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## Behaviour of Inelastic Beams under Cyclic Loading

Comportement de poutres sous l'effet de charges répétées

Verhalten von Trägern unter wiederholten Belastungen

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## 1. INTRODUCTION

In order to use higher strength steels advantageously in structures, many investigations have been carried out heretofore(1),(2). However, most of them have been on the strength and deformation capacity of the members under monotonous loading. In the countries which suffer from earthquakes frequently, such as Japan, the structural steel members are subjected to cyclically repeated loads. From this standpoint, the design methods which take into consideration the energy absorption due to plastic deformation should be adopted to design the structures economically and rationally. For this purpose, it is important to grasp thoroughly the inelastic behaviors of steel members subjected to extreme load reversals such as those which may occur during an earthquake. Investigations on the behavior of structures or structural members under cyclically repeated loads have been done considerably(3), especially in Japan(4). However, there has been little research done under such loads, relating to the phenomena of flange local buckling and lateral buckling, and little work, relating to the steels which have a yield point exceeding that of SM58 class steels.

This paper is a report of laboratory investigation of steel beams subjected to cyclically reversed loads under moment gradient, relating to flange local buckling and lateral buckling.

## 2. INELASTIC LOCAL BUCKLING BEAMS UNDER CYCLIC LOADING

### 2.1 SELECTIONS OF THE SPECIMENS

In Ref. 6, the limitation of width-thickness ratio  $b/t$  (where,  $b$ =one-half of flange width,  $t$ =thickness of flange) of wide-flange sections in the plastic designs is given as 6.00 for SM58(A572) class steels. In the case of beams under moment gradient, M.G. Lay has proposed next equation(8);

$$b/t = 1.78 / \sqrt{\sigma_y (3 + 1/Y) (1 + E/5.2E_{st}) E} \quad (2.1)$$

where,  $Y = \sigma_y / \sigma_u$  = yield ratio of material ( $\sigma_u$ =tensile strength)

In Ref.7, the limitation of the ratio  $b/t$  in the elastic design is given as  $b/t = 11.85$  for SM58(A572) class steels. The mechanical properties of materials are shown in Table 2.1, where  $\epsilon_{st}$ =strain at initial strain hardening. Using the values of this table,  $b/t = 4.20$  is obtained from Eq.(2.1).

From the basic data as described above, for the width-thickness ratio  $b/t$  of the test specimens, The values of 4.2, 6.0, 9.0, and 12.0 were used.

## 2.2 EXPERIMENTAL ARRANGEMENT AND METHOD

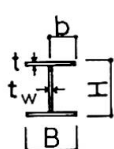
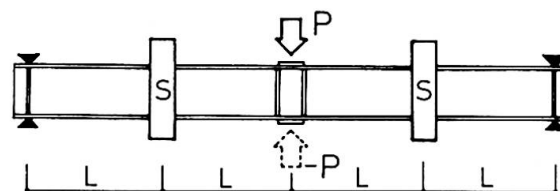
Local buckling tests were carried out on single-span simply supported beams with central concentrated loads applied on the top flanges, as shown in Fig. 1.1 (a). The test specimens are beams having the cross section shown in Fig. 1.1 (b). In Fig. 1.1 (a), the notation S shows the lateral support.

Unsupported length of a beam,  $L$ , is designed as the value of  $L/r_y$  is less than that recommended in Ref.6, i.e.  $L/r_y = 43.9$  for the steels of  $\sigma_y = 5.32t/cm^2$ . The sectional dimensions of the test beams are given in Table 2.2, where,  $I$ =geometrical moment of inertia, and  $Z$ =section modulus. The program was made referring to experimental results in Ref.5.

According to Ref.5, in the beams having the same width-thickness ratio  $b/t$ , rotation capacity of the beam on which a monotonous loading test was performed showed the maximum value for all methods of loading, while the value of the beam subjected to cyclically repeated loads by controlling the deformations of the beam had the minimum value. In this report, therefore, three welded built up beams having the same ratio  $b/t$  were used (See Table 2.2). In Table 2.2, each beam is referred to by a code number such as SM9-3, in which SM stands for the materials of SM58, 9 means a width-thickness ratio of 9.0, and 3 means the third beam. The first beam test was carried out under monotonous loads, the second, under cyclically repeated loads by controlling the loads of the beam, i.e. the ratio of the positive moment to negative one was the ratio of 1:0.6 (See Fig. 2.2, 2.4 and 2.6), and the third, under cyclically repeated load by controlling the deformations of the beam (See Fig. 2.3, 2.5, 2.7 and 2.9).

