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**X****Formal Failure Probability and Observed Failure Rate**

Probabilité formelle de ruine et fréquence observée des défaillances

Formale Versagenswahrscheinlichkeit und beobachtete Versagenshäufigkeit

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**SUMMARY**

Formal probabilistic reliability theory is an indispensable basis for choosing structural dimensions whatever be its ability to predict real failure rates. Most failures are caused by gross errors. It is argued that a measure of proneness to failure due to gross errors can play only a secondary role for this choice. However, it is essential for choosing the structural lay-out.

**RESUME**

La théorie formelle des probabilités est une base indispensable au choix des dimensions structurales, quelle que soit son aptitude à prédire des défaillances réelles. La plupart des défaillances sont dues à des fautes graves. Il est démontré qu'une évaluation de la tendance à commettre des fautes graves joue un rôle secondaire dans ce choix. Cette évaluation est cependant essentielle au choix de la conception des structures.

**ZUSAMMENFASSUNG**

Die formale Zuverlässigkeitstheorie ist eine unentbehrliche Grundlage für die Festlegung von Tragwerksabmessungen auch wenn sie nur begrenzt geeignet ist, die wirkliche Versagenshäufigkeit vorauszusagen. Die meisten Versagensfälle gehen auf grobe Fehler zurück. Es wird gezeigt, dass ein Mass für die Versagensempfindlichkeit von Tragwerken aufgrund grober Fehler im Hinblick auf diese Festlegung nur eine zweitrangige Rolle spielen kann. Für die Wahl des Tragwerkkonzepts ist dieses Mass jedoch wesentlich.



Discussions among professionals about safety with respect to structural failure are often revealing great confusion about the concept of theoretical reliability. It is a fact that the failure rate of several important types of real structures is orders of magnitude higher than the rate predicted by probabilistic reliability theory, [2]. Thus it is quite natural that the value of this theory is questioned by engineers who have been taught to judge theories by their ability to predict the real world's behavior. A typical reaction is that of H. Rüschi\*), [7]:

"Damit könnte man sich zufrieden geben, wenn diese operative Wahrscheinlichkeit zu der unter wirklichkeitsnahen Annahmen berechneten in einem bekannten und konstanten Verhältnis stünde. Dies ist aber wenig wahrscheinlich. Die Frage wurde auch nie untersucht. So ist es verständlich, dass man als Prüfstein für die von der neuen probabilistischen Sicherheitstheorie abgeleiteten Bemessungsverfahren nur die Frage wählen konnte, ob sich die Ergebnisse mit jenen den bisherigen deterministischen Verfahren annähernd decken oder nicht. Viele werden mit Recht fragen, wozu der ganze Aufwand, wenn letzten Endes für die Praxis nur neue, aber kaum bequemere Krücken angeboten werden.

Der Verfasser hat schon seit Jahren die Sicherheitstheoretiker gebeten, sich mit der Abgrenzung des Gebiets der groben Fahrlässigkeit zu beschäftigen. Er hat auch Lösungsmöglichkeiten aufgezeigt, die als Instrument die subjektive Statistik benutzen, welche sich auf dem Gebiet der Meinungsforschung bewährt hat.

Diese Vorschläge stiessen aber auf kein Interesse. Das Hauptargument der Ablehnung war, man dürfe eine strenge Sicherheitstheorie nur auf statistischen Verteilungen aufbauen, die auf objektiv messbaren Sachverhalten beruhen. Dem entsprechen z.B. Festigkeitswerte, Lasten oder Toleranzen. Nur subjektiv belegbare Werte zu verwenden, wurde als unwissenschaftlich angesehen".

The engineer's primary job is, however, to make decisions rather than to make predictions. This fact is in contrast to the contents of most traditional engineering educations in which methods of rational decision making are parenthetic in comparison with the weight given to theories of physical behavior of materials and structures.

Probabilistic reliability theory is an indispensable decision element in the process of making rational design rules that utilize available empirical evidence on material strengths and environmental actions in combination with models that predict structural behavior. The result of using these rules is a set of drawings and a set of specifications containing a linguistic description of the structure including both general and specific quality requirements. Theoretical reliability is attached to this ideal description of the structure and not to the realized structure. The key point is that probabilistic reliability theory is a formalism which is sufficiently rich of concepts and variables to be well suited for combining information on uncertain quantities that may be of importance for the safety of the real structure. Several widely different sources may contribute to this information. This is in contrast to the naive de-

\*) The pessimism of late professor Rüschi with respect to getting the reliability theorists to accept applications of subjective probabilities was, perhaps, a little exaggerated. Engineering judgement elements are, in fact, essential in several recommendation documents based on the concept of "operative Wahrscheinlichkeit", see [4,5].

terministic formalism of safety factors as handled without any reference to a formal probabilistic framework of reasoning. The deterministic formalism simply has no rules for using essential elements of information of direct consequence for the structural safety.

