Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 11 (1980)

Artikel: Approximate analysis and safety of structures

Autor: Baratta, Alessandro

DOI: https://doi.org/10.5169/seals-11406

X

Approximate Analysis and Safety of Structures

Méthodes de calcul approchées et sécurité des structures

Näherungsberechnungen und Tragwerksicherheit

ALESSANDRO BARATTA
Dr. Ing., Assist. Prof.
University of Naples
Naples, Italy

SUMMARY
The influence of errors involved by approximations in structural design is discussed in the context of the probabilistic approach to structural safety philosophy. A definition of the ,,design load'' is proposed, and distinction between ,,design'' and ,,service'' loads is related to error estimates. The ,,reliability error''is also defined, and a practical example is dealt with for a comparison of the upper bound to the actual value of the reliability error.

RESUME
L'influence des erreurs induites par les approximations de calcul est discutée dans le contexte de la philosophie probabiliste de la sécurité structurelle. On propose une définition de ,,charges de projet'' et on introduit la distinction entre charges de projet et charges d'exploitation, en relation avec l'évaluation des erreurs. L',,erreur en sécurité'' est également définie; un exemple numérique permet d'en déterminer la valeur supérieure.

ZUSAMMENFASSUNG
Der Einfluss von auf Näherungsberechnungen beruhenden Fehlern wird im Zusammenhang mit dem wahrscheinlichkeitstheoretischen Ansatz der Tragwerksicherheit diskutiert. Eine Definition der ,,Bemessungslast'' wird vorgeschlagen, wobei der Unterschied zur eigentlichen ,,Nutzlast'' auf Fehlerschätzungen beruht. Der sog. ,,Zuverlässigkeitsfehler'' wird ebenfalls definiert und in einem praktischen Beispiel sein oberer Grenzwert abgeschätzt.

1) INTRODUCTION

The probabilistic approach to structural safety, while originating many questions concerning research of suitable techniques to deal with random variables and/or random functions in the area of structural analysis (for a review of such problems, see for instance Ref. [1]), also enhances the role of interactions between the solution of mathematical problems involved by structural design and the use that can be made of the results of computations. Really, the main difference between the engineering approach to continuum mechanics problems and the analogous treatment by mathematical physics, should be found in the circumstance that mathematical results are not employed directly, but are always filtered, and often neglected in the details, by the engineer's judgement that enters into the rationale (i.e.: the set of rules) of structural design and analysis as a decisive factor, often conditioning even the output of seeming pure mathematical procedures.

As a matter of fact, behind the visible ease by which the "analysis pattern" is usually set up in regard to design loads, admissible stresses, structure geometry, etc., a somewhat more complex reality can be found, that most times could only be modeled by a multiplicity of situations, rather than by a single pattern.

In front of the above considerations, it is quite spontaneous to believe that exact mathematical results may be a too severe requirement, inadequate in view of the fading connections between the real structure and the analysis pattern, that can only be viewed at as a "conventional" description of the expected situation. Nevertheless, errors in analysis may be decisive to cause structural malfunctions, and the control of allowable approximations should be required and founded on well defined rational criteria.

A possible approach to the question is provided by the probabilistic theory of structural safety: since the safety certification is the main objective of the (civil) engineer, the influence of approximations on the safety indices can be investigated after having recognised the conventional character of the mathematical model.

In the treatment presented in the paper, the problem is simplified by assuming that the only source of uncertainty is the service load, often not predictable in detail, that is conventionally replaced by the design load in the analysis pattern.

The latter is supposed to be quite adherent to the real structure except, precisely, as for the loading condition, and a possible philosophy to evaluate the additional safety coefficient to be applied in consequence of approximation is explained.

2) BASIC ASSUMPTIONS

Assume that the service loads on the structure are constant in time and are applied once at the beginning of the structure's lifetime. Consider the structure to have (or to have been reduced to) a finite number of degrees of freedom, say n, and let F be the set of n-dimensional load vectors possibly acting on the structure. Let $\underline{f}$ be any possible load vector, $\underline{u}$ the structure response vector (for instance the displacements) and $\underline{A}$ the characteristic operator of the structure, so that the response equation is established as follows

$$\underline{A}\,\underline{u} = \underline{f} \tag{2.1}$$

and assume that such equation has one and only one solution for any $\underline{f} \in F$. Let $\bar{\underline{u}}(f)$ be an approximate solution of (2.1), and put

$$\bar{\underline{f}} = \underline{A}\ \bar{\underline{u}}(f) = \underline{D}\ \underline{f} \qquad (2.2)$$

$\bar{\underline{f}}$ is named the emerging load associated to $\underline{f}$. It is assumed that an approximate solution $\bar{\underline{u}}(f)$ can be found for every $\underline{f} \in F$, and that the set of emerging loads $\bar{\underline{f}}$ covers the whole F, when $\underline{f}$ varies in F.