Each so-called plastic cycle is defined here as the process of loading the beam downward until reaching  $\delta/\delta_p = 2.5$  (where,  $\delta$  =the deflections of the beam section at the midspan), unloading, then reverse loading the beam upward until reaching  $\delta/\delta_p = -2.5$ , in the case of the third beam. From the second excursion, in every process of loading the beam downward, the deflection of the beam at the midspan was increased with  $\delta/\delta_p = 0.5$ . The

deflections and the rotations of the beam at the midspan were measured with the dial gages and the strains were measured by means of wire strain gages for plastic region.



(a)

S shows the lateral support.

(b)

Fig. 1.1

Table 2.1  
Mechanical Properties of Materials

$\sigma_y$ $^{kcm^2}$	$\sigma_u$ "	$\sigma_y/\sigma_u$	$\epsilon_y$ %	$\epsilon_{st}$ "	$E$ $^{kcm^2}$	$E_{st}$ "	$E/E_{st}$
5.32	6.19	.86	.252	2.06	2,111	25.2	83.7

Table 2.2  
Sectional Dimensions

	H cm	B "	t "	t_w "	b/t	I cm <sup>4</sup>	Z cm <sup>3</sup>
SM12-2	21.76	21.56	.90	.61	11.98	4,629	425.4
3	21.80	21.59	.91	.61	11.86	4,695	430.7
3	21.73	21.56	.89	.62	12.11	4,580	421.5
SM9-2	21.80	16.17	.92	.61	8.79	3,649	334.8
3	21.83	16.18	.92	.62	8.79	3,669	336.0
3	22.30	16.17	.92	.62	8.79	3,845	344.8
SM6-2	21.86	10.79	.91	.60	5.93	2,559	234.1
3	21.77	10.77	.92	.62	5.85	2,564	235.6
3	21.83	10.78	.92	.60	5.86	2,569	235.4
SM4-3	9.74	7.58	.90	.59	4.21	292	60.0
3	9.76	7.57	.90	.58	4.21	293	60.0

Table 2.3 Experimental Results

	$P_m$ "	$P_{mp}$ "	$P_m/P_{mp}$	$\delta_m$ cm	$\delta_p$ "	$\delta_m/\delta_p$	$\theta_m/\theta_p$
SM12-2	53.7	50.03	1.07	2.54	.86	2.95	1.99
3	53.0	50.65	1.05	2.13	.86	2.48	1.86
3	53.4	49.63	1.08	2.15	.86	2.50	1.93
SM9-2	45.1	39.75	1.13	3.15	.87	3.62	2.54
3	43.7	40.20	1.09	3.01	.87	3.46	2.42
3	44.1	41.32	1.07	2.99	.86	3.48	2.22
SM6-2	32.5	28.17	1.12	4.49	.90	4.99	3.40
3	32.1	29.82	1.10	3.80	.90	4.22	3.31
3	33.0	29.22	1.14	3.30	.90	3.67	2.50
SM4-3	19.6	15.44	1.23	9.76	.62	15.74	5.93
3	17.8	15.44	1.15	3.75	.62	6.05	3.69

