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II

Progrès dans l'optimisation structurale

Fortschritte in der Optimierung von Tragwerken

Progress in Structural Optimization

II a

Concepts et techniques d'optimisation

Grundlagen und Methoden

Optimization Concepts and Techniques in Structural Design

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Optimum Design of Steel Frame Subjected to Dynamic Earthquake Forces

Calcul optimal de cadres métalliques soumis aux forces dynamiques des tremblements de terre

Optimierung von Stahlrahmen unter dynamischer Erdbebenlast

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1. INTRODUCTION

The mathematical programing technique has already been adopted for the optimization of the structures subjected to the dynamic excitations.[1,2] Most of these optimizations were dealt with beams, trusses or frames, subjected to simple excitations such as harmonic waves or shock waves, and designed under rather simple elastic constraints.

However, in case of earthquake loadings it becomes important to estimate the dynamic forces correctly using the available model for the elastic design, and to take into account the inelastic behaviour of structures during the very strong ground motion.

Considering these problems, this paper presents a method for the automated minimum weight design of wide-flange steel frames which gives the optimum distribution of the moment of inertia of used members.

2. DYNAMIC ANALYSIS

An idealized dynamic model consist of bedrock, ground layres and a structure is considered (see Fig. 1). Ground excitations are given by the model presented by Kanai and Tajimi, and the dynamic response of the structure to this ground motion is estimated by means of the random vibration theory and Davenport's equation which gives the expected maximum value of a random process.

2.1 Vibration of Ground Surface

Kanai and Tajimi has presented the idea that spectrum observed at bedrock is characterized by a constant pattern (white noise), while the spectrum at the ground surface is amplified by the vibration property of the ground layre and showed a power spectrum of this ground surface as follows:[3]

$$S(p) = \sum_{k=1}^{r} \frac{1 + 4h_{gk}^{2}\left(\frac{p}{w_{gk}}\right)^{2}}{1 + (4h_{gk}^{2}-2)\left(\frac{p}{w_{gk}}\right)^{2} + \left(\frac{p}{w_{gk}}\right)^{4}} s_k S_o \qquad (1)$$

where h_{gk} and w_{gk} are ground damping factor and predominant frequency, respectively, S_o is a constant power spectrum density function and where s_k is a factor which measures predominence of each component. This excitation of ground surface becomes Gaussian process of zero mean.

2.2 Dynamic Response of Structure

The variance of elastic response of the structure subjected to the ground motion mentioned above can be obtained by means of random vibration theory. Let σ_Q and $\sigma_{\dot{Q}}$ be the variance of story shear force and its time derivative, respectively.

Following Davenport[4], the mean value of possible maximum elastic response of story shear force can be given as

$$Q = (2\ln vT)^{\frac{1}{2}} + \frac{0.5772}{(2\ln vT)^{\frac{1}{2}}} \qquad (2)$$

where $v = \frac{1}{2\pi}\frac{\sigma_{\dot{\theta}}}{\sigma_{\theta}}$

and T represents the duration of the strong earthquake excitation which is fixed 10 seconds in this paper.

For very strong ground motion, the response of the structure is considered to be inelastic, and the relative displacements of each floor are estimated following the idea of Newmark and et al.[5] Equating the inelastic potential energy of deformation to the elastic one which can be obtained supposing that the structure responses elastically, the maximum ductility factor of floor drift, μ, can be obtained as follows (see Fig. 2);

$$\dot{\mu} = \frac{1}{2} + \frac{1}{2}\left(\frac{Q}{Q_y}\right)^2 \qquad (3)$$

where Q_y may be thought of as the yield level of the story shear force, and can be obtained by means of a simple plastic analysis assuming the mechanism of beam collapse type or column collapse one for each story.

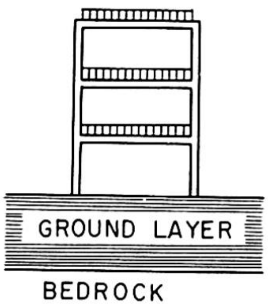

Fig. 1 Dynamic Model

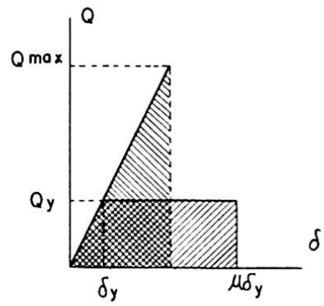

Fig. 2 Definition of Ductility Factor

3. DESIGN CONSTRAINTS

For the moderate earthquakes which give such a dynamic force as usually presented in the design code, the members of the frame are designed elastically in accordance with the design code of steel structure of Architectural Institute of Japan (A.I.J). On the other hand, for the very severe earthquake, which is rarely expected during their service lives, the frame is designed plastically relying on the energy absorption which due to their inelastic deformation. In this design procedure, the maximum ductility factor given by Eq.(3) is constrained less than the allowable value which is fixed 4 in this paper.

To satisfy these ductility requirements, it is necessary for the frame to prevent the weakening of the load-deflection curves caused by the lateral or local buckling of members and P - Δ effects.

These problems are taken into account according to the plastic design code of steel structure of A.I.J.. Namely, lateral buckling is prevented by the correctly designed stiffners, and local buckling is prevented by selecting the members which are on market to satisfy the width-thickness ratio of plate elements imposed by the code mentioned above, or designing each member in-accordance with these requirements after the optimum stiffness distribution of frame member is decided. Moreover to avoid the excessive P - Δ effects, the slenderness ratios and the axial compressive forces of columns are ristricted by the code requirements.

