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Structural System Optimization Based on Suboptimizing Method of Member Elements

L'optimisation du système structural basée sur la suboptimisation d'éléments

Optimierung der Tragstrukturen auf Grund der Suboptimierungsmethode der Teilelemente

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1. INTRODUCTION

The complexity and difficulty arised in the optimization procedure of a practical structural system are caused mainly by the various characteristic and numerous design variables and constraints involved in a structural system. The methodological expansion on the treatment of such design variables and constraints has been expected for the efficient optimum design method of the structural systems. This paper presents practical optimization methods intended to solve the problems based on suboptimization of structural elements.

In the optimum design methods presented herein, suboptimization of the structural elements are performed first for the range of possible loadings and design variables, then suboptimized relationships between an intensive design variable and design constraints, objective function etc. are introduced. Using these relationships logical reductions in the number of design variables and constraints, and introduction of material selection variables may be possible. Objective function is also simplified, and geometrical and discrete variables can be treated easily. The optimum solutions are found by sequential linear programming algorithm and graphical approach. Examples of cost minimization problems of highway girders and minimum weight design of trusses are presented. Using the methods direct optimum design diagrams for highway girders have been established.

2. OPTIMUM DESIGN USING SUBOPTIMIZATION OF STRUCTURAL ELEMENTS AND SLP METHOD

2.1 Girder Problems

Problem Formation - The cost minimization problems of constant-depth highway welded plate girders are solved by SLP method using suboptimization of girder elements. The design variables are assumed as cross sectional dimensions, length, ℓ, and steel type, M, to be used for each girder segment. Design criteria imposed in the steel girder section are constraints on allowable stresses, plate thicknesses for stability of the girder and minimum rigidities of vertical and horizontal stiffeners which are taken from "Specifications for Steel Highway Bridges". (Ref. 13) Discrete constraints on commercial availability of plate thicknesses are also considered.

Total cost of the girder, TCOST, is assumed to consist of material cost, CM, fabrication cost, CFF×(1+FF), and welding cost, CWM + CWF×(1+FF), which are evaluated with reference to "Tables of Prime Costs for Steel Highway Bridges".(Ref. 14)

$$TCOST = \sum_{i=1}^{NM} COST_i \times \ell_i = \sum_{i=1}^{NM} [\ CM_i + CFF_i \times (1+FF) + CWM_i + CWF_i \times (1+FF)\ ] \times \ell_i \qquad (1)$$

in which FF = factor of indirect fabrication cost, CWM = cost for welding materials, CWF = welding cost.

Suboptimization of Girder Elements - In the girder problems, behavior variables are determined by the arrangement of moment of inertia, I, and length, ℓ, of each girder segment and usually dimensions of a girder section are determined by applied maximum bending moment. For this reason suboptimization of the girder sections are performed first for various combinations of steel types, M, web heights, WH, and bending moments, BM, by taking into account all of the design variables and constraints.

The mixed-discrete nonlinear optimization problems of the girder sections may be solved quite effectively by a modified branch and bound algorithm and SLP method, where the order to branch and bound of discrete variables is preassigned according to their importance for the design of girder section, and only two adjacent discrete values to the continuous optimum solution are examined for their optimality. Macro flow chart of the algorithm is shown in Fig. 1. The results of suboptimization of girder elements are arranged in terms of moment of inertia and I-RBM, I-COST, I-SDIM, RBM-GW relationships for each steel type and web height are introduced, where RBM = maximum resisting bending moment, COST = minimum cost per unit length, SDIM = optimum sectional dimensions, GW = girder weight per unit length. I-RBM and I-COST relationships shown in Fig. 2 may be expressed as

$$RBM(I) = a \cdot I + b, \quad COST(I) = c \cdot I + d \quad (2)$$

The coefficients a, b, c and d are all constants for the particular range of I, M and WH. Since flange plate thicknesses are increased discretely as applied bending moment increases, unit price of the steel plate and size of the fillet welding are changed also discretely and I-COST relationships are varied discontinuously at such points. On the contrary I-RBM relationships are varied linearly and may be expressed by several linear equations accurately.