Consider then that the degree of safety of the structure is expressed by the safety index β, substantially as proposed by Hasofer and Lind [2] with a slight modification in order to neutralize the dimension effect.

Let $\underline{f}$ be the generic load vector, $\underline{f}_m$ the expected load vector, and S'the boundary of the strength domain of the structure in F, $\underline{C}_f$ the covariance matrix of the load vector, and put

$$\underline{\sigma}_f = \sqrt{\underline{C}_f} \qquad (2.3)$$

Consider then the n-dimensional vector space X of reduced load vectors

$$\underline{x} = \underline{\sigma}_f^{-1}\ (\underline{f} - \underline{f}_m) \qquad (2.4)$$

and define the biased (by the dimension effect) and the unbiased safety indices β_n and β respectively, putting

$$\beta_n = \min_{\underline{x} \in S'_x} |\underline{x}|\ ; \quad \beta = \sqrt{\chi_1^{-1}\{\chi_n(\beta_n^2)\}} \qquad (2.5)$$

S'_x being the boundary of the strength domain S_x in the space of reduced variables, and χ_n the chi-square distribution.

Now, the conventional character of the design load $\underline{f}_d$ should be explicitly stated. It is assumed that coupling exact structural analysis with correct design and building rules, if the structure resists $\underline{f}_D$ it will also resist any possible service load, except possibly a sufficiently small number, whose probability of occurrence is low enough.

In symbols

$$(\underline{f}_D \in S) \Rightarrow (\beta \geq \beta_p) \qquad (2.6)$$

otherwise, $\underline{f}_D$ cannot be taken as the design load. Note that in the present treatment the circumstance is neglected that design requires sometimes the action of two or more design loads.

3) PRELIMINARY REMARKS

Let S be the actual safe domain (Fig.1.a). It is obvious that if the structure cannot be solved exactly, this domain remains unknown. Approximate analysis being possible, a different domain $\bar{S}$, the approximate safe domain, can be investigated. The same applies in the space of reduced variables (Fig.1.b), where the domains are named respectively S_x and $\bar{S}_x$. It is obvious then that only the seeming safety indices $\bar{\beta}$ can be controlled

$$\bar{\beta}_n = \min_{\underline{x} \in \bar{S}'_x} \underline{x}$$
$$\bar{\beta} = \sqrt{\chi_1^{-1}\{\chi_n(\bar{\beta}_n^2)\}} \qquad (3.1)$$

the actual safety index remaining unknown. Note however that, as proved in [3]

$$\bar{\underline{f}} \in S' \Leftrightarrow \underline{f} \in \bar{S}'$$

remembering that S' denotes the boundary of S. If the definition of emerging load $\underline{\bar{x}}$ associated to $\underline{x}$ is extended to reduced variables by the position

$$\underline{\bar{x}} = \underline{\sigma}_f^{-1} (\underline{\bar{f}} - \underline{f}_m) \qquad (3.2)$$

the same applies to domains $S_x, \bar{S}_x$

$$\underline{\bar{x}} \in S'_x \Leftrightarrow \underline{x} \in \bar{S}'_x \qquad (3.3)$$

Hence, everywhen the structure is analyzed by the approximate procedure under any load $\underline{f}$, and it is found that $\underline{f} \in S$ (i.e. the structure resists $\underline{f}$), really $\underline{f} \in \bar{S}$, and it is the emerging load that actually falls in S. Then, the difference between $\underline{f}$ and $\underline{\bar{f}}$, the vector $\Delta\underline{f} = \underline{f} - \underline{\bar{f}}$ provides the difference between S and $\bar{S}$. Accordingly, the difference $\Delta\underline{x} = \underline{x} - \underline{\bar{x}}$ provides the difference between S_x and $\bar{S}_x$.