4. OPTIMIZATION

Wide-flange steel members on market are supposed to be mainly used in the design. The moment of inertia of them, I, are the design variables and objective function is the total weight of structural members. The emprical relationships between member properties which are required in the design code and moment of inertia of economical series of the steel wide-flange section was obtained by plotting them. The calculation was proceeded using these equations and treating the moment of inertia as continious design variable.

Sequential linear programming (S.L.P.) technique was successfully adopted for the optimization of the frames. Objective function and constraint equations were approximately linearlized, and using linear programming technique, the optimumly modefied design variables were obtained at each design step. Repeating this procedure, the optimum solusion, namely the distribution of moment of inertia of members which gives the minimum weight of structural members, was obtained.

5. SENSITIVITY ANALYSIS

To optimize the structure by means of S.L.P. technique, the change of member stress and deformation caused by the modefication of each members must be quantitively estimated as the first order derivative of these values with respect to the design variables.

Let P be the vector of external nodal forces of global coordinate, and X and K be the corresponding nodal displacement vector and stiffness matrix. Using these notations

$$X = K^{-1} P \tag{4}$$

Therefore, the derivative of nodal displacements with respect to design variable, I, is obtained as follows;

$$\frac{\partial X}{\partial I} = \frac{\partial}{\partial I} K^{-1} \cdot P + K \frac{\partial}{\partial I} P \tag{5}$$

The second term of the right hand side of the above equation contain the derivative of the dynamic loads which vanish in the static problems. If these values are obtained, the sensitivity coefficients of the stresses and deflections can be evaluated applying the same procedure adopted for the static problems.

As the dynamic loads which is evaluated by means of random vibration theory become the explicit function of natural frequencies and mode[6] vectors of structure, if the sensitivity coefficients of these values are evaluated, then that of these dynamic loads can be obtained without difficulty.

6. NUMERICAL EXAMPLE

The method previously mentioned is applied to the design of three-story frames of equal span length, 6m, and equal story height, 3m, with uniformly distributed load, w, on beams, subjected to the four types of ground motions whose characteristics are decided by the parameters presented in Table 1. Frames are designed both elastically for the power S_0 of E_q.(1), and plastically for the power of α times of S_0 so that the story drifts should be less than allowable ductility factor 4, and beam collapse type mechanism is

TYPE	T_{g1}	h_{g1}	β_1	T_{g2}	h_{g2}	β_2
I	0.3	0.6	1.0			
II	1.0	0.6	1.0			
III	0.1	0.3	0.2	1.0	0.3	0.8
IV	0.1	0.3	0.2	1.5	0.3	0.8

Table 1 Ground Parameter

considered for the calculation of yield levels of story shear forces. Steel used is SS41 whose yield stress is 2.4 ton/cm^2.

6.1 Three story one bay frames subjected to the ground motion of type I is optimized for W = 5 ton/m and S_o = 5 cm/rad/sec^3. In Fig. 3, the maximum stresses and the maximum ductility factors of each story corresponding to the final design are presented for α equal 5 and 7 respectively. Where the maximum stress is defined as the value in the most severely violated constraint equation for elastic design whose allowable limit is normalized as unity. For the case of α equals 5, the member size is desided by the elastic constraints and the response ductility factors of each story are scattering. On the other hand, for the case of α equals 7, the beams are not fully stressed for elastic design constraints and for the plastic design constraints they are equally fully constrained. Therefore it can be pointed out that for the optimum design of earthquake resistance structures, it become important to consider the constraints for the inelastic deflection expected during the very strong earthquakes.

6.2 Three story one bay frame subjected to the ground motion of type III and IV is optimized for w = 2 ton/m, S_o = 2 cm/rad/sec^3 and α = 7. The maximum stresses of each member defined previously and the maximum ductility factors for the final design are presented in Fig. 4. This shows that the optimum member size restricted by both elastic and plastic constraints.

The acceleration response spectrum to these ground motions is presented in Fig. 4 with the values of the spectrum correspond to the fundamental frequencies of the structure of initial and final design. This shows that even if the initial design is at the valley of the response spectrum, or final design is at the vicinity of the maximum, this optimization technique can be successfully adopted.

Neglecting the derivative of dynamic forces which is used in Eq. (5), the optimization is also carried out for the same model. The final result obtained starting from the same initial design mentioned above is presented in Fig. 6. Compared with the above analysis, much more iterative calculations are carried out and the real optimum solusion can not be obtained. This too happen for the optimization of the structure subjected to the ground motion which have more moderate response spectrum showing the importance of sensitivity analysis of dynamic forces for these analysis.

6.3 Three story one bay and three bays frames are optimized for α = 7 by changing the parameters concerend with the distributed load and ground motion. The ductility factors of story drift correspond to the final design are shown in Table 2 with these parameters. Each story yield almost equally fully restricted by the constraints of plastic deformation. Therefore it can be pointed out that for this kind of structures, the optimum design correspond to such a structure whose response ductility factors against very strong ground motion are almost equal for all story.

7. CONCLUSION

As a result of this study, following conclusions can be pointed out.

(1) The analysises of some examples shows the validity of the optimization technique mentioned above together with the importance of the sensitivity analysis of dynamic forces.