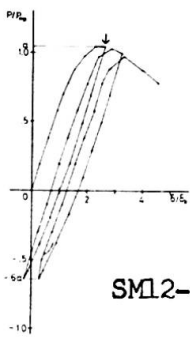


Fig. 2.2

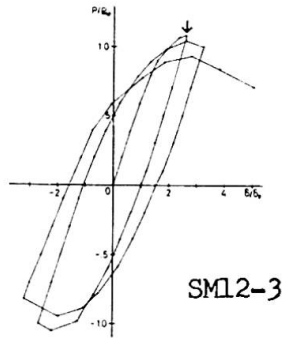


Fig. 2.3

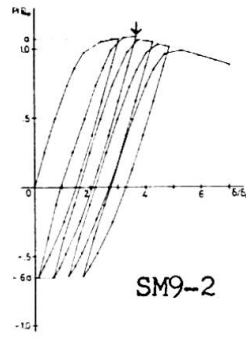


Fig. 2.4

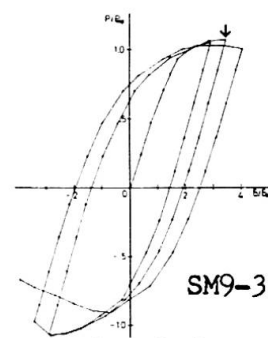


Fig. 2.5

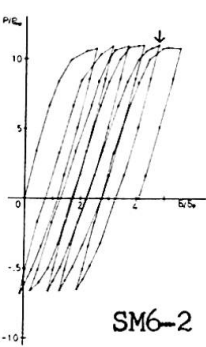


Fig. 2.6

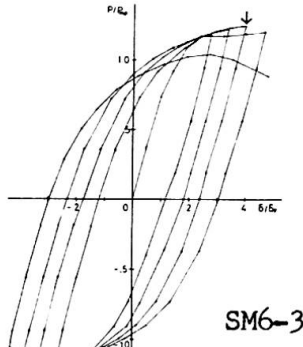


Fig. 2.7

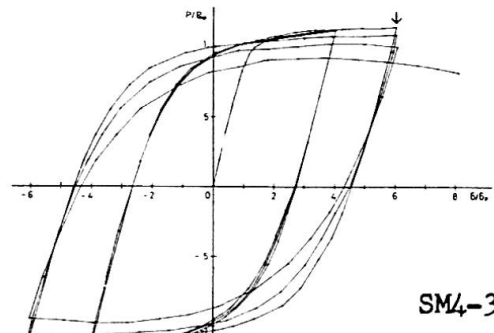


Fig. 2.8

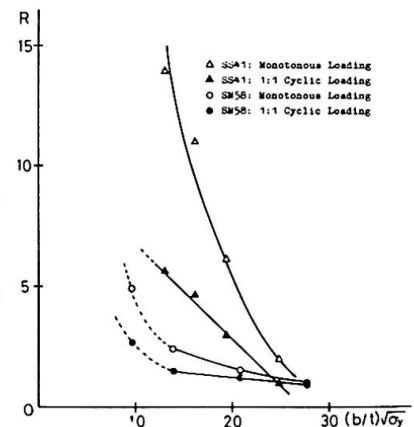
### 2.3 EXPERIMENTAL RESULTS

Experimental results are shown in Table 2.3, where  $P_m$  = maximum load,  $\delta_m$  and  $\theta_m$  = the deflection and rotation corresponding to  $P_m$ , respectively,  $P_{mp}$  = the load corresponding to full plastic moment,  $\delta_p$  and  $\theta_p$  = elastic deformation and rotation corresponding to  $P_{mp}$ , respectively. The  $P$ - $\delta$  curves are shown in Fig. 2.2 to Fig. 2.8. In each figure, the arrow shows the point of beam failure. From these figures, the rotation capacity of the beam in the plastic range was found to be very small. Especially, the negative slope of  $P$ - $\delta$  curves in the third beam became steep comparing with that of the first beam (monotonous loading test).

In Fig. 2.9, the relationships between  $R$  and  $(b/t)\sqrt{\sigma_y}$  are shown together with the experimental

results of SS41 class steels. Fig. 2.9 shows that the rotation capacity of SM58 class steels is extremely small than that of SS41. In the case of the beams having the ratio  $b/t$  exceeding 6.0, as shown in Fig. 2.9, the rotation capacity of the beams decreases linearly as  $b/t$  becomes larger. However, in the case of the beam having the ratio  $b/t=4.2$ , the rotation capacity of the beams under monotonous loading was found to be 4.9, and 2.7 under cyclically repeated load. Therefore, the investigations relating to the beams having the ratio  $b/t$  between 4.2 and 6., need to be performed further. The rotation capacity of the beam under cyclically repeated loads was found to be very small, compared with that under monotonous loading. From the results, it is presumed that there are many problems in regard to the use of high strength steels to earthquake-proof structures.

It is very practically important to know the behavior of the members consisting of high strength steel and further research in this field is needed.



$R = \theta_m / \theta_p - 1$  = rotation capacity

Fig. 2.9 Relationships between  $R$  and  $(b/t)\sqrt{\sigma_y}$

### 3. INELASTIC LATERAL BUCKLING UNDER CYCLIC LOADING

#### 3.1 SPECIMENS AND EXPERIMENTAL METHOD

In this chapter, experiments of cyclic bending beams in which deformation capacity is determined by the combination of local and lateral buckling are presented. The experiments reported in this chapter were made by the Working Group of Fatigue in Low and High Cycle Range in Metals Subcommittee (The chairman is Prof. Morihisa Fujimoto) in Research Committee on Safety of Structural Materials (RCSSM, the chairman is Prof. Takeo Naka). Beam specimens were made of three kinds of structural carbon steel. Properties of the material are given in Table 3.1. Beam specimens were built up with steel plate of 9 mm by welding and the scope of the welding is illustrated in Fig. 3.1. After the specimens were welded, they were not treated by stress relief annealing. Dimensions of the specimens are given in Table 3.2. Ratio of width to thickness of flange,  $b/t$ , is restricted to one on each grade of steel, taking into consideration the experiments by G.H. Lay(9), T. Suzuki and so on, and recommendations of ASCE(6) and AIJ(7). Ratio of width to thickness of web,  $d/t_w$  is determined by the loading equipment, but it is within the values of the above recommendations.

