System and Geometrical Optimization for Linear and Non-Linear Structural Behaviour

Optimisation des systèmes et des dimensions pour des comportements structuraux linéaires et non-linéaires

Optimierung der Systeme und der Abmessungen bei linearem und nicht-linearem Verhalten des Tragwerkes

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1. INTRODUCTION

Many discussions in the I.A.B.S.E. proceedings have considered structural optimization. For example, Professor Courbon defined optimization as designing and construction a structure at the lowest cost, with the object of fulfilling a well defined purpose.(1) In particular, cost consideration must also be given to safety, service life, maintenance and future adaptability. Since all of research and practice in structural engineering is aimed towards such a goal the activity known as structural optimization must be defined in a unique way. That is, the development and application of a priori directed and automated techniques for improving designs within well defined cost contexts and recognized constraints. Within this definition, therefore, questions of design creativity and ingenuity are put aside in favor of quantitative comparisons among a vast array of acceptable yet competing designs. Thus, in much the same way as computer methods of structural analysis, the techniques of structural optimization become an aid to the designer for rapidly proportioning structural details and evaluating design alternatives to obtain the best design among given choices. In this way, when the engineer arrives at comparing quite different conceptual designs for the same application he is fairly certain of intelligently comparing these alternatives and not unfavorably biasing one alternative by poor proportioning of its details.

In the field of structural optimization the computer becomes central as a tool for searching and sorting through the similar design concepts and proportioning the element details for the most economical design. Naturally, it arrives at a design which the engineer could equally have obtained if he were prepared to invest the time and money to directly

search among the design alternatives. The principal advantage of the optimization methods is therefore its saving in design time and cost. A further consideration with the expanding usage of computer analysis programs to calculate structural behavior is that an optimization program can eliminate much of the input-output and the costly data handling effort. Since the methods result in a proportioned structure which satisfies applicable codes it becomes unnecessary to review the vast amounts of structural analysis output. Instead, the optimization program produces design details as an output and in some cases could be further programmed to produce drawings and material and fabrication specifications.

2. RAISING THE DESIGN HIERARCHY

Since structural optimization over the last decade and a half has successfully concerned itself with computer aided proportioning of design details it is easy to ignore further applications. In fact, some recent work has shown the possiblity of introducing into the automated computer procedures design variables which had previously been thought to be either in the realm of creative decisions or else difficult to program for computer selection. These additional design variables have described geometry and shape of structures, material choice, complete building design including comparison of basically different element types and design selections including overall fabrication costs and material availability. Before embarking on a description of several such examples it is worth considering from this framework the historical developments of optimization applications.

There is in this regard an analogy between computer developments in both structural analysis and optimization. When digital computers first became available civil engineers who were among its early intensive users simply programmed classical methods of analysis such as slope deflection and moment distribution. In a similar way the first structural optimization applications were programs using such well known iterative design methods as structural index, stress-ratio, fully stress and other optimality criteria.(2)

A second development of computer applications were matrix analysis

for specific structural types such as trusses, frames and grillages. Paralleling this was specialized optimization programs for these same structural types using methods such as gradient directions or other heuristic design search procedures.(3) The current stage of development includes general purpose computer programs usually available from machine manufacturers or time-share agencies. In analysis this often means finite-element packages for linear and non-linear behavior. The same evolution for design has been program packages each able to handle a wide array of different structural elements and systems. It is important to investigate in detail these current developments in optimization since they include the methodology for extending applications in both structural system and geometry optimization.

3. GENERAL PURPOSE OPTIMIZATION PROGRAMS

To explore the available methodology for structural optimization the author prefers to divide iterative structural decision problems into three categories. These are: 1) element design, 2) interconnected structural systems and 3) discrete decision variables. Each of these categories will now be considered. It is recognized that other areas could be added particularly as we move further into conceptual and creative decision variables but the three categories will suffice to cover the needs of a computer optimization library for a typical design practice.