(2) The constraints concerened with the plastic deformation against the very strong ground motions must be considered together with the constraints for the elastic strength.

(3) For the type of structure dealt with in this paper, the minimum weight design correspond to such a structure whose response ductility factors against very strong ground motion are almost equal for all story.

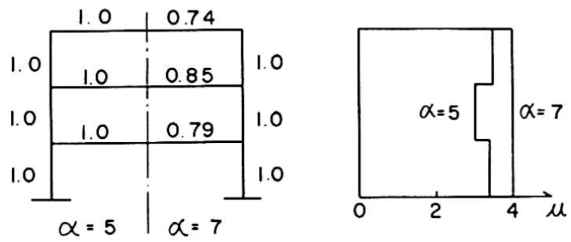

Fig.3 The Maximum Stress and Ductility Factor

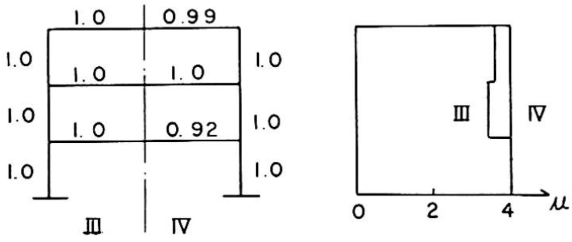

Fig.4 The Maximum Stress and Ductility Factor

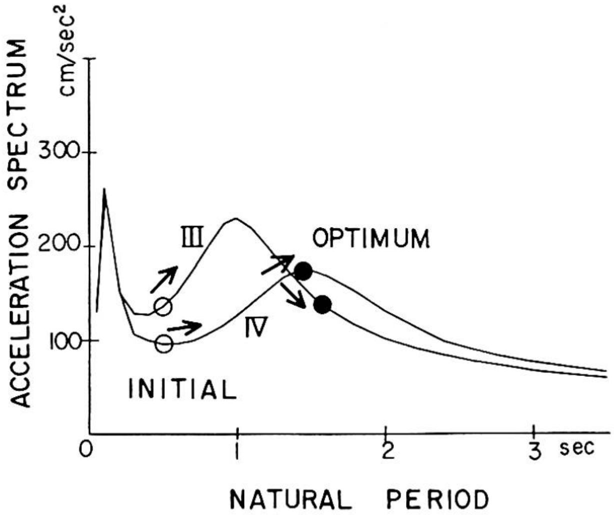

Fig.5 Optimization Process for Different Type Ground Motion

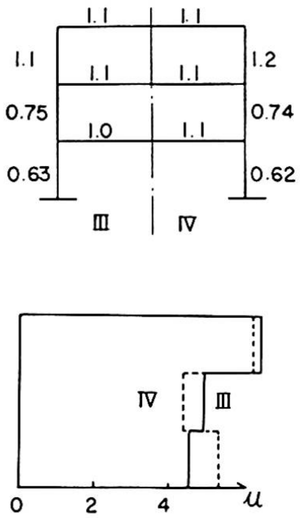

Fig.6 The Maximum Stress and Ductility Factor

SPAN	ω	TYPE	S_o	μ		
				1	2	3
1	10	I	5	4.0	4.0	4.0
1	50	I	5	4.08	4.08	4.05
1	50	II	5	4.04	4.04	4.04
1	5	III	2	4.0	4.0	4.0
1	5	IV	2	4.0	3.68	3.92
1	2	IV	2	4.0	4.0	4.0
1	50	IV	5	3.99	3.82	3.96
3	10	II	5	4.04	4.08	4.08
3	30	I	5	3.92	3.96	3.92
3	30	II	5	4.12	4.04	4.04

Table 2 The Maximum Ductility Factor

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SUMMARY

The minimum weight design of unbraced steel frames subjected to dynamic earthquake loads is presented. Random vibration theory is adopted to elastic member strength and plastic story deflection, the sequential linear programming technique is successfully adopted to obtain the optimum design. Several examples are presented with the analysis and comparisons are drawn.

RESUME

On présente le dimensionnement, pour un poids minimum, de cadres métalliques soumis aux forces dynamiques des tremblements de terre. La théorie des vibrations aléatoires permet de déterminer le comportement "dynamique" de la structure. La programmation linéaire séquentielle donne le dimensionnement optimal dans des conditions de comportement élastique des éléments et de comportement plastique du cadre soumis à la déflection.

ZUSAMMENFASSUNG

Für unausgesteifte Stahlrahmen, die durch Erdbebenwirkung beansprucht sind, wird die Berechnungsmethode des "minimalen Gewichts" abgeleitet. Die "Random"-Vibrationstheorie erlaubt es, das dynamische Verhalten des Tragwerks festzustellen. Unter Annahme "elastischer" Kräfte und plastischer Verformungen liefert die fortschreitende lineare Programmierung das gesuchte Optimum. Beispiele werden gezeigt und Vergleiche angestellt.

Optimization Techniques under Random Loading Effects

Techniques d'optimisation et effets des charges aléatoires

Optimierungstechnik bei Wirkung von Zufallsbelastungen

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1. INTRODUCTION

The developments that have taken place in the last few years in the field of optimization techniques applied to structural problems were restricted mainly to structures subjected to deterministic loadings. The reasons for the lack of research activities towards the analysis of structures under the effects of random loadings could be attributed to the mathematical complication involved in the procedure and the non-availability of sufficient and reliable data regarding the past histories of the random exciting force.

