Structural Behaviour of Reinforced Concrete at Transient High Temperatures

Comportement structural de béton armé à de hautes températures passagères

Strukturelles Verhalten von Stahlbeton gegenüber vorübergehenden hohen Temperaturen

MASAO INUZUKA
Ph. D., D.I.C.
Hokkaido Institute of Technology
Sapporo, Japan

1. Introduction

When a reinforced concrete is subjected to heating, all structural conditions can be time-dependent. The structural behaviour of composant materials is influenced by many parameters such as stress and temperatures. Concrete deformation, in particular, would involve many parameters. Therefore structural analysis on reinforced concrete in such condition can not but depend on rather crude approximations in order to avoid an entirely empirical approach which can lead to either uneconomical processes or dubious results.

Finite theory in which the continuity of quantity can be ignored has been used in a fire research defining the fire resistance by hours. This philosophy may be as well applicable to the definition of a relationship of a cross section element between a bending moment, an axial force and deformations in a linear structure (beams and columns) at given time---there can be only a finite number of cases in the relationship.

2. Sectional Theory

It is common in the structural analysis of reinforced concrete to take the cross section as the smallest element in linear structures. It may be convinient to divide the section into parts which may be subjected to different stress conditions according to their positions. Such a part can be represented by an imaginary linear element. Although these linear elements are distributed in three dimensions, two dimensional elements will be discussed, since the temperature change in the axial direction can be small compared with that in other directions. In the section these elements should comply with both the compatibility and the equilibrium. From the compatibility condition, the strain distribution should be continuous. From the equilibrium the resultants on the section should be zero with respect to certain point in it. The collection of all sections in a structure can give the behaviour of the whole structure. The behaviour of the linear element is based on the constitutive equations of material composing the element.

3. Behaviour of Linear Element

The knowledge on mechanical characteristics of the linear element should be obtained by the experiments with respect to composant materials. In a normal condition, this may be given as a stress-strain relationship. However, many parameters can be related to the transient deformation. Therefore the constitutive equation of the stress and that of the stored energy may take the following parameters in the function f( ) and g( ). respectively:

stress: $\sigma = f(\varepsilon, T, t, H)$ ....(1)

where ε = strain at time in question
T = temperature at time in question
t = time in question
H = historical term between the birth of material and the time in question

stored energy per unit volume: $E = g(\sigma, \varepsilon, T, t, H)$ ....(2)

Today these equations have not been fully cleared with actual structural materials, let alone these on concrete at transient high temperatures. Therefore these may have to be constructed combining experimental results previously offered by many different workers, in so much as the unique value of the stress and the stored energy can be given with all conditions defined between the material birth and the time in qestion.

4. Structural Conditions of Sectional Behaviour

The deformation of section elements, on which the thermal and the historical conditions can be difined, is due to the resultant of bending moment and normal force on the section of area A, when the stored energy is as follows, refering to Fig.1:

$$\int_A dP_e = P_i \qquad ....(3)$$

$$\int_A dM_e = M_i \qquad ....(4)$$

$$\int_A dU_e = U_i \qquad ....(5)$$

where dP_e = normal force applied to the linear element
P_i = normal force resultant on the section i
dM_e = bending moment due to the linear element with respect to a given point
M_i = bending moment with respect to a given point
dU_e = energy stored in the linear element
U_i = energy stored in the section

Therefore we have three equations to give a strain distribution. A pattern of strain distribution on the section should comply with these three mechanical conditions in a given thermal condition. On top of it, three assumptions may be taken---(1) the temperature distribution can be independent from the stress distribution, (2) the strain distribution can be expressed in a finite number of parameters, (3) some parameters can be digitallized or---these parameters can be expressed in an integer number corresponding to a value of the parameter in their range. We are to find out the pattern of the strain distribution out of possible patterns. Since the available equations (1) and (2) would innevitably contain some vagueness, the rigorous mathematical solution for the strain distribution may not be practical. Under these assumptions, the equations (3), (4) and (5) can be written as follows:

$$\sum \Delta P_e - P_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\sum \Delta M_e - M_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$U_i = \sum (\Delta U_e) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where U_i = stored energy in section i

5. Numerical Example

This is an example which has an uniform cross section and expansion restriction. Conditions of a section at time t are as shown in Fig.1. An arbitrary linear element at position (y,z) is subjected to the stress corresponding to its strain $\varepsilon_t(y.z)$ temperature $T_t(y,z)$ and its history $H_t(y,z)$. The following assumptions are taken:

(1) plane theory on the strain distribution,
(2) deformation of the section on y-z plane can be ignored,
(3) temperature is monotonically increasing,
(4) only the maximum stress in the past after heating began is taken into account as the historical stress of the history effects and
(5) time elapses in the interval of 0.15 minutes(not continuously).