The experimental purpose is to discuss the evaluation of deformation capacity of inelastic beams under cyclic loading, changing the variables such as slenderness factor  $\lambda (=l_b/i_y)$ , grade of steel and loading type. Conditions of loading and supporting are illustrated in Fig. 3.2. The specimen supported on rollers was loaded with a single concentrated load applied at the center, and deformable except for restricting to fall down sideways at loading and supported points. In fact, lateral buckling occurred symmetrically in all of specimens in two half-waves and the load reached maximum.

#### 3.2 EXPERIMENTAL RESULTS

Fig. 3.3(SM50,  $\lambda=55$ ) shows  $M/M_p-\theta/\theta_p$  curves in cyclic loading as the amplitude was a rotational angle  $\theta_m$  at maximum load  $M_m$  by monotonous loading, where  $M$  is applied moment,  $M_p$ , full plastic moment of the specimens,  $\theta$ , rotation measured at supported point and  $\theta_p$ , elastic rotation corresponding to  $M_p$ . And  $M/M_p-\theta/\theta_p$  curve in cyclic loading between  $\theta_m$  and  $-0.6\theta_m$  is plotted in Fig. 3.4. And the curves in the case of SM50,  $\lambda=45$  are plotted in Fig. 3.5 and 3.6. The  $M/M_p-\theta/\theta_p$  curves (SM58,  $\lambda=50$  and  $\lambda=40$ ) in cyclic loading between  $(1/2)\theta_m$  and  $-(1/2)\theta_m$  are also plotted in Figs. 3.7 and 3.8. The endurance of beam in the case of SM58,  $\lambda=50$  did not fall after 8 cycles loading, and that of SM58,  $\lambda=40$  reached the maximum load in each cycle.  $M/M_p-\theta/\theta_p$  curves (HW70,  $\lambda=40$  and  $\lambda=30$ ) under cyclic loading with amplitude between  $(1/2)\theta_m$  and  $-(1/2)\theta_m$  are plotted in Figs. 3.9 and 3.10.

After local buckling occurred at flange portion of every specimen, lateral buckling occurred. At the maximum load, local buckling the web portion was observed. It seems that the negative slope of  $M/M_p-\theta/\theta_p$  curve after reaching the maximum load is determined by local buckling at web portion, especially under cyclic loading. The above experimental results indicated that the stability of hysteresis loop and deformational capacity of inelastic beams under cyclic loading are related to  $d/t_w$  as well as  $\lambda$  and  $b/t$ , when deformation capacity is prescribed by the combination.

Table 3.1

Chemical composition and Mechanical properties of Materials

Grade of steel	Chemical Composition (%)										Tension Test			Check Test					
	C	Si	Mn	P	S	Cu	Cr	Mo	V	B	Eq	Y.P.	T.S.	El.	$\sigma_y$	$\sigma_b$	Est.	$E_u$	Est.
	x100				x1000					x100	1000	1000	kgf/mm <sup>2</sup>	kgf/mm <sup>2</sup>	(%)	kgf/mm <sup>2</sup>	kgf/mm <sup>2</sup>	(%)	kgf/mm <sup>2</sup>
SM 50	16	34	130	17	18							33	51	28	36.2	51.3	2.08	2.23	45.9
SM 58	14	24	140	22	6						38	62	69	25	60.9	66.7	1.75	7.44	27.6
HW 70	11	27	89	11	3	24	91	32	4	1	53	84	88	25	84.0	88.9		7.58	