In this paper a simplified approach is reported to deal with the structural optimization problems under non-stationary loadings by making use of the upper bound probability of failure of the structure. The analysis is carried out in two phases:

(A) to obtain an expression for the probability that the response of the structure at a critical zone reaches for the first time an upper limit value with time-dependent control-barriers, interms of their rate of upcrossings; and

(B) to seek an approximate solution to the optimization problem, using the result obtained in phase (A), with the probability of failure, the natural frequency of vibration and the frequency response function of the system as restraints.

2. PHASE (A).

The estimation of the upper and lower bound probabilities of failure of a structure in a closed internal of time, has been a field of great interest among engineers dealing with random vibration problems. J.J Coleman[1] for the first time, suggested an approximate solution to estimate the upper bound value interms of the expected rates of the threshold crossings of the response process at positive and negative slopes. However, the process of independent arrivals of failure, as assumed by Coleman, is unacceptable especially for narrow band random process, such as the response of lightly damped dynamic systems. Besides, for low damped structural systems, crossings of response process tend to occur in 'clumps' of dependent crossings and hence the expected rate of threshold crossings should be replaced by the average clumping rate. M Shinozuka[2] has developed a method applicable to stationary and non-stationary cases as well, to estimate the upper and lower bounds for the probability of the first excursion failure within an arbitrary semi-closed time internal (o, t) and constant barriers without the assumption of independent threshold crossings. When the computed values of the upper and lower bounds are sufficiently close to each other, they are just as valuable as the mathematically exact values of the probability as a basis for making engineering decisions. In a paper[3] published later, Shinozuka has further extended his solution to take into account the effects of time-dependent barriers also.

The solution to the above problem with time-dependent barriers, presented in this paper is a modification to Shinozuka's approach with a different interpretation, interms of the expected rate of crossings of the response-barriers.

Following Shinozuka's expression for the upper - bound probability of failure of the structure,

$$P_r[t; -Y_2(t), Y_1(t)] < P_r[t; -Y_2(t), \infty] + P_r[t; -\infty, Y_1(t)] - P_r[\{x(t_1) < -Y_2(t_2)\}\{x(t_2) > Y_1(t)\}] \quad \text{----(1)}$$

where x(t) represents the response of the system at a critical zone and the failure of the system, for the first time, is defined as when $x(t) \geq Y_1(t)$, or $x(t) \leq -Y_2(t)$, in which $Y_1(t)$ and $Y_2(t)$ are positive barriers of response process.

Let $N[Y_1(t), t]$, hereafter referred as N_1, represents a random variable denoting the number of crossings of $Y_1(t)$ from below during the internal (0,t). The probability that $N[Y_1(t), t]$ takes a value 'r' during (0,t), $P_r[N_1 = r]$, can be expressed as:

$$P_r[N_1 = r] = P_r[N_1 = r; x(0) \geq Y_1(0)] + P_r[N_1 = r; x(0) < Y_1(0)] \quad \text{----(2)}$$

Also,

$$P_r[t; -\infty, Y_1(t)] = P_r[x(0) > Y_1(0), N_1 \geq 0] + P_r[x(0) < Y_1(0), N_1 \geq 1] + P_r[x(0) = Y_1(0), N_1 \geq 0] \quad \text{--------(3)}$$

Equation (3) can further be simplified as :

$$P_r[t; -\infty, Y_1(t)] = P_r[x(0) > Y_1(0)] + P_r[x(0) < Y_1(0)]\, P_r[N_1 \geq 1 \mid x(0) < Y_1(0)]$$
$$\leq P_r[x(0) > Y_1(0)] + P_r[x(0) < Y_1(0)] \sum_{s=1}^{\infty} s\, P_r[N_1 = s \mid x(0) < Y_1(0)] \quad \text{-(}$$

Equation (4) with the help of equation (2) finally reduces to,

$$P_r[t; -\infty, Y_1(t)] \leq P_r[x(0) > Y_1(0)] + E[N_1] - P_r[x(0) \geq Y_1(0)]\, E[N_1 \mid x(0) \geq Y_1(0)] \quad \text{---(5)}$$

in which E denotes the expected value.

If $N[-Y_2(t), t]$, hereafter referred as N_2, represents a random variable denoting the number of crossings of $-Y_2(t)$ from above during an internal $(0,t)$,

$$P_r[t;-Y_2(t),Y_1(t)] < P_r[x(0)<-Y_2(0)] + P_r[x(0)>Y_1(0)] + P_r[x(0)>-Y_2(0)]\,E[N_2|x(0)>-Y_2(0)] + P_r[x(0)<Y_1(0)]\,E[N_1|x(0)<Y_1(0)] - P_r[\{x(0)<-Y_2(0)\}\{x(t)>Y_1(t)\}] \quad \text{----(6)}$$

Equation (6) in effect represents the best upper bound probability of failure of the structure interms of the rate of crossings of the time-dependent barriers of response process.

In case the response process starts from zero origin, such that $P_r[x(0)=0]=1$, equation (6) further simplifies to :

$$P_r[t;-Y_2(t),Y_1(t)] < E[N_1]+E[N_2]-P_r[\{x(t_1)<Y_2(t_1)\}\{x(t_2)>Y_1(t_2)\}] \quad \text{..(7)}$$

The approach presented above, to estimate the upper bound value becomes significant in dealing with those problems where a stationary process for a finite time internal is observed, as in certain control system problems.