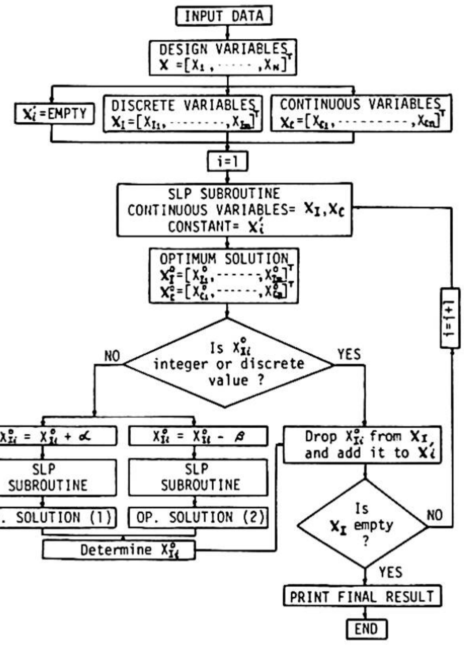

Fig. 1 Macro Flow Chart of Modified Branch and Bound Method with SLP Subroutine

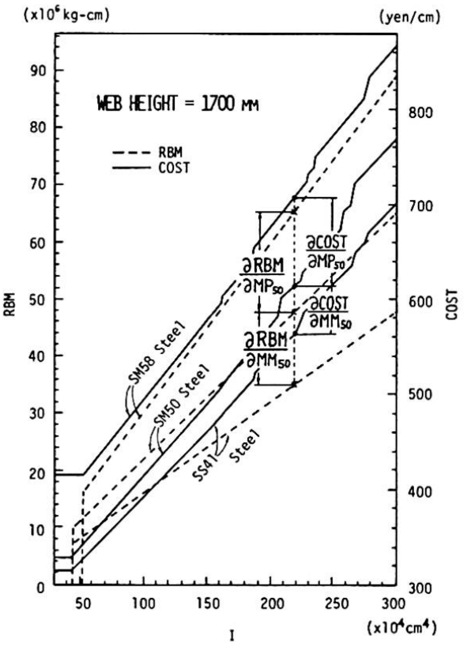

Fig. 2 I-RBM, I-COST Relationships for Girders with Web Height = 1700 mm

Simplification of Problem and Introduction of Material Selection Variables - I-RBM relationships introduced by this method express the allowable upper limit of resisting bending moments of the girder sections to satisfy all of the constraints. Minimum costs of the girder sections with I, WH and M may be evaluated directly from related I-COST relationships. Therefore by using these relationships I of each girder element may be considered as a new design variable instead of all of the sectional dimensions if web height is preassigned as a design parameter and BM ≤ RBM relationship comes to a new intensive constraint in place of all of the restrictions. This reduction in the number of design variables and constraints to be considered simultaneously gives significant advantages to solve complex structural optimization problems, such as simplification of the problem formulation and evaluation of the sensitivities, reduction of the core size and computation time, improvement of the convergency to the optimum solution. Furthermore the differences of values between two material types at a value of I in the I-RBM and I-COST relationships may be considered as the partial derivatives with respect to the design variable for selecting optimum steel type to be used for each girder element.(Fig. 2) The material selection variables M are introduced based on this concept, which consist of MP and MM. The former are provided for selection of the stronger steel type and the latter are for the weaker. MP and MM are treated as independent continuous variables same as I and ℓ.

Optimization by SLP Method - The girder is analysed by the displacement method and the behavior variables and their partial derivatives with respect to I, ℓ, M are evaluated by using the influence line analysis. Partial derivatives of RBM and COST with respect to I, ℓ, M are also evaluated from related I-RBM and I-COST relationships. The nonlinear optimum design problem is approximated with a linear programming problem by the first order terms of Taylor series expansion and an improved solution is determined by Simplex algorithm. Adaptive move limit constraints on the changes of design variables are also added to ensure the convergency to optimum solution. Since the material selection variables are assumed here as continuous variables, which are modified to the nearest discrete steel types at every iteration of analysis. If a solution comes closely to the optimum solution, all steel types are fixed as most profitable and material selection variables are eliminated from a set of the design variables. Then the problem is reanalysed until optimum solution is obtained. The optimum sectional dimensions for each girder element may be decided directly from the related I-SDIM relationships.