Table 3.2 Dimension of Specimens and Experimental Results

Grade of steel	Specimen NO.	$l$ (mm)	$R$ (mm)	$b$ (mm)	$b/t$	$d$ (mm)	$d/t$	$l_b$ (mm)	$i_y$ (mm)	$l_y/i_y$	$M_p$ (t·m)	$\theta_p$ (rad)	$M_u/M_p$	$\theta_u/\theta_p$	Loading condition
SM50	H5755M	8.86	9.8	63.0	7.10	282	31.9	1378	24.6	56.0	19.2	0.90	1.18	4.79	Monotony
	H5755C1	8.86	9.1	63.0	7.10	282	31.9	1378	24.7	55.9	19.0	0.90	1.16	4.47	$\theta_u \sim -\theta_u$
	H5755C2	8.86	10.5	63.2	7.13	282	31.9	1378	24.7	55.8	19.3	0.90	1.16	4.26	$\theta_u \sim -0.5\theta_u$
	H5745M	8.86	10.3	63.1	7.12	282	31.9	1130	24.7	45.8	19.3	0.74	1.22	7.77	Monotony
	H5745C1	8.86	10.2	62.9	7.10	283	31.9	1130	24.6	46.0	19.2	0.74	1.24	6.81	$\theta_u \sim -\theta_u$
SM58	H5745C2	8.86	10.4	63.2	7.13	282	31.8	1130	24.7	45.7	19.2	0.74	1.22	6.36	$\theta_u \sim -0.5\theta_u$
	H6650M	9.40	9.4	54.1	5.76	198	21.1	1130	22.3	50.7	19.5	1.79	1.05	3.51	Monotony
	H6650C1	9.40	9.2	54.4	5.79	198	21.1	1130	22.4	50.4	19.5	1.79			$0.7\theta_u \sim -0.7\theta_u$
	H6640M	9.40	10.2	53.9	5.73	198	21.1	904	22.1	40.9	19.6	1.43	1.10	5.53	Monotony
HW70	H6640C1	9.40	10.7	53.9	5.73	198	21.1	904	22.1	41.0	19.7	1.43			$0.6\theta_u \sim -0.6\theta_u$
	H7540M	9.48	11.7	45.4	4.79	126	13.3	805	19.6	41.0	14.4	2.63	1.06	4.88	Monotony
	H7540C1	9.48	10.4	45.3	4.78	126	13.3	805	19.7	41.0	14.0	2.60			$0.6\theta_u \sim -0.6\theta_u$
	H7530M	9.48	10.7	45.3	4.78	126	13.3	604	19.7	30.6	14.1	1.95	1.15	7.24	Monotony
	H7530C1	9.48	8.8	45.2	4.76	126	13.3	604	19.7	30.6	13.7	1.96			$0.6\theta_u \sim -0.6\theta_u$