3. PHASE (B).

An approximate solution to the structural optimization problem is attempted in this phase, making use of the results obtained in phase (A), with the probability of failure of the structure and the system-characteristics as restraints.

Let $Z(d)$ be the objective function to be minimised subject to the condition,

$$P_r\left[\bigcup_{i=1}^{k}\{S_i(x(d,t)) \geqslant r_i\}\right]_j \leqslant [p_f]_j \quad \text{..........(8)}$$

and $$S_j(x(d,t)) \leqslant r_j \quad \text{..........(9)}$$

and $$\omega_{il} \leqslant \omega_i \leqslant \omega_{iu} \quad \text{..........(10)}$$

where $S_i(x(d,t))$ is the frequency response function of the system; $x(d,t)$ represents the response (stress, strain or displacement) at a critical zone to random excitation;

ω_{il}, ω_{iu} are the lower and upper limits of the natural frequency of vibration of the structure, respectively;

$[p_f]_j$ denotes the upper limit of the probability of failure under mode j.

Let $$P_r[S_i(x(d,t)) \geqslant r_j] = p_i(d) \quad \text{..........(11)}$$

For example, if the safety of the structure is analysed on the basis of the external load acting on it and its internal resistance, say F and R respectively, both treated as statistically independent normal distributions, then,

$$p(d) = \frac{1}{\sqrt{2\pi}}\int_{\rho}^{\infty} e^{-x^2/2}\,dx, \quad \text{..........(12)}$$

where $$\rho = \frac{\bar{R}-\bar{F}}{\sigma_R}\;\frac{1}{\sqrt{1+(\sigma_F/\sigma_R)^2}} \quad \text{----------(13)}$$

in which $\bar{R}$ and $\bar{F}$ are respectively the mean value of the resistance and the load; σ_R^2 and σ_F^2 are their variance.

Equation (8) now reduces to,

$$P_r \left[\bigcup_{i=1}^{k} \{ S_i(x(d,t)) \geq r_i \} \right]_j = \left[\sum_{i=1}^{k} p_i(d) \right]_j , \quad \ldots\ldots (14)$$

the limit of summation of the time variable being from $-\infty$ to ∞.

It follows,

$$\left[\sum_{i=1}^{k} p_i(d) \right]_j \leq [p_f]_j , \quad j = 1, 2, \ldots n . \quad \ldots\ldots (15)$$

In the case of non-stationary random excitations, for example, ground acceleration due to earthquakes, the left hand side of equation (15) may be replaced by the upper bound value of the probability of failure of the structure as obtained in phase (A).

4. CONCLUSIONS.

Since a knowledge of the rate of crossings of the time-dependent response-barriers is an essential pre-requisite to the present analysis, a rigorous statistical analysis of the past records of the random exciting force is warranted to achieve a high level of accuracy. A large class of optimization problems in control system engineering could be advantageously studied using this method.

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SUMMARY

A general solution to deal with structural optimization problems under non-stationary random loadings is presented, with the upper bound probability of failure of the structure within time-dependent barriers and the system characteristics as restraints.

RESUME

Une technique générale d'optimisation des structures est présentée pour le cas de charges aléatoires. Les caractéristiques du système et les valeurs supérieures de la probabilité de ruine en fonction du temps sont prises en considération.

ZUSAMMENFASSUNG

Es wird eine allgemeine Lösung der Bauoptimierungsprobleme für nicht stationäre Unfallsbelastungen dargestellt, mit der oberen Grenze der Versagenswahrscheinlichkeit innerhalb zeitabhängiger Grenzen und den Systemcharakteristiken als Einschränkungen.

Optimisation des structures: Critères prépondérants et méthode de prédimensionnement en structure métallique

Optimierung von Tragwerken: Entscheidende Kriterien und Verfahren zur Vorbemessung im Stahlbau

Structural Optimization: Prevailing Criteria and Proportioning Approach in Steel Structures

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1 - INTRODUCTION

L'idéal que cherchent à atteindre tous ceux qui sont associés à l'art de construire est de réaliser l'ouvrage qui donnera les meilleures garanties de service dans des conditions requises de sécurité et au meilleur prix.

L'optimisation envisagée ainsi n'est aujourd'hui pas accessible par des méthodes déductives. Elle demeure un art. Cependant, pour les démarches qu'il doit faire en vue de cette optimisation, l'ingénieur dispose de moyens de plus en plus élaborés. Les critères qu'il faudra respecter dans ces choix sont dans la pratique imposés par les autorités responsables de la sécurité, par les maîtres d'ouvrage et par les maîtres d'oeuvre. On les trouve exposés soit dans les textes règlementaires [1,2] , soit dans des cahiers des charges.

Par utilisation des techniques de programmation linéaire, le projeteur peut dans la pratique optimiser sa structure en poids, tout en satisfaisant un certain nombre de critères aux états limites ultimes. Un programme de dimensionnement optimal de structures à barres, visant ces objectifs, a été réalisé dans le cadre de travaux entrepris au CTICM et nous montrerons un certain nombre d'exemples qui mettent en lumière l'influence que peut avoir le respect des critères de vérification sur l'optimisation de la structure.