Element Design - This design problem is characterized by a well defined code of practice for the constraints and a relatively direct method of calculating the element loading such as moment, shear, torsion and axial load. Some examples of application are shown in Figure 1 including welded wide flange, unsymmetrical box girder and prestressed beams.(4-6) Design variables are typically depths, thickness, shape, reinforcement ratios and other design details which are often part of the tedious aspects of design and for which economic selection rules are not always available. Other published examples include welded columns, gabled frame, stiffened ship plates, shear walls, prestressed plates and reinforced concrete beams.(7-11) These examples typically have a small number of independent design parameters (say less than 10)

but relatively complex functional code constraints such as allowable
tension, compressive and shear stresses, buckling and displacement
constraints. In one example using the A.I.S.C. code the constraints
were due to lateral buckling and were discontinuous representing
a transition from elastic to inelastic behavior.(4) In some appli-
cations it becomes necessary to repeat the design for a large number
of different elements. For example, in the case of box girder sections
a particular overhead crane manufacturer using such sections needed
over 5000 specified designs.(5) This obviously required an efficient
design program.

Several computer packages have been developed to automate the
solution to element optimization. The author prefers to use programs
based on the penalty technique which combine the criteria function
(cost or weight) and the constraint into a single expression to be
minimized.(12) This transforms the more difficult non-linear pro-
gramming problem with cost and constraints into a more tractable
unconstrained minimization for which many straightforward solution
methods are available. Other methods have also been successfully used
on some problems including geometrical programming and linear programming.(13)
As was mentioned earlier, element design is also characterized by a
direct calculation of the forces on the elements. This may either be
for statically determinate structures or even for complex frameworks
in which a matrix structural analysis is used to solve for element forces.
It is assumed in the latter case that the element size does not affect
the force distribution within the structure; if this effect is significant
then several cycles of iteration may be necessary to converge both the
force distribution and element design. Since an element design optimization
usually involves comparing many alternatives it would be computationally
difficult to repeat the force analysis each time a design parameter were
changed.

System Optimization - This refers to structures where there is a
major physical interaction between different elements or there exists
design constraints based on total structural behavior such as stability,
stiffness, vibration and dynamic responses. Furthermore, system opti-
mization arises when some of the design variables relate to more then
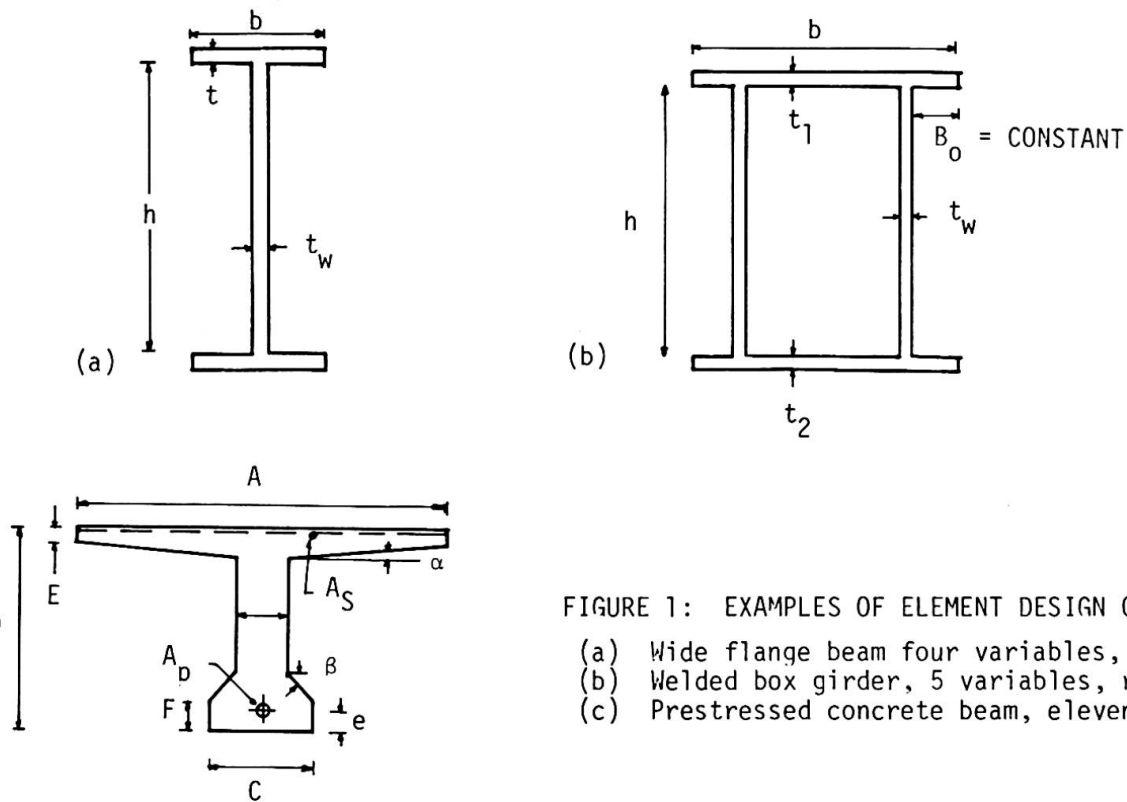
FIGURE 1:   EXAMPLES OF ELEMENT DESIGN OPTIMIZATION