Define now the numerical error ε as follows

$$|\underline{f} - \underline{\bar{f}}| \leqslant \varepsilon |\underline{f}| \quad \forall \underline{f} \in F \qquad (3.4)$$

and note that for most approximate techniques of solving structural models, ε can be actually calculated. It can be conceived that availability of bounds on $\Delta\underline{f}$ can be used to get similar bounds on $\Delta\underline{x}$, and that such bound can be used in turn to get a bound on β_n. In a previous paper [4], the Writer has obtained the following lower bound for the actual biased index

$$\beta_n \geq (1 - r\,\varepsilon)\bar{\beta}_n - \frac{\varepsilon r}{V_f} \qquad (3.5)$$

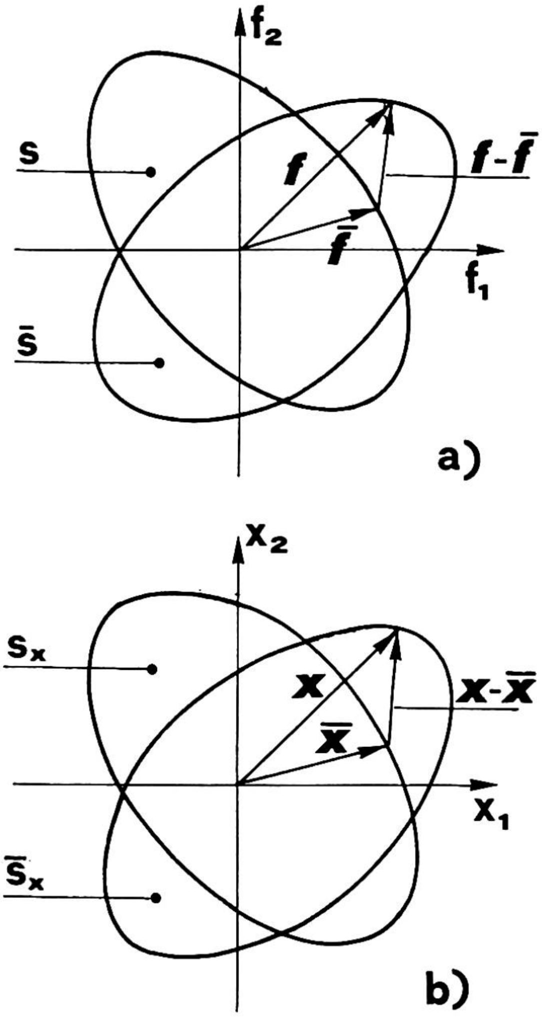


Fig. 1

where r is the condition number of the matrix $\underline{\sigma}_f$, and $V_f = |\underline{\sigma}_f^{-1} \underline{f}_m|^{-1}$ is a parameter that essentially specifies the coefficient of variation of loads. From eq. (3.5), the unbiased index β can also be bounded in an obvious way, and a condition for $\bar{u}$ to be considered an approximation of the true response is established in the form $r\,\varepsilon \leq 1$.

4) THE RELIABILITY ERROR

It is now necessary to specify a parameter allowing to evaluate the significant error introduced by approximations, in accord with the considerations presented in the Introduction.

Let γ^* be the coefficient to be applied to the design load $\underline{f}_D$ in order to neutralize errors in the solution procedure as regards the safety index, i.e. such that

$$\gamma^* \underline{f}_D \in \bar{S} \Rightarrow \beta \geq \beta_p \qquad (4.1)$$

$\bar{S}$ being the erroneous strength domain that could be calculated by approximate methods.

The <u>reliability error</u> ε^* is defined by the position

$$\varepsilon^* = \gamma^* - 1 \tag{4.2}$$

In Ref. [4] , it is proved that, if β_p is the prescribed value of the safety index, the following upper bound ε^*_u can be established for ε^*

$$\varepsilon^*_u = \frac{\varepsilon}{1-\varepsilon} \; \frac{1+\beta_{pn}V_f}{\beta_{pn}V_f} \tag{4.3}$$

This upper bound enhances some valuable features that can be attributed to the reliability error; namely, confusing ε^* with ε^*_u:

i) The (upper bound on the) reliability error does not depend on the error in load effects, but only on the error in applied and emerging loads, as defined in [3,4] .

ii) ε^* is a decreasing function of the product $\beta_{pn}V_f$, i.e. it is smaller when applied loads are affected by increasing uncertainty (larger V_f) and it is smaller when high reliability is required for the structure (larger β_{pn}), a result that agrees with the well known circumstance that the diagram of the failure probability versus the load factor becomes steeper and steeper as the failure probability decreases.

iii) ε^* is proportional to the numerical error , a result in agreement with numerical experiments.

iv) if $\beta_{pn}V_f = 0$, ε^* has a finite value only if the numerical error $\varepsilon = 0$. In other words, approximations would not be allowed if no margin of safety was guaranteed ($\beta_{pn} = 0$), or if the design philosophy rested on exact, deterministic, prediction of applied loads ($V_f = 0$).

This is probably due to the upper-bounding procedure used to obtain ε^* independently from analysis of load effects; in such case, analysis of the propagation of the error on load effects cannot be avoided.

