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**Structural Behaviour of Reinforced Concrete at Transient High Temperatures**

Comportement structural de béton armé à de hautes températures passagères

Strukturelles Verhalten von Stahlbeton gegenüber vorübergehenden hohen Temperaturen

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**1. Introduction**

When a reinforced concrete is subjected to heating, all structural conditions can be time-dependent. The structural behaviour of componant materials is influenced by many parameters such as stress and temperatures. Concrete deformation, in particular, would involve many parameters. Therefore structural analysis on reinforced concrete in such condition can not but depend on rather crude approximations in order to avoid an entirely empirical approach which can lead to either uneconomical processes or dubious results.

Finite theory in which the continuity of quantity can be ignored has been used in a fire research defining the fire resistance by hours. This philosophy may be as well applicable to the definition of a relationship of a cross section element between a bending moment, an axial force and deformations in a linear structure (beams and columns) at given time---there can be only a finite number of cases in the relationship.

**2. Sectional Theory**

It is common in the structural analysis of reinforced concrete to take the cross section as the smallest element in linear structures. It may be convinient to divide the section into parts which may be subjected to different stress conditions according to their positions. Such a part can be represented by an imaginary linear element. Although these linear elements are distributed in three dimensions, two dimensional elements will be discussed, since the temperature change in the axial direction can be small compared with that in other directions. In the section these elements should comply with both the compatibility and the equilibrium. From the compatibility condition, the strain distribution should be continuous. From the equilibrium the resultants on the section should be zero with respect to certain point in it. The collection of all sections in a structure can give the behaviour of the whole structure. The behaviour of the linear element is based on the constitutive equations of material composing the element.

### 3. Behaviour of Linear Element

The knowledge on mechanical characteristics of the linear element should be obtained by the experiments with respect to composant materials. In a normal condition, this may be given as a stress-strain relationship. However, many parameters can be related to the transient deformation. Therefore the constitutive equation of the stress and that of the stored energy may take the following parameters in the function  $f(\cdot)$  and  $g(\cdot)$ , respectively:

where  $\epsilon$  = strain at time in question

T = temperature at time in question

t = time in question

H = historical term between the birth of material and the time in question

$$\text{stored energy per unit volume: } E = g(\sigma, \varepsilon, T, t, H) \quad \dots \dots \dots (2)$$

Today these equations have not been fully cleared with actual structural materials, let alone these on concrete at transient high temperatures. Therefore these may have to be constructed combining experimental results previously offered by many different workers, in so much as the unique value of the stress and the stored energy can be given with all conditions defined between the material birth and the time in question.

#### 4. Structural Conditions of Sectional Behaviour

The deformation of section elements, on which the thermal and historical conditions can be defined, is due to the resultant of bending moment and normal force on the section of area A, when the stored energy is as follows, referring to Fig.1:

where  $dP_e$  = normal force applied to the linear element

$P_i$  = normal force resultant on the section  $i$

$dMe$  = bending moment due to the linear element with respect to a given point

$M_i$  = bending moment with respect to a given point

$dU_e$  = energy stored in the linear element

$U_i$  = energy stored in the section

Therefore we have three equations to give a strain distribution. A pattern of strain distribution on the section should comply with these three mechanical conditions in a given thermal condition. On top of it, three assumptions may be taken---(1) the temperature distribution can be independent from the stress distribution, (2) the strain distribution can be expressed in a finite number of parameters, (3) some parameters can be digitalized or---these parameters can be expressed in an integer number corresponding to a value of the parameter in their range. We are to find out the pattern of the strain distribution out of possible patterns. Since the available equations (1) and (2) would inevitably contain some vagueness, the rigorous mathematical solution for the strain distribution may not be practical. Under these assumptions, the equations (3), (4) and (5) can be written as follows:

where  $U_i$  = stored energy in section  $i$

## 5. Numerical Example

This is an example which has an uniform cross section and expansion restriction. Conditions of a section at time  $t$  are as shown in Fig.1. An arbitrary linear element at position  $(y, z)$  is subjected to the stress corresponding to its strain  $\epsilon_t(y, z)$  temperature  $T_t(y, z)$  and its history  $H_t(y, z)$ . The following assumptions are taken:

- (1) plane theory on the strain distribution,
- (2) deformation of the section on y-z plane can be ignored,
- (3) temperature is monotonically increasing,
- (4) only the maximum stress in the past after heating began is taken into account as the historical stress of the history effects and
- (5) time elapses in the interval of 0.15 minutes(not continuously).