(a)   Wide flange beam four variables, ref. 4
(b)   Welded box girder, 5 variables, ref. 5
(c)   Prestressed concrete beam, eleven variables, ref. 6
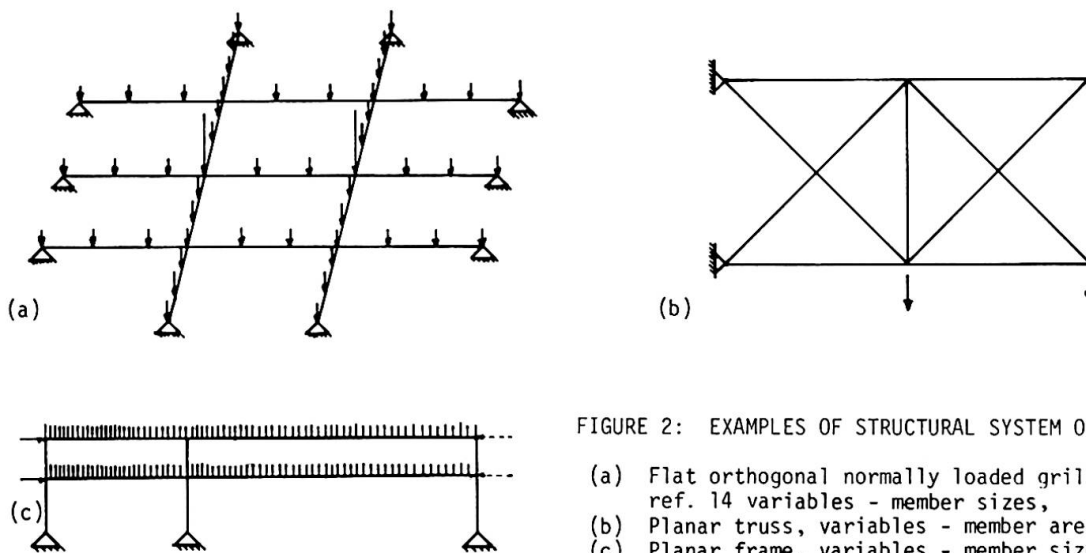