5) NUMERICAL PERFORMANCE OF THE UPPER BOUND

In order to have an idea of the difference of the upper bound (4.3) to the true reliability error, the results obtained by the Author in Ref. [5] are considered, where the frame in Fig. 2 under stochastic loading (25 independent load components) was analyzed and designed in the elastic range, and exact solution of the classical equilibrium equations written by the displacement method was compared with the iterative solution of the same system, obtaining different levels of approximation by stopping the procedure after 1,2.., n iterations. The actual numerical error ε , and the reliability error were calculated by a Montecarlo procedure, for V_f = 5%, 10%, 15%, 20%. Here, the calculated ε^* is compared with the corresponding ε^*_u obtained by eq. (4.3), and the results are presented for V_f = 10% in Fig. 3, where h denotes the number of

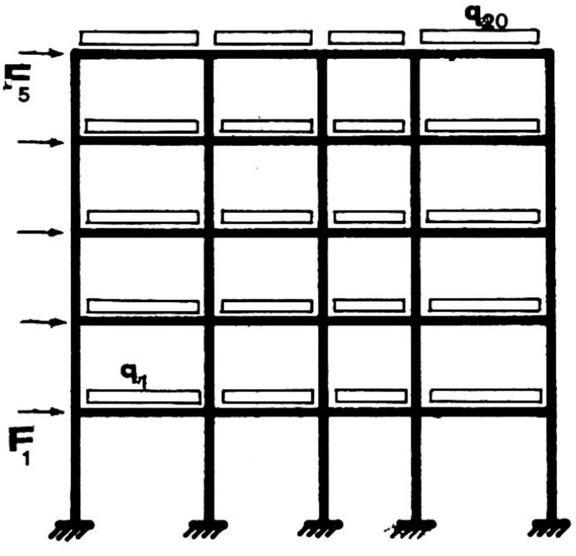
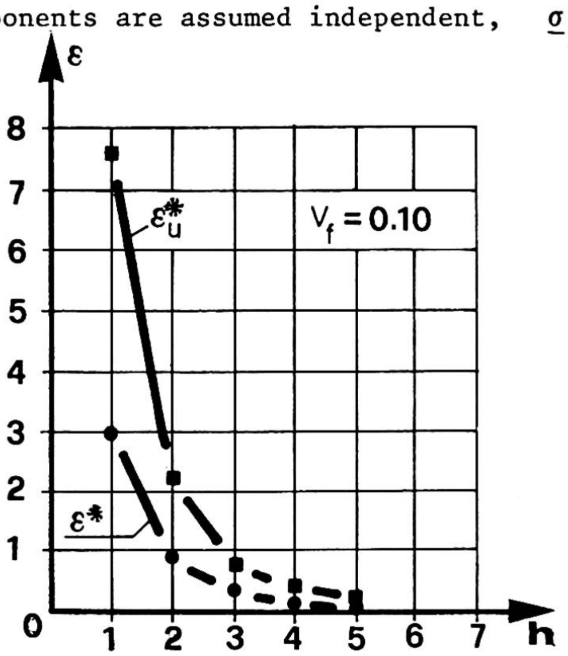


fig. 2

iterations. Note that, since the load components are assumed independent, $\underline{\sigma}_f$ is a diagonal matrix, and its condition number is equal to unity, and that β_{pn} is calibrated on the calculated collapse probabilities corresponding to different values of V_f in Ref. [5] .It should also be evidentiated that, in the case considered and for all values of V_f that have been investigated, the ratio $\varepsilon^*/\varepsilon^*_u$ is not much different from 2.



ACKNOWLEDGEMENT: Research supported by Italian C.N.R.

REFERENCES

1. AUGUSTI, G., BARATTA, A., CASCIATI, F.: Structural Response under Random Uncertainties, Proc. 3rd Int. Conf. "Applications of Statistics and Probability in Soil and Structural Engineering", 1979,vol. III, pp. 280-328.

2. HASOFER, A.M., LIND, N.C.: Exact and Invariant Second-Moment Code Format. Proc. ASCE, vol. 100, No. EM1, 1974.

3. BARATTA, A.: Non-linear Truss Reliability by Montecarlo Sampling, Proc. 3rd Int. Conf. "Applications of Statistics and Probability in Soil and Structural Engineering", 1979, vol. I, pp. 136-148.

4. BARATTA, A.: "Engineering" Error Evaluation in Approximate Structural Analysis", In printing.

5. BARATTA, A.: Criterio Probabilistico per la Sicurezza Strutturale e Metodi Approssimati di Calcolo. Proc. IV Not. Conf. Italian Ass. Theoretical and Applied Mechanics, vol. II, 1978, pp. 45-55.