Earthquake-Resistant Design of the Tower and Pier System of Suspension Bridges

Dimensionnement contre les tremblements de terre du système de pylône et pile des ponts suspendus

Die Erdbebenbemessung des Systems von Pylon mit Sockel bei Hängebrücken

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1. INTRODUCTION

Economical applications of mathematical programming methods in structural optimization are limited to specific structures as mentioned in Introductory Report.[1] In the case of structures with relatively simple and bulky dimensions, the mathematical programming method could be applied efficiently even if the structures are designed under relatively complicated design conditions. Dynamic loading problems are not treated in Introductory Report, and loading conditions appearing in optimal design have been mostly limited to the static ones. Studies on aseismic design of long-span suspension bridges were carried out for many years in Japan, and the results of investigations were published as the official or individual reports. According to the studies on aseismic design of the suspension bridges, design of the tower and the pier is very important.[2] These parts of the bridge must be investigated as a system because of the interaction of these parts during the earthquake. The tower and pier system of suspension bridges involves rigid, massive, and large pier and relatively flexible and slender tower, so that the system has very complicated interaction.[2] The combination of the methods of mathematical programming and dynamic structural analysis is in fact well suited to the aseismic design of the tower and pier system of suspension bridges.

To formulate earthquake action for aseismic design, the method of response spectrum is employed in the design codes of the long-span suspension bridges in Japan. In this paper, the response spectrum method is mainly applied in the dynamic analysis and design of the system. Another approach based on more probabilistic concepts using power spectrum density of earthquake action and random vibration theory is possible using design constraints for reliability. Some approximation concepts[3] are used to save the computing time and to decrease the design variables in this paper.

2. THE STRUCTURAL SYSTEM

The system to be designed is the tower and pier system of the suspension

bridges as shown in Fig.1, height h_T of the tower and h_P of the pier are determined from the environmental attribute of the bridge, and width b_1 of the pier is determined from geometrical relation with the bridge width. The design variables in global sense are, therefore, the longitudinal width b_2 of the pier and the stiffness of the tower. The combination of these two variables induces very complicated dynamic properties of the system.[2]
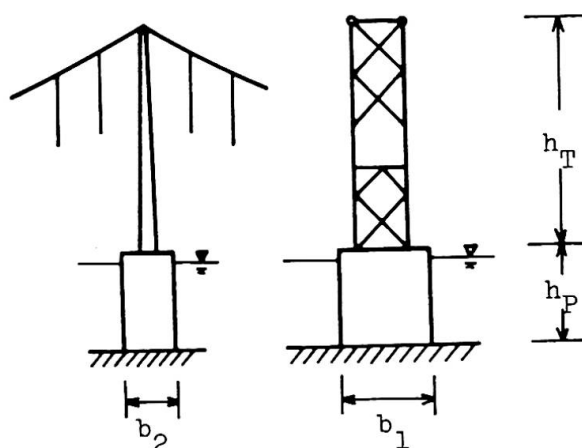

Fig.1 Structural System

3. DYNAMIC ANALYSIS OF THE SYSTEM

Analytical Model   The analytical model of the tower and pier system of the suspension bridge treated in this paper is shown in Fig.2. The tower is assumed to be the lumped mass system, and the following assumptions are made:
  (1) The foundation has elastic property.
  (2) The reaction of the cable at the top of the tower is taken into account by applying the equivalent axial thrust and using an equivalent spring for the cable.[4]
  (3) The pier is assumed to be perfectly rigid and to be a single-degree-of-freedom capable of rocking motion.

Model of Earthquake Excitation   Earthquake excitation is represented by response acceleration spectrum. In this study, the standard spectrum as shown in Fig.3 is used which is authorized by Honshu-Shikoku Connection Bridge Authority of Japan. In this figure, the longitudinal axis refers to be response magnification factor β, and standard acceleration in this design is 180 gal.