FIGURE 2:   EXAMPLES OF STRUCTURAL SYSTEM OPTIMIZATION

(a)   Flat orthogonal normally loaded grillage,
      ref. 14 variables - member sizes,
(b)   Planar truss, variables - member areas, ref. 15
(c)   Planar frame, variables - member sizes, ref. 16

one element such as geometry, material or topology. In these cases, the force distribution invariably requires a matrix or finite element analysis. Computer costs usually restrict the number of possible trials that the computer can sequentially examine as it searches for the optimum to often under 10 trials. Some cases that have been optimized include foundation or ship grillages, trusses and frames and are illustrated in Figure 2.(14-16) The most efficient search techniques for systems usually utilize some form of linear programming steps since in this way large changes in design may be made with only a single analysis interaction.(16,17)

Discrete Selection Parameters - This category of optimization is considerably more complex then either the two first categories. Discrete design variables are not always clearly defined or constant in number and the cost function may be extremely complex and discontinuous. The discrete nature of variables often requires heuristic or intuitive search methods of solution which make general purpose programs irrelevant. One method however, which has solved a variety of such problems is the dynamic programming technique which is easy to program if the basic problem formulation meets its definition.(18) Among the reported structural design problems solved by dynamic programming are the minimum cost of continuous coverplated highway bridge girders, single story building selection of different roof, column and foundation elements, spacing of supports of multi-span girders, thickness variations in ship plate components, reinforcing bar arrangement in continuous reinforced concrete beams and girder selection for minimum material, detailing and fabrication costs.(19-24) These examples have in common discrete variable selection and more important a sequencing of decisions into stages which satisfy the dynamic programming criterion. As an example in Figure 3, the single story optimization starts separately with the roof and then includes the column and finally the foundation and bay spacing. Since roof cost is independent of the supporting columns and foundation this sequencing of decisions is possible. The same notion of sequencing is true of the other dynamic programming examples mentioned.

4. GEOMETRY AND SHAPE OPTIMIZATION

In extending the design optimization hierarchy beyond the variables associated with design details the variables associated with shape and geometry have arisen. This is a natural continuation of much of the early optimization work on truss and frame structures which optimized

the force distribution within the structure or element optimization which finds the best details in a specific cross section. Naturally much greater cost savings can be made by optimization of geometry variables than by member selection alone since the force distribution is relatively insensitive in many cases to the latter variables.

An example of this is illustrated in Figure 4 with a section of a transmission tower.(25) With a fixed initial geometry as in Figure 4a little savings can be realized in structure weight by optimizing the element force distribution. The difference between total optimized weight and say the weight obtained by a traditional direct iterative analysis and design approach is quite small even over a wide array of load conditions and even displacement constraints. However, when geometry variables are introduced the structural weight is reduced 18% as in Figure 4b. The geometry variables were the location of joints and the width of the base support. In this shape optimization the geometry design variables were separated from element design variables. For each change in geometry the minimum element design was found by a direct design method such as stress ratio. The changes in geometry were found by gradient methods.

A broader generalization of this search for optimum geometry or "best shape" structure was reported by Zienkiewicz and Campbell.(26) Starting with a finite-element analysis program an extension was programmed to automate the calculation of derivatives of structural behavior such as stresses and displacements with respect to various structural shape parameters. Optimum changes in geometry were then carried out by linear programming. Applications of this approach have been reported for arch dam geometry, dam cutouts and plates. Vitiello reported a similar program for beam shapes, gravity dams and seismic loading.(27) By properly fitting the structural behavior with polynomial functions of the geometry he was able to do the minimization with a penalty function program. An example showing the gravity design variables and the finite element modelling is given in Figure 5.