### 5.1. Constitutive Equations

Concrete equation: from Malhotra's experimental results(Ref.1), the strength  $f'$  at  $T$  (difference between material temperature and constant room one in the centigrade unit) with  $t=0$  (heating time in the unit of minutes) is:

where  $f_r$  = strength at  $T=0$  and  $t=0$

From Furumura's work(Ref.2), the virgin stress  $\sigma$  may be expressed in a second degree equation with respect to the strain :

where  $\varphi = -f' / (\varepsilon_r - \varepsilon_x)^2$   
 $\varepsilon_r = 0.002 + 0.007 \{ 1 - \exp(-t/100) \}$ : strain corresponding to the strength  $f'$  according to Rüsch(Ref.3).

$\xi_x = -10^{-5}T$  : thermal expansion

$$\varepsilon_x \leq \varepsilon \leq \varepsilon_T$$

When the stress in question is lower than the historical stress (corresponding to its strain):

Also from Furumura's research work the stored energy in a unit volume may be as follows:

Steel equation: since its time effect due to relaxation is small compared with that of concrete, the equation may be simplified as follows: referring to Harada's results(Ref.4):

$$\sigma = \begin{cases} -f & \text{when } E(\varepsilon - \varepsilon_x) < f_r \\ E(\varepsilon - \varepsilon_x) & \text{when } -f_r \leq E_s(\varepsilon - \varepsilon_x) \leq f_r \\ f & \text{when } E(\varepsilon - \varepsilon_x) > f_r \end{cases} \quad (13)$$

where  $E_s = 2.1 \times 10^6 \text{ Kg/cm}^2$  : Young's modulus  
 $f_r = 5000 - 5T \text{ Kg/cm}^2$  : yield stress

Similarly as in the concrete equation, the stored energy may be expressed as:

$$4U_i \equiv \sigma^2/2E\epsilon \quad \dots \dots \dots \quad (14)$$

Covering a diminutive area  $\Delta A$ , a linear element will give rise the followings.

$$\Delta P_i = \sigma \Delta A, \quad \Delta M_i = y \sigma \Delta A, \quad U_i = \sum \Delta A_i \sigma U e_i \Delta x_i$$

Temperature distribution at time  $t$  is that in the author's past work (Ref. 5), in which the ambient temperature rise was given in the time interval according to the time temperature curve of IF code. A solution, corresponding to a given moment distribution and normal force, can be found by the trial-error method with two strain values at each section element, since the plane theory is employed and the two interim values of strain can give strain of all other linear elements in the same section. Equations (6) and (7) would have some discrepancy from zero at their rights even for the most plausible strains,

where  $\omega_1$  and  $\omega_2$  are discrepancy from zero

A possible pair of two strains should make the least of  $S_i$  from the least square sum principle. At the same time the stored energy  $U_i$  should be minimum. However, the moments and the normal forces used in the above are the function of deformations of all section elements. Therefore, these also must be found by trial in order to comply with the restraining condition at the ends of the structural member---the total compatibility should be kept. In this example, the distance between the two ends should be constant. Thus, the moment and normal force at each section may be computed by trials. The value of stress and strain at all linear elements should be stored for the next time step, once the solution is found.

Each section element is subjected to a different thermal condition. Thus the ratio between the length of heated parts and the rest should be taken into account. The flow chart of this procedure is shown in Fig.2, the section analysis being applied between  $s-1$  and  $s-N$ . Fig.3 shows the calculated results of heat resistance against the applied moment in three cases.

## 6. Conclusion

Even the simplest structural member can behave as the complicated machinery when heated. The results of the numerical example shows that the complete restriction against the expansion can reduce the heat resistance considerably. When the end rotation is restrained, the restriction can normally increase the resistance, because the whole structural system will be changed and the structural redundancy can support after the first fracture took place at the particular section. Therefore the end effects should be divided into two: one against the axial expansion, the other against the rotation. However the assessment of resistance has to take the loading condition into account. Therefore the simple example could suggest that the most severe end effects may be caused by the complete expansion restraint, keeping rotation free, and that the higher the rate of the heated part in the structure, the shorter the heat resistance. Since it is difficult to obtain the experimental condition similar to that in the actual fire condition, particularly on the full-scale specimens, the assessment of the behaviour and the resistance should be based on the data on both the temperature distribution change and the constitutive equations on materials.