Dynamic Response Analysis   The equation of motion for this multi-degrees-of-freedom-system can be written as:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-[\bar{M}]\{\ddot{z}\}$$

where $[M]$ is a mass matrix, $[C]$ is a damping matrix, $[K]$ is a stiffness matrix, $\{y\}$ is a displacement vector, $\{\ddot{z}\}$ is an earthquake acceleration vector. With the aid of modal matrix $[\Phi]$ and the generalized displacement vector $\{q\}$, where $\{y\}=[\Phi]\{q\}$, then the equation of motion rewritten in the following form assuming proportional damping.

Fig.2 Analytical Model

$$[I]\{\ddot{q}\}+[\diagdown(2h_i\omega_i)\diagdown]\{\dot{q}\}+[\diagdown(\omega_i)^2\diagdown]\{q\}=-\{P\}$$

where ω_i refers to natural frequency, and h_i is damping constant of i-th mode.

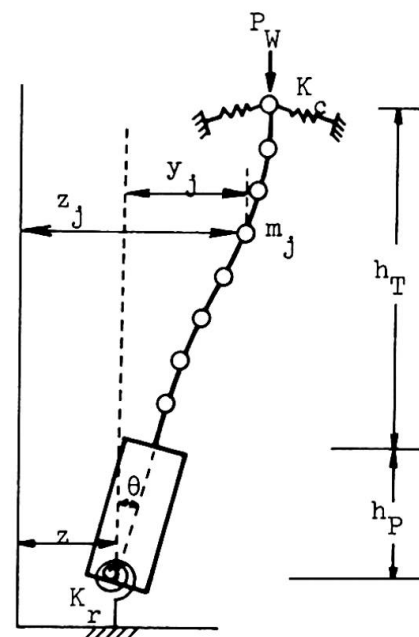The maximum displacement of point j, $y_{max}^{(j)}$, can be evaluated by root mean square method:

$$y_{max}^{(j)}= \sqrt{\Sigma(F_i \phi_i^{(j)} q_{i\ max})^2}$$

where F_i refers to the participation factor of i-th mode, $\phi_i^{(j)}$ refers to the relative displacement of point j in i-th mode. $q_{i\ max}$ is obtained from response spectrum given in Fig.3:

$$q_{i\ max}=\beta_i \ddot{z}_{max}/\omega_i^2$$

where $\ddot{z}_{max}$ is the maximum earthquake acceleration.

Fig.3 Earthquake Excitation Model

4. DESIGN MODEL

Approximation Concepts of the Tower   To save culculation time and to improve reliability of solution, two design variables of the system are selected: One is the moment of inertia of the tower, the other is the longitudinal width of the pier. Other variables of the system are defined by approximation concepts.[3]

Let I, A and Z refer to the moment of inertia, the cross sectional area and the section modulus respectively, the empirical relation such as following may hold:

$$A=1.21*I^{0.33}$$

$$Z=0.55*I^{0.75}$$

The moment of inertia of the tower can be varied along the height in two ways: One is linearly varied; the other is stepwise varied into two portions. These design models are shown in Fig.4.

Fig.4 Design Model

Foundation Model   The modulus of elasticity of the foundation is denoted by E. In the result of the past studies[2] complicated dynamic phenomena due to the foundation condition, width of the pier, and the rigidity of the tower were observed. In the cases where two of the natural frequencies are very close, the coupled vibration of the tower and pier occurs, and the structural systems of such cases should be avoided. In this study, the modulus of elasticity ranges from $10*10^4$ ton/m^2 to $150*10^4$ ton/m^2 taking into account wide variety of foundation conditions.

Damping Constant   The damping constant is assumed to be 0.1 for the mode where the vibration of the pier is predominant, and to be 0.02 for the vibration of the tower. For the coupling modes 0.05 for both modes is assumed.