2 - RAPPEL DES DIFFERENTS CRITERES A SATISFAIRE AUX ETATS LIMITES ULTIMES

Un état limite ultime est atteint lorsqu'un des phénomènes suivants se produit :

a) perte d'équilibre de la structure
b) transformation de tout ou d'une partie de la structure en un mécanisme
c) instabilité de forme :
  - d'ensemble de la structure,
  - individuelle d'une barre
d) déformations excessives
e) cumul de déformations sous charges répétées
f) rupture d'un élément (fragilité ou par fatigue).

Un état limite d'utilisation est atteint lorsque la structure devient inapte aux fonctions normales pour lesquelles elle est conçue, en particulier lorsque les déformations excessives entraînent une interruption du service normal de la structure ou des désordres dans les éléments non structuraux.

Dans le cadre actuel français de la philosophie de la sécurité, pour vérifier la sécurité vis-à-vis des états limites, le projeteur multiplie les valeurs (caractéristiques ou nominales) des actions par des facteurs appelés coefficients de pondération. Les valeurs de ces coefficients dépendent de l'état limite considéré (état limite d'utilisation ou état limite ultime) du type d'action envisagé (actions permanentes ou variables) et de la combinaison d'actions étudiée (intervention simultanée d'actions variables).

Ainsi, pour la vérification à l'état limite ultime, on est conduit à considérer les plus défavorables des combinaisons d'actions données dans le tableau ci-dessous :

Tableau 1

SYMBOLES

- Majuscules

  Q = action permanente
  G = surcharge
  S = neige
  W = vent

- Indices

  e = extrême
  r = réduite

Cas de combinaisons d'actions		
prenant en compte une des trois actions Q, S, W.	$\frac{3}{2}Q + \frac{4}{3}G$ $\frac{3}{2}S + \frac{4}{3}G$ $\frac{3}{2}W + \frac{4}{3}G$	$\frac{3}{2}Q + G$ $\frac{3}{2}S + G$ $\frac{3}{2}W + G$
prenant en compte simultanément deux des trois actions Q, S, W.	$\frac{17}{12}(S_r+W) + \frac{4}{3}G$ $\frac{17}{12}(W+Q) + \frac{4}{3}G$ $\frac{17}{12}(Q+S) + \frac{4}{3}G$	$\frac{17}{12}(S_r+W) + G$ $\frac{17}{12}(W+Q) + G$ $\frac{17}{12}(Q+S) + G$
prenant en compte simultanément trois des actions Q, S, W.	$\frac{4}{3}(Q + S_r + W + G)$	$\frac{4}{3}(Q+S_r+W) + G$
prenant en compte les actions climatiques extrêmes	$Q + S_{re} + W_e + G$ $Q + S_e + G$	$Q + S_{re} + W_e + G$ $Q + S_e + G$

A l'état limite d'utilisation, la vérification doit être effectuée en considérant les combinaisons les plus défavorables des actions non pondérées.

Selon l'état limite considéré, la vérification consiste en particulier à contrôler si la structure satisfait aux critères de déformations, d'instabilité ou de résistance.

Il a paru utile, dans le cadre de cet article, de bien souligner les principes sur la manière de prendre en compte la sécurité dans l'optique des règlements actuels français. Car il est de l'opinion des auteurs que ces considérations sont de nature à avoir une influence très importante, non seulement sur la façon dont on entend poser le problème de l'optimisation, mais aussi sur la nature des résultats de cette optimisation.

L'étude et la mise au point d'un projet de construction passent toujours par trois phases essentielles, à savoir :

. le choix des dispositions générales de la construction,
. la détermination des dimensions de tous les éléments composants,
. la vérification que les dimensions adoptées sont acceptables et -en particulier- confèrent à la construction un degré de sécurité suffisant.

En ce qui concerne la première phase, on admet généralement que seul le choix des dispositions générales de l'ouvrage et de sa conception constitue oeuvre d'imagination créatrice, pour laquelle l'intuition et l'expérience de l'architecte et du constructeur jouent un rôle essentiel.

La question qui nous préoccupe dans le cadre de cet exposé est de savoir s'il existe des méthodes pratiques qui permettent de déterminer un choix préalable des sections ou composants d'une structure quelconque et qui, d'une part satisfont à l'ensemble des critères de vérification que nous venons de décrire brièvement et d'autre part, conduisent à une optimisation de poids de la structure.

3 - TECHNIQUES D'OPTIMISATION DES STRUCTURES

Le cadre réduit de cet article ne nous permet pas d'exposer les fondements de la méthode utilisée ni le détail de sa formulation en termes de programmation linéaire. Cette étude a fait l'objet de plusieurs publications [5,6,7] où l'on trouvera la formulation du problème de prédimensionnement optimal en termes de de programmation linéaire, avec le choix de la fonction objective (que l'on peut linéariser) et la prise en compte, d'une part de l'interaction effort normal-moment fléchissant et d'autre part du flambement.

L'approche utilisée par les auteurs se distinguent d'autres méthodes itératives [3,4] de type "heuristique", qui abordent le problème de la recherche d'un optimum au travers d'un processus complexe "d'itération-contrôle-modification" permettant de prendre en considération de nombreux critères de vérification (contrainte, stabilité, déformabilité) de la structure étudiée. Ces méthodes présentent, à défaut d'un manque de généralités et d'une incertitude sur l'optimum atteint, l'avantage d'avoir été pensées comme un programme module (PLADS-I PLASTIC ANALYSIS AND DESIGN SYSTEM, écrit dans un système général de langage orienté : ICES INTEGRATED CIVIL ENGINEERING SYSTEM). A ce titre, il a le mérite d'être immédiatement disponible et utilisable par l'ingénieur de bureau d'études.