Examples - The method has been applied to many cases of simple span, 2 ~ 3-span continuous constant-depth highway girders and three examples are presented in Table 1 in which the solutions are compared with the results by graphical approach described later. In Table 1 BW = bridge width, SL = span length, WH = web height, P_ℓ = a concentrated live load, q_ℓ = uniformly distributed live load, q_d = distributed dead load which differs with each girder segment, but averaged value in the girder is shown in the table.

Table 1 Optimum Solutions by SLP Method and Graphical Method

	Seg	SLP Method			Graphical Method			Design Condition
	No.	L (cm)	I (cm⁴)	M*	L (cm)	I (cm⁴)	M*	BW= 8.00 m
1 - SPAN**	1	296.7	1389224	41	293.7	1376687	41	SL= 30.0 m
	2	701.2	1507252	58	710.0	1520473	58	WH= 200 cm
	3	1500.0	2113595	58	1500.0	2113532	58	P_ℓ= 17.990 t
	Min.TCOST	1643675(YEN)			1643622 (YEN)			q_ℓ= 1.259 t/m
	CPU TIME	150~200(sec)			10 ~ 16 (sec)			q_d= 2.310 t/m
	No.of Iter.	15 ~ 20			3 ~ 5			
2-SPAN***	1	323.1	779103	50	285.0	697606	50	BW= 8.00 m
	2	725.7	1430407	50	618.8	1288148	50	SL= 30.0 m
	3	1997.0	1695642	50	1962.0	1712473	50	WH= 170 cm
	4	2683.0	1085665	50	2702.2	1136303	50	p_ℓ= 17.955 t/m
	5	3000.0(s)	1429441	58	3000.0(s)	1431270	58	q_ℓ= 1.257 t/m
	Min.TCOST	2891515(YEN)			2893060 (YEN)			q_d= 2.300 t/m
	CPU TIME	60~100(sec)			3 ~ 4 (sec)			
	No.of Iter.	20 ~ 25			5 ~ 8			
3 - SPAN***	1	248.8	888658	41	233.8	846592	41	BW= 8.00 m
	2	559.9	1250520	50	546.5	1238921	50	BL= 90.0 m
	3	1850.0	1700003	50	1805.6	1723537	50	WH= 190 cm
	4	2486.0	1217784	50	2501.5	1217830	50	Span Ratio =
	5	2812.5(s)	2180159	50	2812.5(s)	2128666	50	1 : 1.2 : 1
	6	3153.0	2180159	50	3153.5	2128666	50	$p_{\ell 1}$= 18.042 t
	7	3841.0	1159787	50	3898.8	1112415	50	$p_{\ell 2}$= 17.747 t
	8	4500.0	1486529	50	4500.0	1529952	50	$q_{\ell 1}$= 1.263 t/m
	Min.TCOST	4241036(YEN)			4224079 (YEN)			$q_{\ell 2}$= 1.242 t/m
	CPU TIME	300~450(sec)			10 ~ 15 (sec)			q_{d1}= 2.030 t/m
	No.of Iter.	25 ~ 35			8 ~ 12			q_{d2}= 2.031 t/m

** Calculated by FACOM 230-28
*** Calculated by HITAC 8800/8700
(s) indicats intermediate support

* 41 = SS41 (JIS) Steel
50 = SM50 (JIS) Steel
58 = SM58 (JIS) Steel

Approximate convergency to the optimum solution including material selection is quite well by using move limit constraints, but computation time and number of iteration of reanalysis required for the optimum solution are increased so much as number of design variables and constraints increases as seen in Table 1. Comparisons of several solutions with different initial inputs should be made for confirmation of the global optimum solution.

2.2 TRUSS PROBLEMS

The truss problems are solved as weight minimization problems and cross sectional dimensions of the member and coordinates of the panel points are considered as design variables. The steel is fixed as SS41 (JIS) only.