Ramakrishnan and Francavilla also using a linear programming approach found optimum shape designs for plates, pressure vessel end closures, and a gravity dam.(28)

Loading → ROOF JOIST → GIRDERS → COLUMNS → FOUNDATION → BAY SPACING → Final Design

FIGURE 3: DISCRETE DESIGN OPTIMIZATION
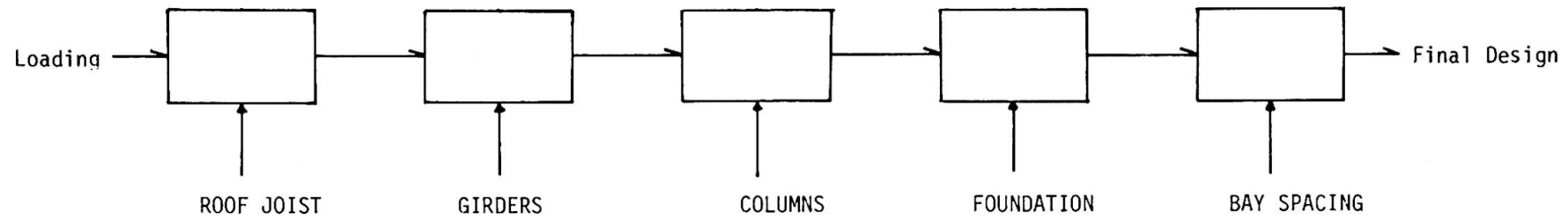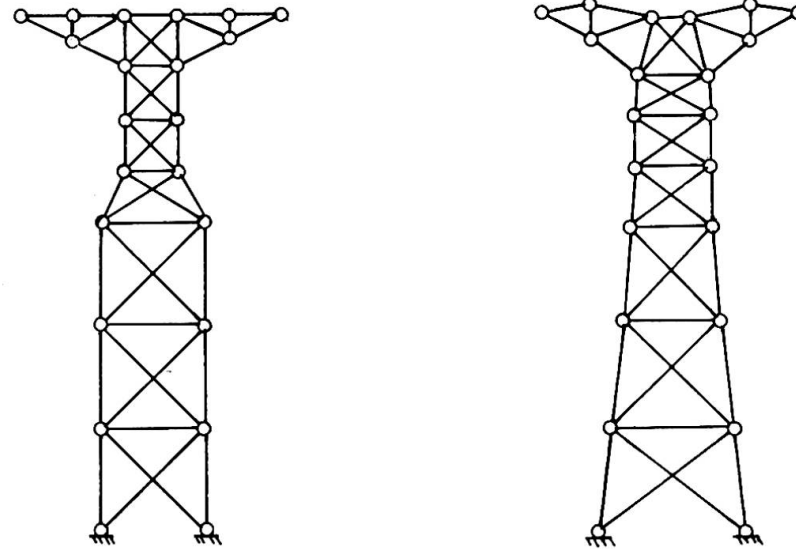
Single story building ref. 20 and extensions
by the author.

(a) Weight = 2306

(b) Weight = 1910

FIGURE 4: OPTIMAL GEOMETRY DESIGN OF TOWERS   (ref. 25)

(a)   Starting Design
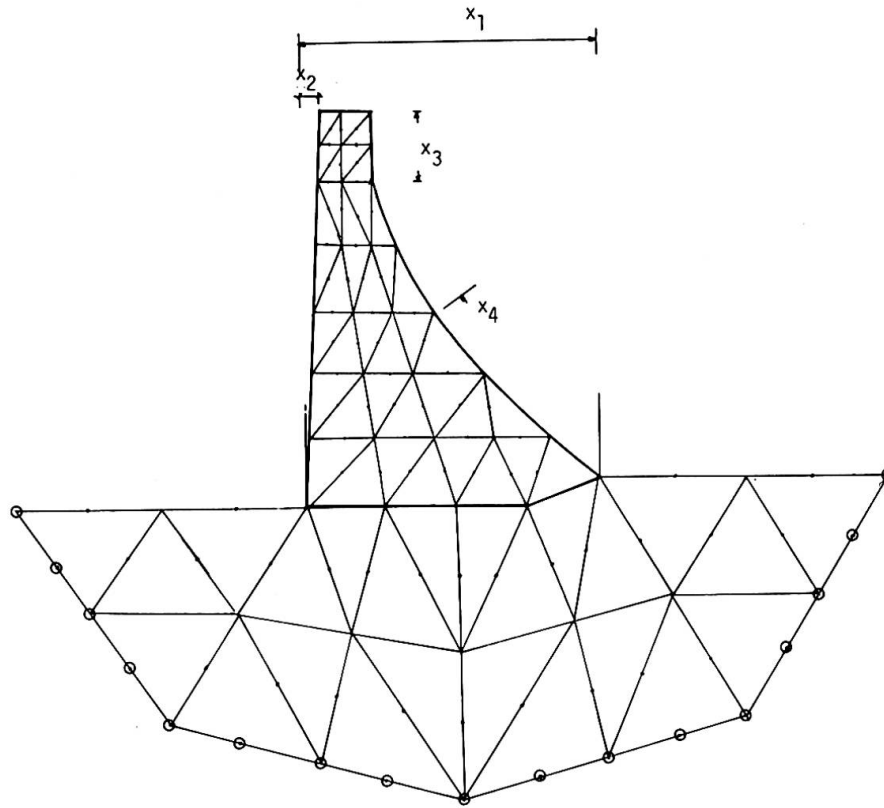(b)   Optimal Geometry