5. OPTIMIZATION

Objective Function   The generalized cost, W, is selected to be the objective function:
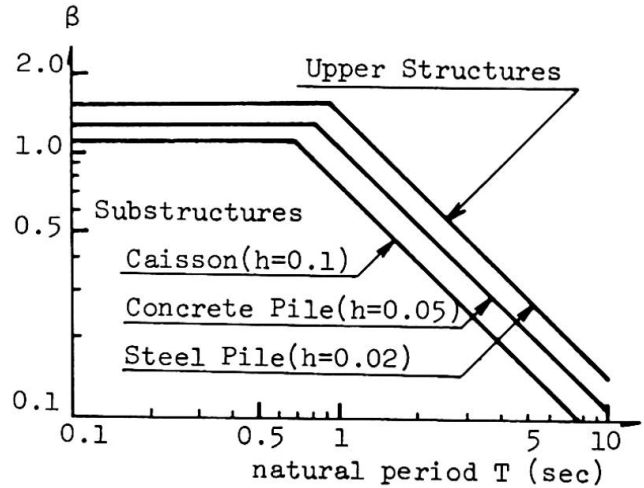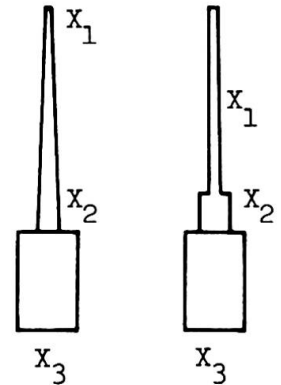
$$W=W_T+k*W_P$$

where W_T represents the weight of the tower and W_P, of the pier, and k refers to the ratio of unit cost of the pier to that of the tower.

Constraints   The following constraints are considered:
  (1) Stress of the tower shaft does not exceed the given allowable stress defined against earthquakes.
  (2) Displacement of the pier top does not exceed the given allowable value.
  (3) Tower shaft is safe against buckling.
  (4) Pier is safe against overturning.
  (5) Other geometric conditions.

Optimization Technique   Objective function and constraints obtained in this way become non-linear and undifferential type, so SUMT by Powell's direct search method without differential is employed as optimization technique.

6. NUMERICAL EXAMPLE AND INVESTIGATIONS

As a numerical example, the tower and pier system shown in Fig.5 is considered, and the results of the computation are shown in Table 1,2. These computation were performed using the design model with stepwise varied cross section. In making Table 1, the following data was used:

P_W=29743 ton
K_c=45650 ton/m
h_T=184.0 m
h_P=52.0 m
b_1=50.0 m

Fig.5   Tower and Pier System

        cost ratio 0.2
        maximum acceleration .... 180 gal
        allowable value of pier top displacement .... 0.05 m
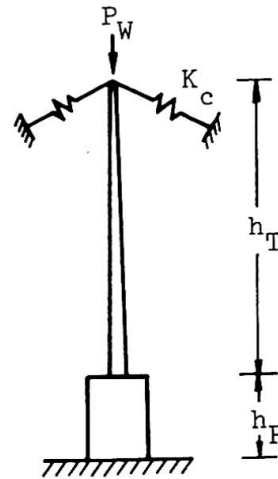   allowable stress of steel .... 37700 ton/m^2

From Table 1, the following investigations may be made:
  (1) When elastic modulus of foundation, E, is small, the design of the system is determined only by the displacement constraint at the pier top. When the value of E is large, it tend to be determined by overturning of the pier and buckling of the tower, and the pier width tends to decrease. It shows that the pier width is closely related with E.
  (2) The generalized cost is greatly affected by the modulus of elasticity of the foundation. Thus, the investigation of the foundation is very important.
  (3) When E is large, the effect of earthquake response tend to decrease, and stiffness of the tower becomes uniform along the height of the tower. From this, when E is large enough, it is not necessary to increase the cross section of the lower part of the tower.
The design is controlled severely by the constraint of the displacement of the pier top in the range of small E. When this constraint is relieved to 0.065 m, the results are shown in Table 2. From these Tables, the following remarks may be made:
  (1) In the range of small E, when the constraint of the pier top displacement is relieved slightly, the generalized cost decrease considerably. This result shows that the allowable value of the displacement of the pier top has a significant effect.
  (2) In the range of large E, the result is not so affected by the constraint on displacement.