Suboptimization of member elements - In the truss problems, suboptimization of the compression and tension members for many combinations of applied loads and member lengths are treated first, then sectional area A - maximum allowable stress σ_a, A - optimum sectional dimensions, SDIM, relationships for various member lengths are introduced. A-σ_a Relationships at any member length may closely be approximated as

$$\sigma a = \{a \cdot (A-b)\}^{1/n} + C \qquad \text{or} \qquad \sigma a = d \cdot A + e \tag{3}$$

in which a, b, c, d, e and n are all constants related to the member length and A. A-σ_a relationships express the allowable upper limits of the stresses of members with A which are guaranteed to satisfy all of the constraints prescribed to the member design.

By using A-σ_a relationships all design variables and constraints imposed in the member design can be replaced only by A and $\sigma \leq \sigma_a$ relationships respectively, moreover the derivatives of σ_a with respect to the geometry variables X_g can be evaluated simply as

$$\frac{\partial \sigma_{ai}}{\partial X_{g_j}} = \frac{\Delta \sigma_{ai}}{\Delta X_{g_j}} \tag{4}$$

in which $\Delta\sigma_{ai}$ represents the change in σ_a at i-th member due to the change in i-th member length. The problem is approximated as a linear programming problem and reanalyzed until optimum solution is obtained.

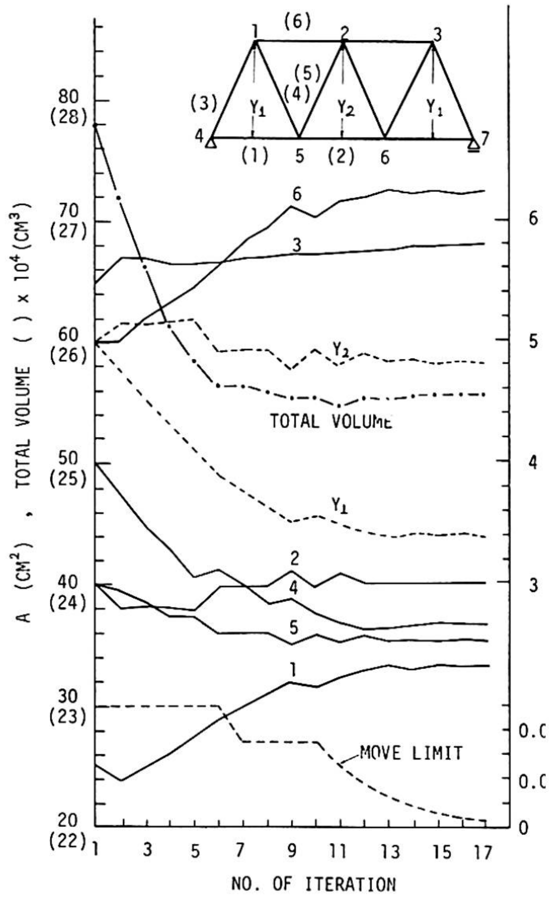

Fig. 3 11-Bar Truss, Moving Loads, $\sigma \leq \sigma_a$ Constraints

Examples - An example of eleven bar truss subjected to the moving loads P=50 ton, q_l=4 ton/m, and the dead load q_d=2 ton/m is shown in Fig. 3. The panel length is fixed as 5 m. Sectional areas of member 1 to 6 and coordinates of panel point 1 and 2, Y_1, Y_2, are assumed as the design variables and only $\sigma \leq \sigma_a$ constraints of the members are taken into account. The initial Y_1 and Y_2 are assumed as 500 cm, however they are reduced finally to 340 cm and 483 cm respectively. Furthermore, members 1, 2, 3, 4 and 6 are fully stressed, while sectional area of member 5 is determined by the maximum slenderness ratio requirement. The minimum total volume obtained is $25.56 \times 10^4 cm^3$ and maximum live loads displacement is 1.17 cm at panel point 5.

In the case maximum live loads displacements of the panel points are limited to 1.0 cm, the optimum solution is found such that the sectional areas A_i are 29.58, 45.45, 66.71, 45.13, 39.30, 62.90(cm^2) respectively and Y_1 = 428 cm, Y_2 = 549 cm with the total volume $27.53 \times 10^4 cm^3$. The total volume increases 7.7% more than previous solution and only member 3 and 6 are fully stressed.