On the other hand, application of probabilistic reliability theory requires that it is possible to formulate a mathematical model of the structural behavior with well defined variables that may be interpreted as random variables, random vectors, or even random processes in time and space. Clearly any such model will be highly idealized relative to the real structure even though the model may be supplemented with variables that are substitutes for the idealization errors to an extent where these may be modeled as if they were drawn from a homogeneous statistical population. Such idealization errors are of several categories. One is due to non-controllable deviations of real dimensions and shapes from specified dimensions and shapes because perfect workmanship is not obtainable and, for economical reasons, perhaps, not even desirable. A completely different category is the type of idealization error which results from adopting a well defined mathematical model of the structural system. The profession has throughout its history developed a set of generally accepted modeling elements and principles for application in the analysis of usual structures. Provided the combination of these standard modeling elements into a structural model is based on competent engineering inspection of the lay-out of the real structure and its loads the acceptance follows from the fact that experience reveals no catastrophic deviations between predicted behavior and real behavior. Deviations are believed to be within control in a sense equivalent to drawing from a well-defined statistical population. The modeling of such a population may be based on statistical evidence from laboratory experiments or it may be based on comparative studies between different models that describe the same phenomenon. It may even be strongly influenced by subjective factors stemming from the general experience and insight of the engineer (or the profession) about the nature of the matter.

The absence of consistency and rationality behind the old deterministic methods make their use a more or less trial and error procedure for example to decrease material consumption of structural elements down to such limits that the rate of failure due to simple uncontrollable statistical deviations of relevant parameters to the unsafe side is still within acceptable limits. Use of tools based on probabilistic reliability theory is in contrast to this "blind man's slow walk" an apprehensible and expedient method of analysis and decision. Perhaps the intellectual rationality of reliability analysis may best be appreciated by the fact that a calculated failure probability always can be taken as a prediction of the relative frequency of the event called failure in a consistently programmed Monte Carlo simulation on a computer. It should be emphasized, however, that this simulated relative frequency rarely, if ever, can be taken as a prediction of the physical failure rate in the corresponding population of real structures. There are three main reasons for this. The first reason is that probabilistic reliability theory explicitly works with a more general probability interpretation than just the narrow relative frequency interpretation. To be useful reliability theory must work with subjective probabilities (or credibilities), that is, degrees of professional belief about the values of parameters of interest. The second main reason is that the real structure may be supplemented with several second-



ary elements (e.g. window frames) which in the safety analysis are neglected as contributors to the carrying capacity but which in reality may contribute significantly. This is in particular so in cases where gross errors in the structural principle either inherent in the design or occurring in the building process of the structure cause significant forces to be transmitted to these secondary elements. This is in spite of the primary design principle of protecting these elements against loads that may cause damage to them. If the secondary elements are able to sustain the loads the result is, obviously, an increased carrying capacity of the structure. Secondary elements may often be allowed to carry loads but their effect is considered difficult to take into account in the structural analysis model or the effect is considered insufficiently reliable to be taken into account. This attitude is in particular characteristic for the old deterministic safety reasoning. In principle the effect of secondary elements can very well be taken into account in probabilistic reliability analysis under due consideration of the model uncertainty and the uncertainty of knowledge attached to the evaluation of the effect. The third main reason that the failure rate of real structures is different from that resulting from a reliability analysis is the occurrence of human gross errors in all stages of the realization and use of a structure including structural lay-out, mathematical model formulation, evaluation of environmental factors, communication (e.g., drawings, descriptions, verbal instructions), building process, material delivery etc.

Some types of gross errors may in principle be described in parameter form and considered statistical. Such errors can be consistently included in the reliability analysis model. Typically they are of "on-off" type like "upside-down" errors. The failure probability given the error may be calculated and knowing the probability of occurrence of the error the product of the two probabilities may be calculated giving an additive contribution to the total failure probability. Clearly, if the error is reasonably rare, the failure probability given the error may be allowed to be considerably larger than the total failure probability before its contribution becomes a dominating part of the total failure probability. By active control procedures or "fail-safe" design the probability of occurrence of such an error may be kept below reasonable limits.