4 - EXEMPLES D'APPLICATION

Le programme de prédimensionnement automatique des structures permet de prendre en compte la stabilité individuelle des barres et une combinaison quelconque d'états de charges pondérées. Il est cependant nécessaire, pour être en conformité avec les règlements de calcul [1,2] , de contrôler que la solution obtenue satisfait les critères aux états limites d'utilisation et de vérifier les conditions d'instabilité d'ensemble de la structure.

Nons donnons ci-après deux exemples qui démontrent que d'une part, la solution optimale recherchée dépend des critères d'états limites adoptés, selon que le dimensionnement se réfère à un règlement de calcul en élasticité [1] ou en plasticité [2], d'autre part le prédimensionnement est d'autant plus proche de la solution finale optimale que l'on considère ou non les conditions d'instabilité individuelle.

Exemple 1 : A titre d'exemple, nous donnons les résultats obtenus sur la structure donnée à la figure 1a. Les schémas 1b et 1c donnent la valeur de deux combinaisons de charges les plus défavorables pour la structure considérée, à savoir charges permanentes + neige et charges permanentes + neige + vent.

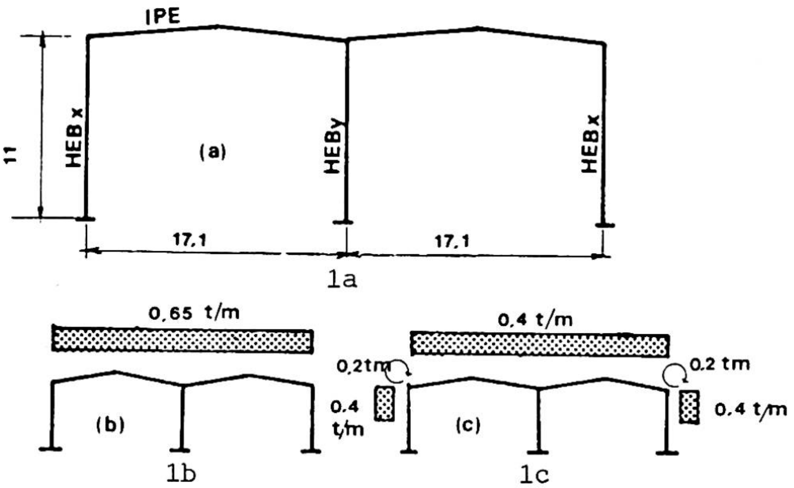

Fig. 1

Les résultats sont résumés dans le tableau de la page suivante.

Tableau 2

		Poids (tonnes)	$\frac{\Delta H}{H}(\frac{1}{150})$	$\frac{\Delta v}{L}(\frac{1}{200})$	Nbre plastification état limite utilisation
Elas. [1]	poteaux HEB 200 traverse IPE 360	3,98	$\frac{1}{199,5}$	$\frac{1}{350}$	0
Plas. [2]	poteaux HEB 200 traverse IPE 300	3,47	$\frac{1}{174}$	$\frac{1}{210}$	2
Pred.	poteaux HEB 180 traverse IPE 360	3,64	$\frac{1}{144}$	$\frac{1}{332}$	

Dans cet exemple particulier pour lequel les conditions d'instabilité au flambement sont vérifiées, l'optimisation est différente selon qu'elle est élastique ou plastique. Dans les deux cas elle satisfait aux conditions de déformabilité aux états limites d'utilisation ; par contre, la présence de 2 rotules plastiques aux états limites d'utilisation n'est pas acceptée en élasticité. Le gain de poids est ici de 12,8%.

Le prédimensionnement initial donnait une solution proche de la solution élastique, mais la condition de déformabilité en tête du poteau n'était pas vérifiée, quoique la condition de flambement du poteau était satisfaisante.

Exemple 2 : Soit la structure donnée en figure 2, avec le cas de charges pondérées considéré. Les résultats du prédimensionnement sont rassemblés dans le tableau 3.

Fig. 2

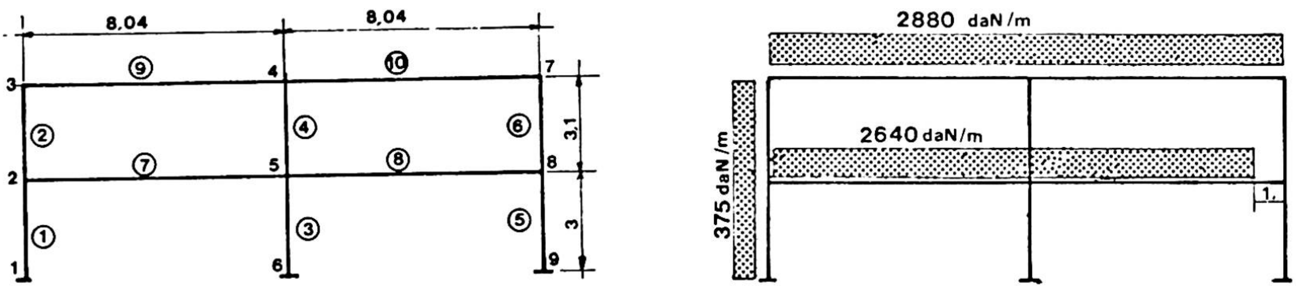

Tableau 3

	Poteaux 1,2,5,6	Poutres 3,4	Poutres 7,8,9,10
sans interaction M, N, ni flambement	IPE 300	HEB 100	IPE 400
avec interaction M, N et flambement	IPE 360	HEB 160	IPE 400
solution opt. selon Elas [1]	IPE 360	HEB 200	IPE 400