Topological Member Arrangement - If the constraints on lower limits of member sections are not imposed, sectional areas of unnecessary members come to 0 cm^2. Then optimum topological member arrangement of truss may also be determined. Several simple examples on this problem are shown in Ref. 2).

3. GRAPHICAL OPTIMIZATION OF HIGHWAY GIRDERS BASED ON SUBOPTIMIZATION OF GIRDER ELEMENTS

SLP method has been used successfully on a wide range of large and complex structural optimization problems, however in the optimization procedures partial derivatives of the behavior variables and objective function with respect to the design variables should be evaluated at every iteration of reanalysis. Therefore as depicted in the previous girder examples computation time is so much increased as number of design variables increases and more efficient methods to solve the large optimization problems are expected. Graphical optimization method, an approximate approach based on suboptimization of girder elements, has been developed for solving such problems and applied to the cost minimization girder problems.

Design Procedure by Graphical Method - In the graphical approach, a minimum cost diagram related to the initial girder arrangement is developed first by using maximum bending moment diagram of the girder and I-RBM, I-COST relationships. Then improvement of I, ℓ and M of each girder segment is performed by investigation of the change in minimum cost at the adjoining two segments due to a change of segment length, $\Delta\ell$. In case of Fig. 4, the change of minimum cost of the girder, $\Delta TCOST_i$, due to a change of $\Delta\ell_i$ can be evaluated as

$$\Delta TCOST_i = \Delta COST_i \cdot \ell_i - \Delta COST_{i+1} \cdot \Delta\ell_i \qquad (5)$$

If $\Delta TCOST_i$ is positive, $\Delta\ell_i$ may proceed to $-\Delta\ell_i$ direction. The improvement due to $\Delta\ell_i$ may be finished when $\Delta TCOST_i$ converges to zero them next improvement on ℓ_{i+1} is performed. After the improvement of all segments is accomplished, the girder is reanalyzed with new I, ℓ and M and the procedure is repeated until a converged solution will be obtained. Three highway girder examples are given in Table 1.

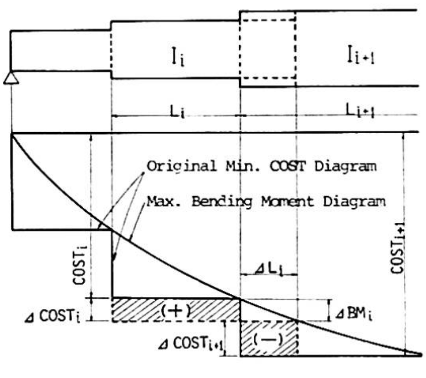

Fig. 4 The Change of TCOST due to ΔL_i

In this approach, attention is paid only to the change of objective function in order to improve the design variables of a girder segment, and effects to the over all behavior variables caused by changes of the design variables are evaluated by reanalysis of the girder. In this sense graphical method is more approximate approach than SLP method, but convergency to the global optimum solution by this method is quite well as seen in Table 1. Computation times required for optimum solution are reduced notably as 3 ~ 5 sec. and 10 ~ 15 sec. on HITAC 8800/8700 for 2 and 3-span continuous girder problems respectively, which are 1/12 ~ 1/30 cpu. time compared with SLP method. Larger reduction in cpu. time is made as number of variables and constraints increases.

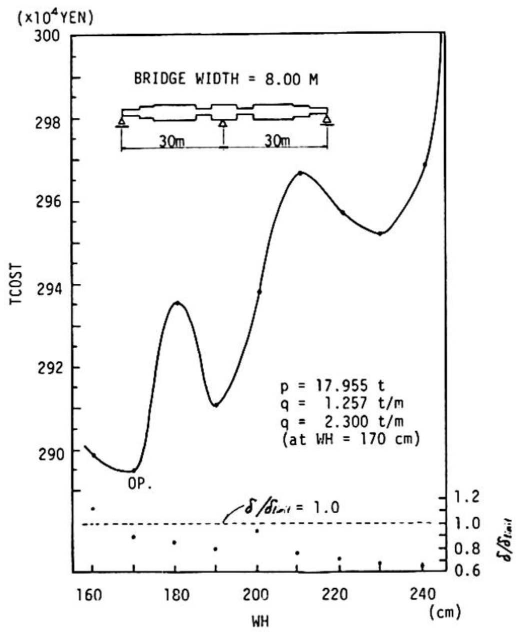

Fig. 5 WH-TCOST, δ/δ_{limit} Relationships for 2-Span Continuous Girder (SL = 30 m, BW = 8.00 m)