FIGURE 5: OPTIMUM SHAPE OF GRAVITY DAM,
64 Element, 262 D. of Freedom F.E. Analysis,
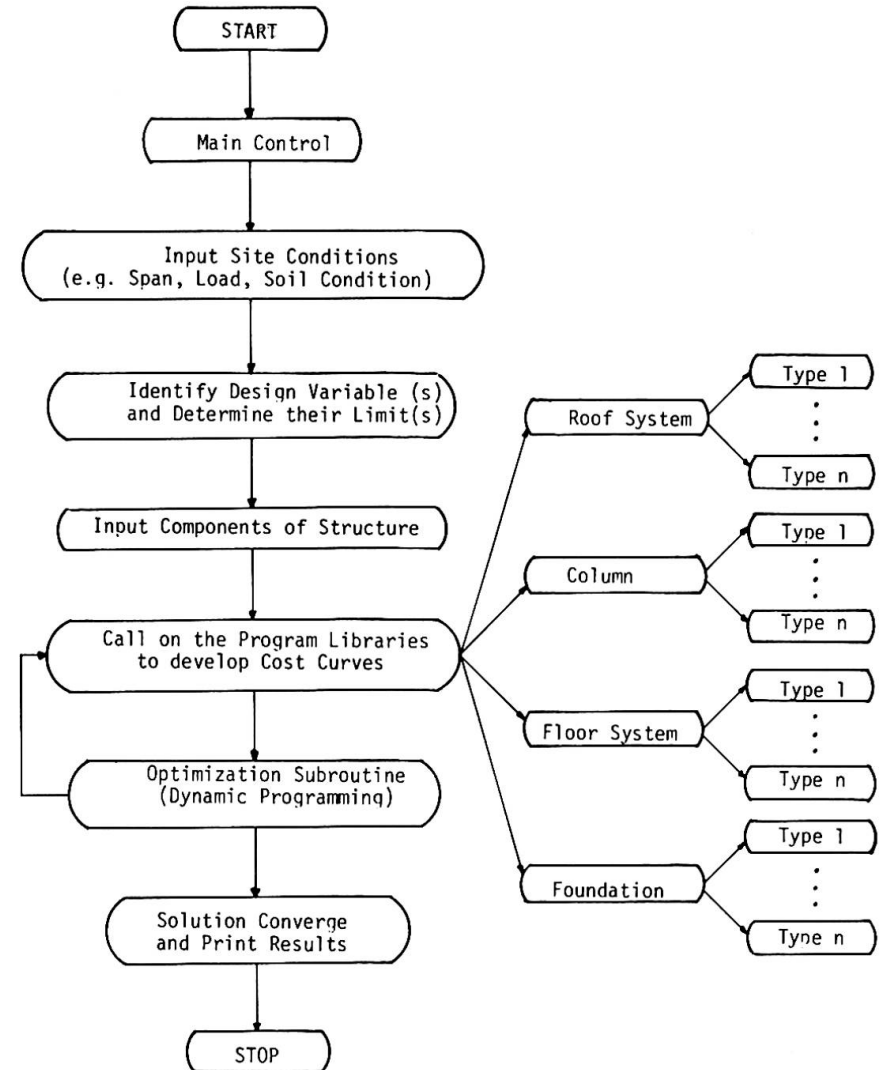4 Design Variables, ref. 27.

