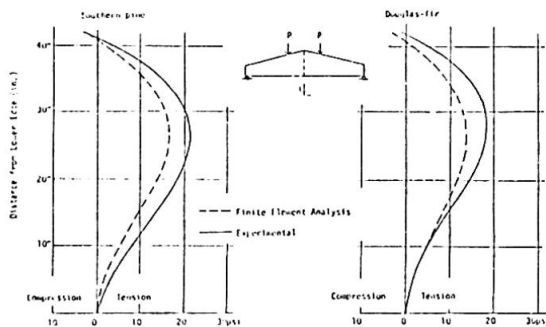

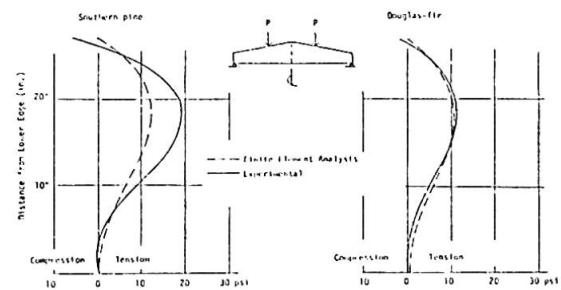Fig. 5   Radial Stress for 42" Beams

Fig. 6   Radial Stress for 27" Beams

Table 2 lists various theoretical stress comparisons for the ultimate load values. All bending stresses are tensile and the absolute maximum in the span.

Table 2   Stresses at Failure

Specimen No.	Failure Load (lbs.)	Maximum Bending Stress at Failure (psi)		Maximum Radial Tension Stress at Failure (psi)	
		FEM	Mc/I	FEM	AITC[15]
DF42DT	52000*	4590	3810	83.2	85.4
SP42DT	54000	5000	4500	91.8	90.9
DF27DT	24800*≠	4180	3620	53.2	47.3
SP27DT	33000	4840	4940	70.4	64.5

* Failed in radial tension (listed bending stress did not control)
≠ DF27DT was reloaded to 29600 lb after initial radial tension failure

As noted earlier, the finite element maximum flexural stress predictions exceed those of the classical Bernoulli-Euler theory. This is true even in the two cases where radial tension governed the failure.

Two design codes [15,16] provide graphical aids for determining K_r for double-

tapered, pitched and curved beams. Values for straight bottom beams can be obtained by extrapolating the curves to the case where d_c/R_m is equal to zero. A comparison of these values with those predicted by the finite element model is included in Table 2. Excellent agreement is obtained for all specimens.

5.2 Deflections

Deflection data recorded in the working range tests were compared with the finite element results and a conventional method. Conventional values were calculated by use of Castigliano's theorem and numerical integration. The MOE values listed in Table 1 were employed.

Table 2.   Centerline Deflections at Working Load Limit

Specimen No.	Centerline Deflection Experimental (in.)	Finite Element Analysis		Castigliano's Theorem	
		Deflection (in.)	% Difference with Expt.	Deflection (in.)	% Difference with Expt.
DF42DT	0.417	0.446	+ 6.9	0.389	− 6.8
SP42DT	0.586	0.583	− 0.5	0.539	− 8.1
DF27DT	0.591	0.623	+ 5.4	0.604	+ 2.3
SP27DT	0.789	0.823	+ 4.2	0.780	− 1.2

It appears that the finite element method overestimates the midspan deflection by as much as 7%. Conversely, treatment of wood as an isotropic material, as inferred in the use of Castigliano's theorem, results in unconservative deflection values. In these cases, the deflection is underpredicted by as much as 8%. Shear deflection ranged up to 16% of the flexural deflection. The 16% values was for DF42DT. The finite element model inherently includes shear deformation and its isolated effect cannot be distinguished from the total.

6. CONCLUSIONS

Several significant findings were noted in this work
- Flexural stress distributions measured or calculated by the FEM are curvilinear in the central region and linear near the ends. Extreme fiber tensile stresses equal or exceed the compressive stress at all sections and are greater than predicted by Mc/I. These observations are in agreement with past findings [17].
- Radial stresses exist in the central region with the maximum at centerline. The stress mode governed the failure of the Douglas-fir specimens. This is significant because current design codes do not require a check of this stress condition. Use of the K_r factor method for $d_c/R_m = 0$ is a possible means for making this check.
- Maximum horizontal shear stress is some distance away from the supports and at the tapered surface. This is contrary to VQ/It but agrees with past work [17].
- Locations and modes of failure were not in agreement with the interaction formula for combined stresses.
- Castigliano's theorem underestimates the midspan deflection.
Detailed discussion of these points can be found in the complete report [20].

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