5.1. Constitutive Equations

Concrete equation: from Malhotra's experimental results(Ref.1), the strength f' at T (difference between material temperature and constant room one in the centigrade unit) with t=0 (heating time in the unit of minutes) is:

$$f' = f_r (1 - T/1000)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where f_r = strength at T=0 and t=0

From Furumura's work(Ref.2), the virgin stress σ may be expressed in a second degree equation with respect to the strain :

$$\sigma = \varphi(\varepsilon - \varepsilon_x)\left\{ \varepsilon - (2\varepsilon_r - \varepsilon_x)\right\} \qquad \ldots\ldots\ldots\ldots\ldots (10)$$

where   where $\varphi = -f'/(\varepsilon_r - \varepsilon_x)^2$,
$\varepsilon_r = 0.002 + 0.007\left\{ 1 - \exp(-t/100)\right\}$:strain corresponding to the strength f' according to Rüsch(Ref.3).
$\varepsilon_x = -10^{-5}T$:thermal expansion
$\varepsilon_x \leqslant \varepsilon \leqslant \varepsilon_r$

When the stress in question is lower than the historical stress (corresponding to its strain):

$$\sigma = 2f'(\varepsilon - \varepsilon_H)/(\varepsilon_T - \varepsilon_x) + \sigma_H \qquad \ldots\ldots\ldots\ldots\ldots (11)$$

Also from Furumura's research work the stored energy in a unit volume may be as follows:

$$\Delta U_i = \sigma^2(\varepsilon_T - \varepsilon_x)/4f' \qquad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Steel equation: since its time effect due to relaxation is small compared with that of concrete, the equation may be simplified as follows: refering to Harada's results(Ref.4):

$$\sigma = \begin{cases} -f & \text{:when } E(\varepsilon - \varepsilon_x) < f_T \\ E(\varepsilon - \varepsilon_x) & \text{:when } -f_T \leqslant E_S(\varepsilon - \varepsilon_x) \leqslant f_T \\ f & \text{:when } E(\varepsilon - \varepsilon_x) > f_T \end{cases} \qquad \ldots\ldots\ldots (13)$$

where $E_S = 2.1 \times 10^6 \, Kg/cm^2$:Young's modulus
$f_T = 5000 - 5T' Kg/cm^2)$:yield stress

Similarly as in the concrete equation, the stored energy may be expressed as:

$$\Delta U_i = \sigma^2/2E_S \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Covering a diminutive area ΔA, a linear element will give rise the followings.

$$\Delta P_i = \sigma \Delta A, \quad \Delta M_i = y \sigma \Delta A, \quad U_i = \sum \Delta A \cdot \Delta Ue \cdot \Delta x_i$$

Temperature distribution at time t is that in the author's past work(Ref.5), in which the ambient temperature rise was given in the time interval according to the time temperature curve of IF code. A solution, corresponding to a given moment distribution and normal force, can be found by the trial-error method with two strain values at each section element, since the plane theory is employed and the two interim values of strain can give strain of all other linear elements in the same section. Equations (6) and (7) would have some discrepancy from zero at their rights even for the most plausible strains,

Thus,
$$\left\{ (\sum \Delta P_e) - P_i \right\}/P_i = \omega_1 \quad \dots\dots\dots\dots\dots\dots\dots(15)$$

$$\left\{ (\sum \Delta M_e) - M_i \right\}/M_i = \omega_2 \quad \dots\dots\dots\dots\dots\dots\dots(16)$$

$$S_i = \omega_1^2 + \omega_2^2 \quad \dots\dots\dots\dots\dots\dots(17)$$

where ω_1 and ω_2 are discrepancy from zero

A possible pair of two strains should make the least of S_i from the least square sum principle. At the same time the stored energy U_i should be minimum. However, the moments and the normal forces used in the above are the function of deformations of all section elements. Therefore, these also must be found by trial in order to comply with the restraining condition at the ends of the structural member--- the total compatibility should be kept. In this example, the distance between the two ends should be constant. Thus, the moment and normal force at each section may be computed by trials. The value of stress and strain at all linear elements should be stored for the next time step, once the solution is found.

Each section element is subjected to a different thermal condition. Thus the rate between the length of heated parts and the rest should be taken into account. The flow chart of this procedure is shown in Fig.2, the section analysis being applied between s-1 and s-N. Fig.3 shows the calculated results of heat resistance against the applied moment in three cases.