Left over beyond systematic mathematical description and, perhaps, imagination there are all kinds of arbitrary gross errors due to mistakes, gross negligence, criminal acts, bad human performance as results of economical or political pressures, haste, lack of "think before you leap"-attitude, haphazard behaviour etc. Imprudent application of new materials, new structural types, significantly changed dimensions relative to traditional dimensions, new erection principles on building site etc. may involve danger of overlooking new significant modes of mechanical behaviour or it may even expose effects outside the knowledge and experience of the profession. A famous case of this is the Tacoma bridge failure.

In view of the existence of non-parametrizable gross errors it may seem hopeless to try to formulate a mathematical rationale which is able to predict the failure rate in the population of a given type of real structures. Something can be done, however. Clearly it is important to be able to identify the circumstances that have potential for producing gross errors and to judge their gravity.



For a complicated project several such circumstances may be present and thus it is important to be able to evaluate their common effect with respect to proneness to damage or failure. Such insight may be the basis for wise decisions about design changes, changes of production, contractors, or use of the structure.

It is important to make clear that an increase of the safety factor level, that is, an increase of the theoretical reliability, has generally very small or no effect at all with respect to decreasing the proneness to failure due to most types of gross errors. The effect is only to increase member dimensions. For example, an upside-down error in placing a prefabricated reinforced concrete beam is not made harmless by any substantial increase of the intended downside reinforcement. What helps is to identify the possibility of the upside-down error and then change the design to eliminate the possibility of the error, that is, to make the design fail-safe in this respect.

Commonly it is argued that the design value of the theoretical failure probability for a given structural lay-out should be fixed at the value which minimizes the total expected costs (where "costs" may be taken in a more general sense than just direct monetary costs), that is, the expected value of the establishing costs plus the operation and maintenance costs plus the costs of damage or failure. The question raises whether such an optimization is reasonable in consideration of the gap between real and theoretical failure rate. Fortunately the answer is confirmative in most cases. To see this let  $p_{th}$  be the theoretical probability of failure and let  $p_{gr}$  be some measure of proneness to failure due to gross errors. The point is that for a given lay-out of the structure (including the entire plan for the building process) the proneness to failure  $p_{gr}$  is in most cases almost unaffected by variations of  $p_{th}$ , these variations only causing variation of the material consumption. Therefore the value of  $p_{th}$  which minimizes the expected cost of the given lay-out is almost unaffected by the expected cost of failure due to gross errors (provided the costs associated with a failure are only slightly dependent on variations of the material consumption). Thus it is rational for each lay-out to choose as design value that value of  $p_{th}$  which minimizes the expected cost of the given lay-out. The validity of this argumentation seems to be the only salvation of probabilistic reliability theory from being just a plaything for university teachers. However, when the question is about choosing between different lay-outs the expected cost of failure due to gross errors must be added, that is, a cost which depends on  $p_{gr}$  must be added. While the widely accepted modern decision theory defines the failure cost as a function of  $p_{th}$  simply as the expected cost with respect to the given probabilistic model there is as yet no generally accepted definition of  $p_{gr}$ . Even with such a definition available it is by no means obvious how to define the expected failure cost as a function of  $p_{gr}$  except, perhaps, that it should be an increasing function of  $p_{gr}$ .

To the author's knowledge the terminology "proneness to failure" together with an attempt to define a numerical measure of it was first suggested by Pugsley, [6]. Blockley, [1], has published a useful checklist for grading the quality of a project with respect to proneness to gross errors in all its stages from design to use. He applies this checklist in a grading of 23 major projects that all turned into disasters. The problem is to cook down all these



gradings to a single appreciable measure of proneness to failure. He applies the fuzzy set concept which was introduced by Zadeh in 1965, [8], with the purpose of giving a precise mathematical interpretation of imprecise linguistic statements and a modeling of relations between such statements. However, in the light of the above discussion, Blockley's attempt in [1] (see also [2]) to "fuzzify" the theoretical failure probability  $p_{th}$  seems inappropriate.

An extension of this paper, [4], analyses the fuzzy set tool with respect to the possibility of establishing a measure of proneness to failure due to gross errors.

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