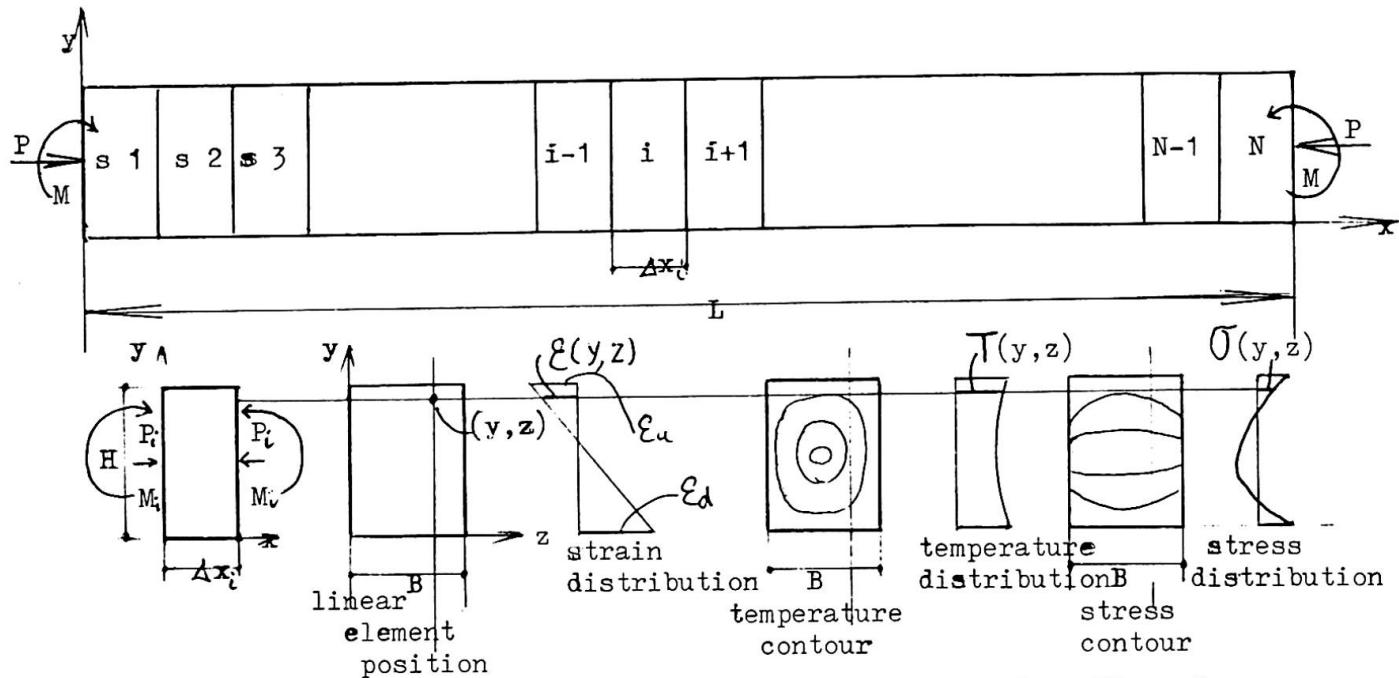


Fig. 1-Condition Diagram of Section Element

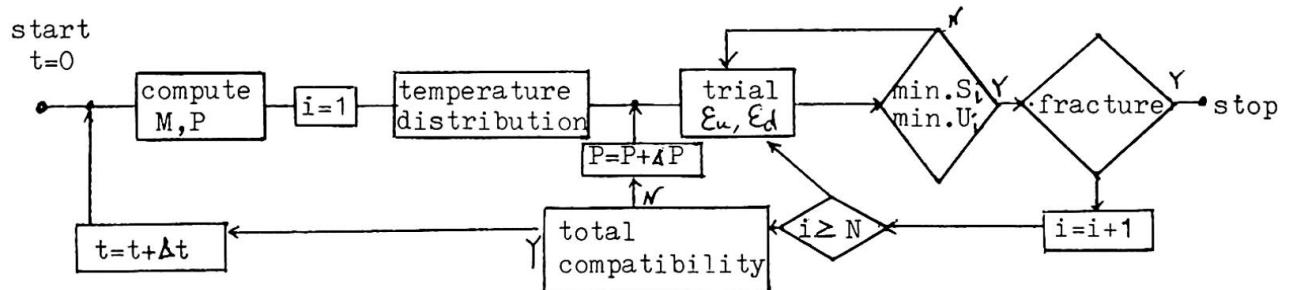


Fig. 2-Flow Chart of Numerical Example

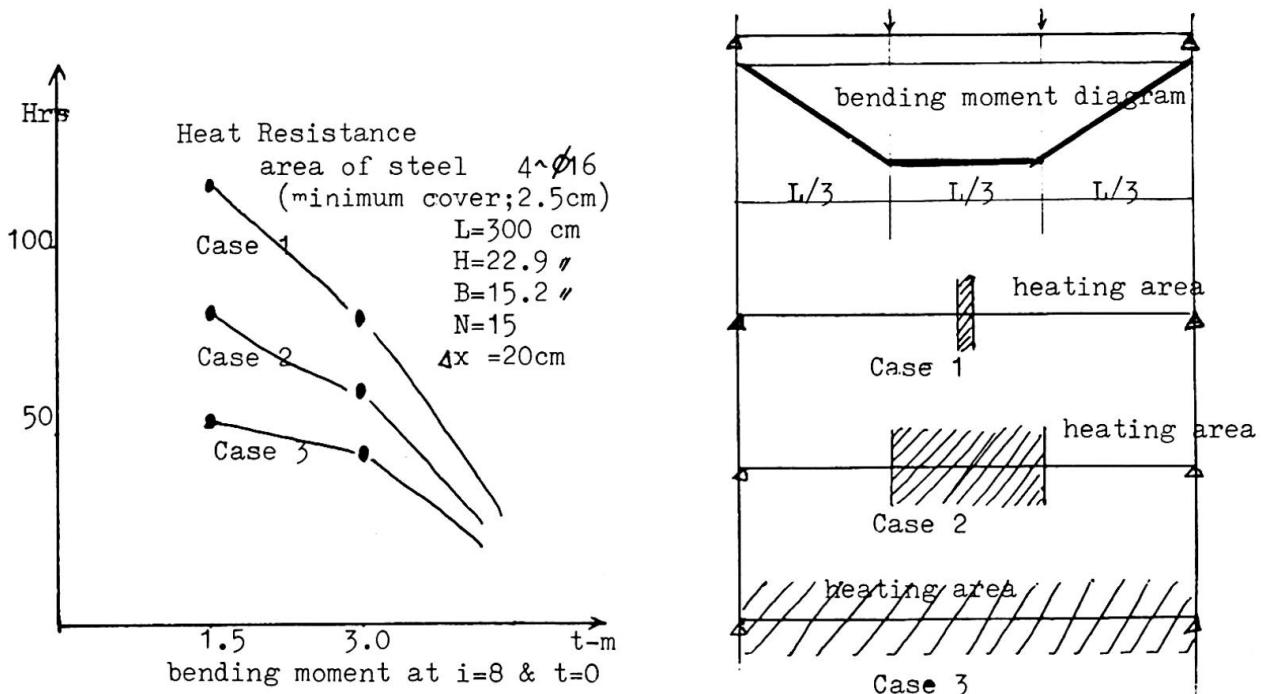


Fig. 3-Case Study on Heat Resistance

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### Bibliography

- (Ref.1) Malhotra, H.L. 'The Effect of Temperature on Compressive Strength of Concrete' Magazine of Concrete Research Vol.8, No.23, Aug.1956, pp.85-94.
- (Ref.2) Furumura, F. 'The Stress-Strain Curve of Concrete at High Temperatures(No.1)' Report for Annual Meeting of Architectual Institute of Japan, 1966.
- (Ref.3) Rüsch, H. 'Researches toward a General Flexural Theory for Structural Concrete' ACI Journal, Proc.57, No.1, July 1960, pp.1-28.
- (Ref.4) Harada, T and Furumura, F. 'The Behaviour of Steel at High Temperatures' Transaction of AIJ Meeting, 1967, pp.93-96.
- (Ref.5) Inuzuka, M. 'Flexural Behaviour of Reinforced Concrete Members at Transient High Temperatures' Thesis for Ph.D. of University of London, March, 1974, pp.189-192.

### SUMMARY

The study centres on the mechanical response of a sectional element in beams and/or columns subjected to incessant changes such as temperature distribution, a bending moment and an axial force. If these changes can be ignored in the analysis, the procedure should give a result similar to the ones in ordinary structural analysis on the behaviour of section elements. For the normal analysis under a constant room temperature is a special case among the cases taking temperatures into account.

### RESUME

L'étude se concentre sur les réactions mécaniques d'un élément de poutre et/ou de colonne sujet à des changements incessants tels que cas de charge, une distribution de température, un moment de flexion et une force axiale. Si ces changements peuvent être ignorés dans le calcul, la procédure devrait donner un résultat similaire à ceux d'un calcul ordinaire sur le comportement d'éléments. Car le calcul conventionnel sous température ambiante constante est un cas spécial dans les cas tenant compte des températures.

### ZUSAMMENFASSUNG

Im Mittelpunkt der Untersuchung steht das mechanische Verhalten eines Elementes in Balken und/oder Stützen, die einem dauernden Wechsel der Temperaturverteilung, des Biegemomentes und der Normalkraft ausgesetzt sind. Wenn diese Änderungen in der Berechnung ausser Acht gelassen werden können, sollte das Verfahren ein ähnliches Resultat ergeben wie bei der üblichen Strukturanalyse über das Verhalten von Elementen. Die normale Berechnung unter konstanter Raumtemperatur ist ein Sonderfall unter den in Betracht gezogenen Temperaturfällen.