6. Conclusion

Even the simplest structural member can behave as the complicated machinery when heated. The results of the numerical example shows that the complete restriction against the expansion can reduce the heat resistance considerably. When the end rotation is restrained, the restriction can normally increase the resistance, because the whole structural system will be changed and the structural redundancy can support after the first fracture took place at the particuler section. Therefore the end effects should be divided into two: one against the axial expansion, the other against the rotation. However the assessment of resistance has to take the loading condition into account. Therefore the simple example could suggest that the most severe end effects may be caused by the complete expansion restraint, keeping rotation free, and that the higher the rate of the heated part in the structure, the shorter the heat resistance. Since it is difficult to obtain the experimental condition similar to that in the actual fire condition, particulaly on the full-scale specimens, the assessment of the behaviour and the resistance should be based on the data on both the temperature distribution change and the constitutive equations on materials.
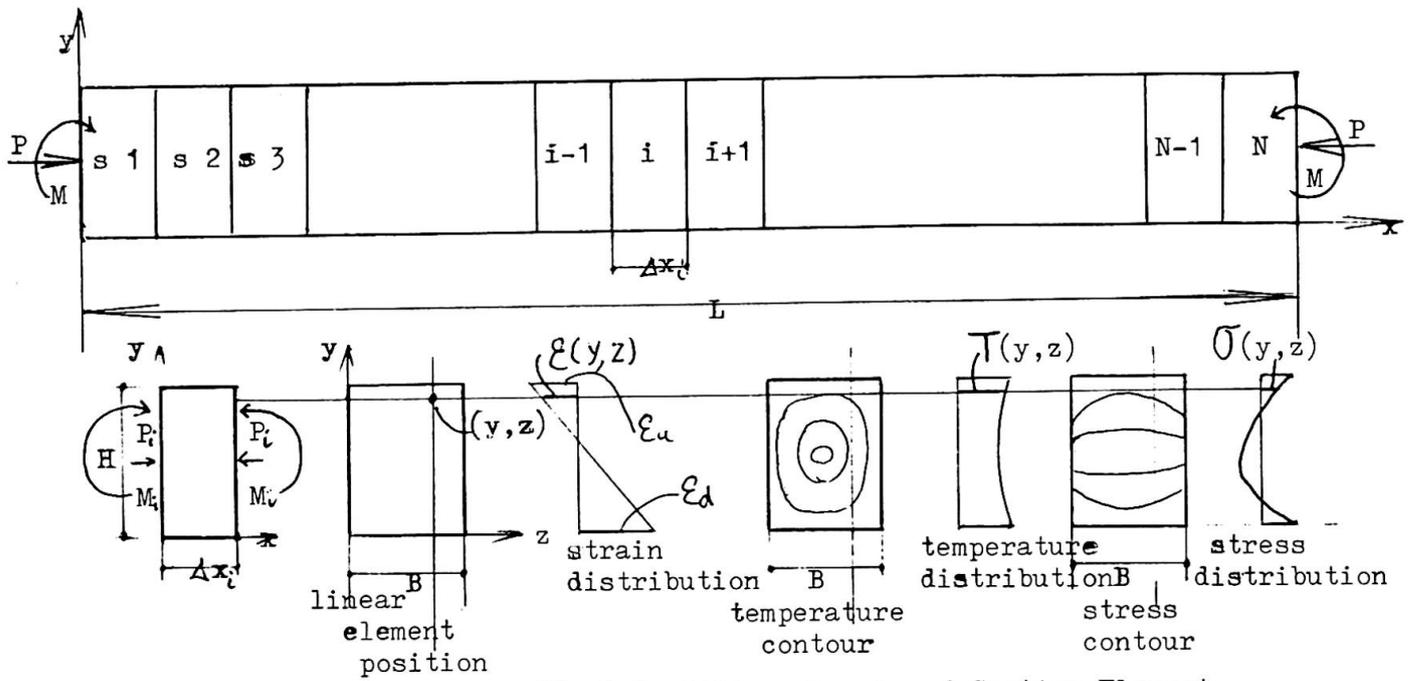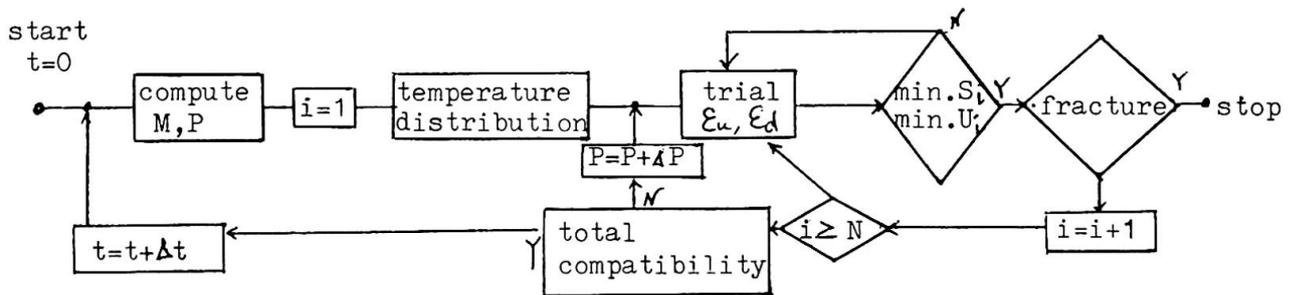
Fig.1-Condition Diagram of Section Element
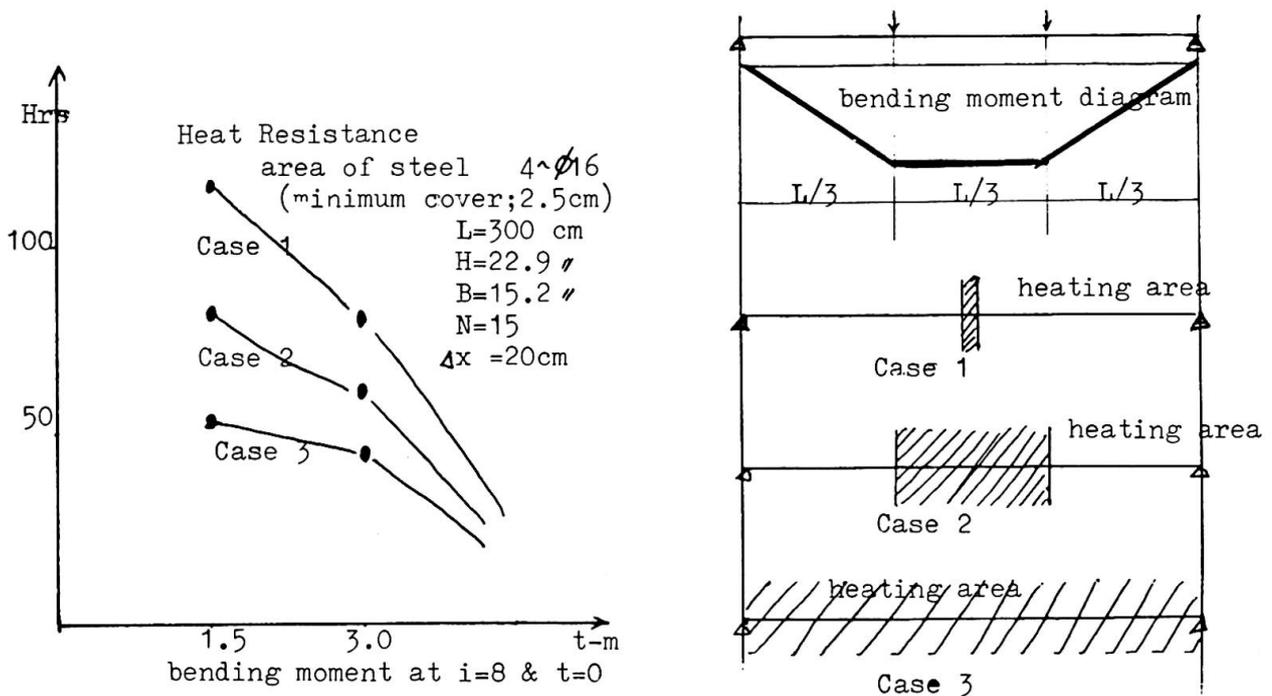