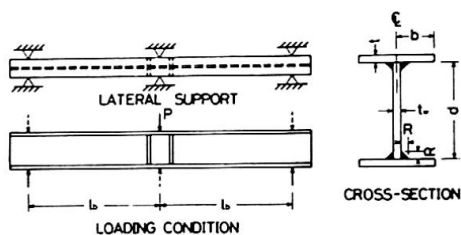


Fig. 3.1

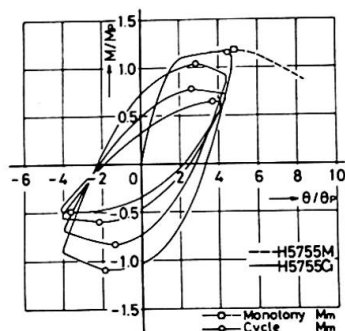


Fig. 3.3

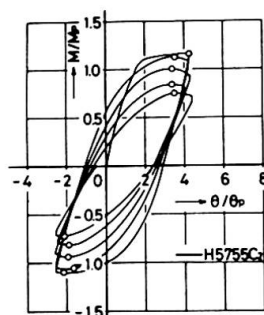


Fig. 3.4

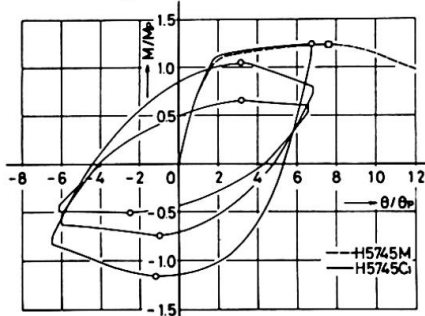


Fig. 3.5

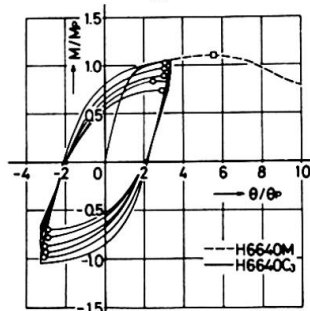


Fig. 3.8

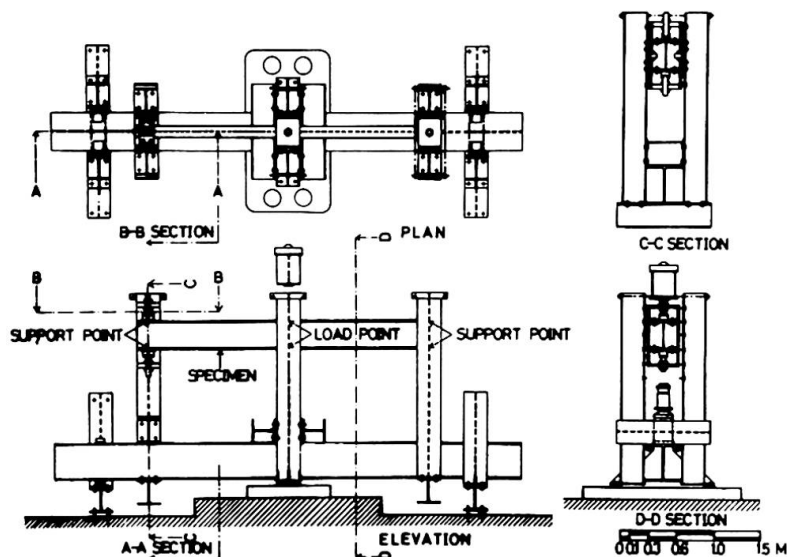


Fig. 3.2

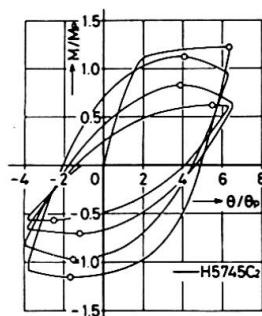


Fig. 3.6

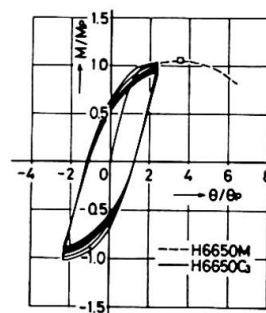


Fig. 3.7

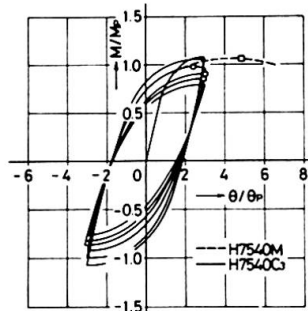


Fig. 3.9

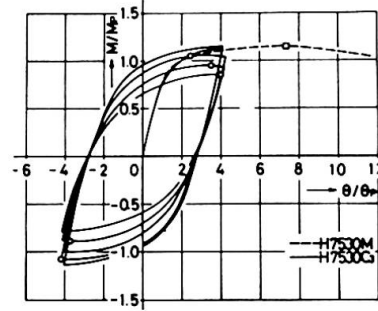


Fig. 3.10

## ACKNOWLEDGEMENT

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## SUMMARY

It was confirmed experimentally that rotation capacity of structural steel members, both mild and high strength steels, was influenced considerably by cyclic loading. The rotation capacity for beams of high strength steels with high yield ratio as compared with that of mild steels is small. The figures indicate that the rotation capacity of beams under cyclic loading is still lower. Therefore, it is desirable that the existing values  $L/r_y$  and  $b/t$  for high strength steels should be limited even more strictly.

## RESUME

L'expérience confirme l'influence considérable de charges répétées sur la capacité de rotation d'éléments en acier doux et même à haute résistance. La capacité de rotation des poutres en acier à haute résistance et à haute limite d'élasticité est plus petite que celle de l'acier doux. Les figures montrent que la capacité de rotation des poutres sous charges répétées est encore plus petite. Par conséquent, il vaudrait mieux que les valeurs  $L/r_y$  et  $b/t$  en vigueur pour l'acier à haute résistance soient limitées sévèrement.

## ZUSAMMENFASSUNG

Durch Versuche wurde festgestellt, dass die wiederholte Belastung auf die Rotationskapazität von Stäben sowohl aus leichtem wie aus hochfestem Stahl eine starke Wirkung ausübt. Die Rotationskapazität von Balken aus hochfestem Stahl mit hohem Streckungsverhältnis ist geringer als jene des weichen Stahls. Die Abbildungen zeigen, dass die Rotationskapazität der Balken unter wiederholter Belastung noch kleiner ist. Somit ist der bestehende Wert von  $L/r_y$  und  $b/t$  für hochfesten Stahl noch strenger zu begrenzen.