Optimum Web Height – To decide the optimum web height at each span length, optimum solutions for several web heights should be compared with each other. Fig. 5 shows an example for 2-span continuous girder with span length 30 m. As seen clearly in the figure, several local minimum solutions exist on web heights and the girder with WH=170 cm gives absolutely minimum cost in this example. For this reason, web height should be treated as a parametric variable in cost minimization highway girder problems.

Optimum Design Diagrams for Highway Girder Bridges – For the purpose of direct optimum design or planning of 1~ 3-span constant-depth highway welded plate girders, various optimum design diagrams and tables such as span length – minimum total cost, optimum WH, I, ℓ, M, and I – SDIM relationships for the girders with nonuniform cross sections, and bending moment – minimum cost, optimum WH, I, M, GW diagrams for the girders with uniform cross sections have been established by using the graphical method, and they will be published soon.[6,7,8]

The optimum design diagrams mentioned above may be utilized as one of the suboptimized structural size design programs in a general purpose system program for highway bridges.

4. CONCLUSIONS

Practical structural optimization methods based on suboptimization of structural elements, SLP and graphical method are presented.

An element size optimization for minimum cost is formulated as a mixed-discrete nonlinear programming problem, and a modified branch and bound algorithm with SLP can be solved the problem effectively. Cpu. time was 1.0 sec. on FACOM 230-75 required for an optimum solution of the girder section.

By using the relationships obtained from suboptimization of structural elements, structural optimization problems may be simplified and be solved effectively. Moreover material selection variables and graphical optimization algorithm have been developed on the basis of this design concepts.

SLP method may be utilized successfully on a wide range of large and complex structural optimization problems and its approximate convergency to the optimum solution is quite well, however computation time and number of iteration of reanalysis increases so much as design variables and constraints increases.

Graphical optimization method is a practical and efficient design method for the cost minimization problems of highway girders. Formation of the computer program is simple, and excellent convergency to the global optimum solution and existence of several local minima on web height have been confirmed. Design diagrams prepared for direct cost minimum design or planning of highway girders have been established by this method. The design diagrams may be utilized as one of the suboptimized structural size design data in a general purpose system program for highway bridges.

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SUMMARY - The optimum design concepts based on suboptimization of structural elements are presented. Large scale and complex structural cost minimization problems may be simplified, and treatments of various types of design variables and constraints such as sizing, material selection, geometry, continuous, discrete come to ease by this concept . SLP method and graphical optimization method are used effectively to find the minimum cost solutions of highway girder and truss examples.

RESUME - Les concepts de l'optimisation basés sur la suboptimisation d'éléments structuraux sont présentés. Cette suboptimisation permet de simplifier des problèmes de minimisation de coût de structures complexes de grande dimension; elle facilite le traitement de variables de projet, de contraintes de types variés telles que dimensionnement, sélection de matériaux, géométrie, continu, discret,... La méthode "SLP" et la méthode d'optimisation graphique s'emploient pour trouver efficacement des solutions permettant de construire, au coût minimum, des ponts et des charpentes.

ZUSAMMENFASSUNG - Das Konzept des optimierten Entwurfes aufgrund der Suboptimierung struktureller Elemente wird dargestellt. Durch dieses Konzept lassen sich die Probleme der Kostenminimierung vereinfachen sowie die Behandlung verschiedener Arten von Entwurfsvariablen und Randbedingungen, wie z.B. Abmessungen, Materialwahl, Geometrie, stetige und unstetige Formen, überdies erleichtern. Die SLP-Methode und die Methode graphischer Optimierung werden verwendet, um die effektiven Minimalkosten eines Brückenträgers une eines Fachwerks zu erhalten.