FIGURE 6: FLOW CHART FOR GENERAL SYSTEM PROGRAM FOR BUILDING OPTIMIZATION

It is apparent that despite some of this work described in shape
optimization that much remains to be done and would accordingly be
very profitable. Certainly, once the geometry of a structure is fixed
there may be little amount of cost or material saving to be achieved
by optimiztion techniques. It is therefore important that the geometry
variables be considered as part of the automated design search. The
most fruitful application appears to be three dimensional heavy structures
such as towers, dams, pressure vessels, containment structures and
storage tanks for which indeed form may follow economy and structure
represents a major part of the total investment. In buildings,
except perhaps for high-rise or other structures in severe seismic or
climatic zones the potential material savings in the structure repre-
sents only a small percentage of the total building cost and therefore
cannot dictate geometry in conflict with other architectural considera-
tions. However, even in buildings particularly in industrial applica-
tions there may still be some gain to introducing geometry variables
concerned with bay dimensions or location of shear walls and stiffening
trusses.

5. SYSTEM OPTIMIZATION

The above discussion has almost exclusively centered on design
variables which describe a structure but are in the main continuous
variables such as element sizes or geometry variables. An exception
has been the example in Figure 3 illustrating a single story building
with discrete variables. What is unique about this latter example is
the selection among alternative structural elements to perform the same
function. Thus not only, say rolled steel beams but welded girders,
reinforced concrete or prestressed beams and even trusses are sequen-
tially compared while finding the optimum. Similarly, for other
components of the structural load carrying system. Thus, the hierarchy
of possible designs entering an automated optimization is considerably
expanded.

Figure 6 shows a flow diagram of a general purpose program for
performing the sequential comparison and optimization. It has been
applied to design of single story buildings but may also be used for
high-rise buildings. The various components required for the structure
i.e., foundation, columns, girders, roof joints etc. are treated
independently. A table is developed which obtains the minimum cost of

each component as a function of the loads, with the range of possible loadings automatically established based on the input data. Each component in such a table may be found only after comparison of different types of elements to carry out the same function. These individual elements may be from a stored list of optimum designs or else generated by element design programs for a specific structure. Any convenient programming method either non-linear optimization or direct design may be used for finding the element table. The entire program is then controlled with a dynamic programming type minimum cost selection scheme to choose the best combination of elements. The input data specifies which element types should be candidates for a particular structure and new elements can be added to a program library.

To make such general programs more accessible to designers simplified programming languages are needed. The input data must be in the form of basic geometrical dimensions and load data. The connecting of components must be inputted in a direct manner as well as specifying the possible element types which are design candidates. Since the variables are so general including element selection and geometry the output need not be a detailed design specification but rather a broad general indication of the element types and geometry which give the minimum cost structure.

6. LIMITATIONS ON OPTIMIZATION APPLICATIONS

Since computers and procedures for automated optimum design have been with us for some time it may be appropriate to reflect on why such methods have not always entered routine office practice. To be sure, a similar question may be raised about matrix analysis procedures that despite readily available programs many structures are built today after being analyzed with only crude approximate techniques. A major reason, at least in the United States, is the lack of incentive for designers to utilize computer methods which cost them money but save the client construction cost. This is one reason why many of the extensive applications of computer optimization have been in areas of design-build operations where there is stiff competition and hence a desire to reduce construction cost in order to obtain a job. Other optimization applications have been to bridge construction where many bridges are designed by state highway departments who are in effect the owners of the structure.