Fig.2-Flow Chart of Numerical Example

Fig.3-Case Study on Heat Resistance

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SUMMARY
     The study centres on the mechanical response of a sectional element
in beams and/or columns subjected to incessant changes such as tempe-
rature distribution, a bending moment and an axial force. If these
changes can be ignored in the analysis, the procedure should give a
result similar to the ones in ordinary structural analysis on the beha-
viour of section elements. For the normal analysis under a constant room
temperature is a special case among the cases taking temperatures into
account.

RESUME
     L'étude se concentre sur les réactions mécaniques d'un élément de
poutre et/ou de colonne sujet à des changements incessants tels que cas de
charge, une distribution de température, un moment de flexion et une force
axiale. Si ces changements peuvent être ignorés dans le calcul, la procé-
dure devrait donner un résultat similaire à ceux d'un calcul ordinaire sur
le comportement d'éléments. Car le calcul conventionnel sous température
ambiante constante est un cas spécial dans les cas tenant compte des
températures.

ZUSAMMENFASSUNG

     Im Mittelpunkt der Untersuchung steht das mechanische Verhalten eines
Elementes in Balken und/oder Stützen, die einem dauernden Wechsel der
Temperaturverteilung, des Biegemomentes und der Normalkraft ausgesetzt
sind. Wenn diese Aenderungen in der Berechnung ausser Acht gelassen werden
können, sollte das Verfahren ein ähnliches Resultat ergeben wie bei der
üblichen Strukturanalyse über das Verhalten von Elementen. Die normale
Berechnung unter konstanter Raumtemperatur ist ein Sonderfall unter den in
Betracht gezogenen Temperaturfällen.