7. CONCLUSION

Table 1

E $(10^4 ton/m^2)$	I (m^4) Upper	Lower	$b_2(m)$	W	Pier (1)	(2)	Tower Top	(3)	Base	(4)
10	11.22	62.26	38.97	52146	X					
20	16.52	65.97	24.45	36077	X					
30	5.81	24.90	19.91	27865	X					
50	4.78	21.80	14.37	20896	X	X				X
70	4.75	4.77	14.35	20509		X				X
150	4.75	4.77	14.35	20509		X				X

(1): Displacement of the pier top (2): Overturning
(3): Stress of the tower shaft (4): Buckling

Table 2

E $(10^4 ton/m^2)$	I (m^4) Upper	Lower	$b_2(m)$	W	Pier (1)	(2)	Tower Top	(3)	Base	(4)
10	8.15	32.71	28.76	39081	X					
20	13.65	24.43	18.56	27053	X					
30	4.75	8.41	14.54	20842	X					X
50	4.77	4.78	14.35	20511		X				X
70	4.77	4.78	14.35	20511		X				X
150	4.75	4.77	14.35	20508		X				X

(1): Displacement of the pier top (2): Overturning
(3): Stress of the tower shaft (4): Buckling

The optimal design of the tower and pier system on the elastic foundation subjected to earthquake excitation is studied by using response spectrum and modal analysis. Investigation in this study shows that necessity or importance of displacement condition of the pier top must be discussed more precisely from the point of safety of the structure in the range of small E, and that necessity of earthquake-resistant design must be discussed more precisely from the dynamic response of the structure in the range of large E.

PROBABILISTIC APPROACH

Probabilistic approach using power spectrum density for earthquake and based on random vibration theory can be formulated as follows.

Earthquake load is represented by power spectrum density function shown in Fig.6.[5] As earthquake is assumed to have zero mean and to be stationary probabilistic process, variances of the displacement and of the velocity can be evaluated based on the random vibration theory.
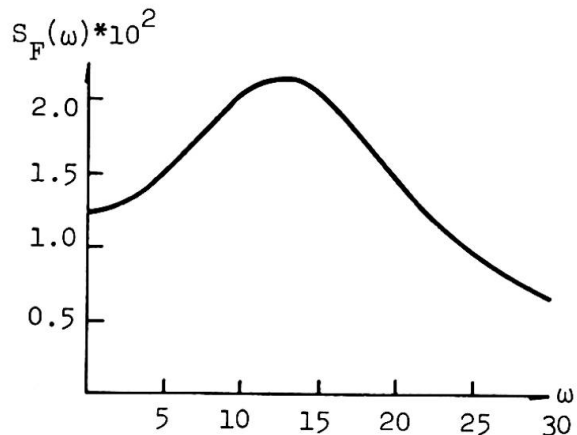
Failure probability can be com-

Fig.6 Power Spectrum Density

puted through dynamic reliability theory using displacement and velocity vari-
ances. Thus, it is possible to formulate optimization by probabilistic
approach using failure probability as constraints.

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SUMMARY
    Effective application of the structural optimization method is limited
to some specific types of structures in civil engineering structures. In
the case of structures with relatively simple and bulky dimensions, the
mathematical programming method could be applied efficiently. In this
paper, the authors carried out the optimal design of the tower and pier
system of suspension bridges on the elastic foundation subjected to
earthquake ground motion using response spectrum and dynamic analysis.

RESUME
    Une application pratique de la méthode d'optimisation structurale est
limitée à certaines structures du génie civil. Dans le cas de structures
relativement simples et de grandes dimensions, la méthode de programmation
mathématique peut être appliquée efficacement. Dans cet article les
auteurs ont fait le calcul d'optimisation du système de pylône et pile
des ponts suspendus sur fondation élastique subissant le tremblement de
terre, à l'aide d'une analyse dynamique.

ZUSAMMENFASSUNG
    Die Anwendung der Tragwerks-Optimierung ist auf einige spezielle
Strukturarten im Bauingenieurwesen begrenzt. Bei Strukturen mit einfachen
und massigen Abmessungen lässt sich das mathematische Programmierungsver-
fahren erfolgreich verwenden. In diesem Aufsatz wird eine Optimierung des
Pylonsystems auf elastischem Untergrund unter Erdbebenlast entwickelt.
Hierbei werden Verhaltensspektren und dynamische Analysen angewendet.