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# Buckling Analysis of Reticulated Cylindrical Shell Roofs

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## SYNOPSIS

It is noted that instability failures such as snap-through are likely to occur in single layer reticulated shells as well as in true shells. Computer analysis approach in this field is usually done by the matrix method but there arise many difficulties due to large-scale degrees of freedom. In this study, we develop a simplified analytical method and apply it to the buckling analysis of single layer reticulated shells.

## EVALUATION OF MEMBER STIFFNESS

As shown in Fig.2, a reticulated shell is considered to be composed of a truss element with rotational springs at its edges. A truss element has only axial stiffness. Bending stiffness and joint rigidity is represented by these springs. Total member stiffness is obtained by the equilibrium method.

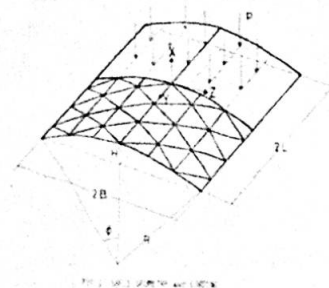


FIG. 1. SHELL GEOMETRY AND LOADING

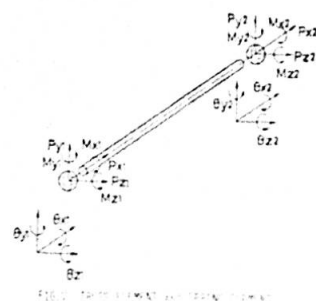
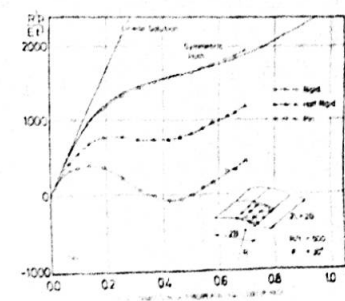
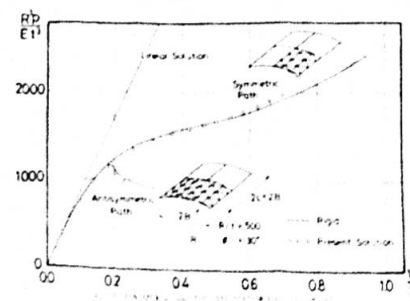


FIG. 2. TRUSS ELEMENT AND ROTATIONAL SPRING

## NONLINEAR BUCKLING BEHAVIORS

Fig.3 shows fundamental equilibrium and post-bifurcation path of a rigidly jointed shell. Loading condition is shown in Fig.1. Post-bifurcation path is pursued by giving small asymmetrical perturbation. Present solutions give good agreement with the solid lines obtained by the conventional FEM method. Fig.4 shows the transition of fundamental path as joint rigidity decreasing. It is shown that snap-through occurs as decreasing of joint rigidity even when no snap-through exists on the fundamental path of perfectly jointed one. Although this is one example, this fact indicates the importance of joint rigidity in single layer reticulated shells. Required CPU time is much deduced by the simplified evaluation of bending stiffness.



# Buckling Analysis of Reticulated Cylindrical Shell Roofs

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## SUMMARY OF THE DISCUSSIONS AND QUESTIONS

We received several questions at the "Poster Session". These questions and discussions are classified into the following

- 1) Iterative procedure in the incremental step, 2) Assembly of the element stiffness, 3) Accuracy of this method, 4) CPU time.

In this analysis, the unbalanced forces appear at each incremental loading step. As mentioned before, these forces are caused by the second-order terms in the axial strain expression. These forces are extinguished by the Newton-Raphson method. In this iterative procedure, the stiffness matrix is always recomposed. Accuracy of this analysis is depend on the validity of ignoring the terms related to initial inner moments. A uniformly loaded reticulated shell is considered to be near the membrane state. In other words, the inner moments are secondary compared with the axial force, and like a 3-hinged truss structure, there may be an equilibrium state by axial force. By these reason, we consider this analytical method effective when applied to the buckling analysis of uniformly loaded reticulated shells. The computing time is much deduced by the reason that solutions converge well and the required time of forming a stiffness matrix is deduced. In comparison with the required CPU time of the conventional FEM, this simplified method need 1/5 ~ 1/10 of that time. In addition to it, these calculations are done by Hitac M-180 which seems to have an equal ability with IBM's large computer. The assembly of the element stiffness is shown in the next section with the formulation process of a stiffness matrix [K].

## FORMULATION OF A STIFFNESS MATRIX

A stiffness matrix of a truss element is lead by the virtual work principle. The finite element formulation of the equilibrium is shown in quadratic form of displacement vector  $\{\Delta u_e\}$ .

$$\begin{aligned} & \iiint [\Delta \sigma_{xx} \delta \Delta e_{xx}^* + \sigma_{xx} \cdot \frac{1}{2} \delta [(\frac{d\Delta v}{dx})^2 + (\frac{d\Delta w}{dx})^2] - \Delta p_i \delta \Delta u_i] dv - \iint \Delta f_i \delta \Delta u_i ds - \delta \Delta W_r \\ & = \delta \{\Delta u_e\}^t \cdot [K_e] \cdot \{\Delta u_e\} - \delta \{\Delta u_e\}^t \cdot \Delta F_{ex} - \delta \{\Delta u_e\}^t \cdot R \end{aligned} \quad (1)$$

where,  $\sigma_{xx}$  = fiber stress,  $p_i$  = body force increment, and  $f_i$  = surface force increment. According to the arbitrariness of the virtual displacement  $\{\Delta u_e\}$ , a stiffness matrix  $[K_e]$  of a truss element and equilibrium equation is obtained. Total flexibility matrix  $[F]$  is obtained using flexibility matrix of spring elements  $[F_1]$ ,  $[F_2]$  and equilibrium matrix.

$$[F] = [H]^t \cdot [F_1] \cdot [H] + [F_m] + [F_2] \quad (2) \quad [F_m] = [K_e]_{jj}^{-1} \quad (3)$$

Total member stiffness matrix is obtained in inverced form of equation (2).