Another limitation has been that many of the computer programs have required detailed knowledge of mathematical programming and computer software techniques. This is changing and as in more recently developed structural analysis programs, the optimization routines in some of the examples cited above do not require the user to be at all familiar with such programming procedures. In fact, in many cases, the user finds these optimum design programs easier to utilize then traditional design tools or programs. It is this attraction, rather then the construction cost saved by optimization which has often decided the user in favor of this approach.

7. CONCLUSIONS

1.   Advances in automated computer techniques for design have reached the stage where many types of detailed design and selection between alternatives for minimum cost can be carried out. Such design tools could be used for lowering cost, increasing standardization of elements and evaluating effects of changing constraints.

2.   Success in achieving programs for element design has suggested that the design variable search be extended to include more significant variables of material and geometry. In particular shape optimization has been used, particularly for massive concrete structures such as gravity and arch dams and containment structures.

3.   System programs capable of data manipulation and automated design of a wide variety of different structural schemes can be expected during the next few years. This should make possible the application of automated design by engineers with little background in programming and software techniques. At the same time there will still be demands for special purpose programs which more efficiently automate the design of a single type of structure. This will be done by organizations which have repeated need for a particular structure and are prepared to invest time and money in computer applications. An example of this latter approach is the GAD system developed by Professor Goble at Case Western Reserve University for the design of continuous welded plate girder highway bridges.(5.19) The program has been in use by the Ohio Department of Transportation for several years. The program reflects the cost data, design details, code specification and construction practices of that organization. However, due to the number of bridges of this type which are built the investment in computer programming was justified.

4. Since the cost of developing optimization programs may be large and the incentive for the design firms to use these programs may be relatively small the advances into practice of such techniques may be thwarted. A mechanism such as a central agency is needed to develop, document and disseminate such programs to insure wide practical utilization.

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SUMMARY

     Structural optimization is defined as directed computer techniques for
improving designs within well defined cost contexts and recognized constraints.
Applications are divided into:
a) element design characterized by code constraints of practice;
b) system optimization involving large numbers of elements and
c) discrete decision variables.
     Solutions and examples are presented for all three categories. Geometry
and shape optimization as well as general programs for optimizing a variety of
different structures is discussed in detail.

RESUME

     L'optimisation structurale a pour but d'améliorer le dimensionnement de
structures au moyen de techniques appropriées d'ordinateur, dans des limites de
coûts et de contraintes bien définies. Le domaine d'utilisation en est le suivant:
a) dimensionnement d'éléments conformément aux règlements de construction
b) optimisation de systèmes composés d'un grand nombre d'éléments
c) variables discrètes de décision.
     Des solutions et des exemples sont donnés pour ces trois catégories. L'opti-
misation de la forme et des dimensions est présentée en détail; des programmes
généraux applicables à diverses structures sont également discutés.

ZUSAMMENFASSUNG

     Die Optimierung von Tragwerken wird definiert als unmittelbare Anwendung der
Computertechnik zum Entwurf und zur Berechnung von Konstruktionen bei genau um-
schriebenen Nebenbedingungen hinsichtlich Baukosten und zulässigen Spannungen. Die
Anwendungsmethoden werden aufgeteilt in:
a) Bemessung von Einzelelementen nach den geltenden Normenvorschriften,
b) Optimierung ganzer Systeme bestehend aus einer grossen Anzahl von Einzelelementen,
c) diskrete Entscheidungsvariable.
     Für alle drei Kategorien werden Lösungen und Beispiele angegeben. Die Opti-
mierung der Form und der Abmessungen sowie allgemeine Programme für die Optimierung
von verschiedenartigen Bauwerken werden eingehend besprochen.

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