L'examen des résultats de ce tableau par le programme de prédimensionnement automatique des structures montre que si le dimensionnement sans interaction M et N est acceptable pour les poutres (c'est-à-dire lorsque la sollicitation de flexion est prépondérante), il n'en est pas de même pour les poteaux où l'effort axial est prépondérant. Il est nécessaire alors d'introduire dans le prédimensionnement les conditions d'interaction entre l'effort normal et le moment fléchissant et les conditions d'instabilité (voir 2ème ligne du tableau 3). L'introduction de ces conditions amène généralement une redistribution des efforts entre les sections et peut conduire aussi à une augmentation des sections simplement fléchies (barres). La 3ème ligne du tableau 3 donne la solution finale compatible avec les exigences d'un règlement élastique [1].

5 - CONCLUSIONS

La méthode mise au point dans le cadre d'études entreprises au CTICM trouve son fondement dans l'application du théorème statique en plasticité et les techniques de programmation linéaire. Elle conduit d'une manière pratique à un prédimensionnement initial correct, à condition toutefois de prendre en considération les conditions d'interaction entre sollicitation de flexion et effort axial et les conditions de stabilité individuelle au flambement des barres.

Il y a lieu cependant de procéder à une vérification de ce prédimensionnement initial, pour contrôler si la structure satisfait aux diverses exigences imposées par les codes de calcul aux états limites d'utilisation.

La fonction à optimiser est le coût total de la structure, c'est-à-dire la somme des coûts des aciers, de la fabrication, du montage et de l'entretien. Une étude factorielle de l'influence de ces divers coûts dans l'établissement d'une fonction économique a été étudiée [8]. Si cette étude a montré qu'il était possible d'améliorer sensiblement la fonction économique, la qualité du dimensionnement n'est cependant pas accrue dans les mêmes proportions. En particulier, du fait de nombreuses hypothèses au niveau de la prise en compte dans le prédimensionnement de l'instabilité individuelle des barres, le gain de précision du à l'amélioration de la fonction économique est illusoire.

Le programme de prédimensionnement automatique des structures est valable quelle que soit la configuration géométrique de la structure et la nature des charges extérieures appliquées. Cependant, le nombre de sections potentiellement critiques choisies et celui des contraintes résultant des conditions de plastification, d'interaction M et N et d'instabilité de flambement des barres comprimées et fléchies, en limitent l'application pratique à des structures relativement simples (portiques simples, portiques accolés, cadres multi-étagés de 2 niveaux, 3 baies).

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RESUME

L'article expose brièvement l'état des conditions à satisfaire dans le cadre d'une philosophie règlementaire aux états limites. Il est actuellement possible de tenir compte des conditions d'interaction effort normal-moment fléchissant et des conditions de flambement dans l'optimisation des structures à barres. Deux exemples montrent qu'il est important de prendre en considération ces critères si l'on veut aboutir à un prédimensionnement valable.

ZUSAMMENFASSUNG

Der Artikel weist kurz auf die Bedingungen hin, die im Rahmen einer vertretbaren Philosophie der Grenzzustände erfüllt sein müssen. Es ist heutzutage möglich, in der Optimierung von Stabtragwerken der gegenseitigen Wirkung zwischen Normalkraft und Biegungsmoment und dem Knicken Rechnung zu tragen. Zwei Beispiele zeigen, dass es wichtig ist, solche Kriterien in Betracht zu ziehen, wenn eine günstige Vorbemessung erreicht werden soll.

SUMMARY

The paper states briefly the conditions to be satisfied within the framework of an ultimate state design philosophy. It is presently possible to improve the optimization of structures by taking into account interaction between normal force and bending moment and buckling conditions. Two examples show that it is important to consider such criteria, if we want to achieve a proper members selection.

Minimum Weight Plastic Design of Multi-story Plane Frames for Five Sets of Design Loads

Calcul plastique, pour un poids minimum, de cadres plans à plusieurs étages, avec cinq groupes de cas de charge

Plastische Bemessung auf Minimalgewicht von mehrstöckigen ebenen Rahmen für fünf Belastungszustände

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1. Introduction Several classes of general solutions to the problem of minimum weight plastic design of multi-story multi-span plane frames subjected to a class of *one* set of practical design loads have been derived by the senior author [1] by applying Foulkes' theory [2] and by extending it to a more general theory [3] which incorporates the axial force-bending moment interaction yield conditions. The present authors have further extended the result of [1] so as to incorporate the reaction constraints in [4]. These analytical general solutions are of theoretical and practical interests. Firstly, they serve to clarify even partially the general features of the minimum weight designs. Secondly, once an analytical method is developed for simpler problems based upon the moment yield condition [1], their general solutions would provide a good lead to the general solutions to more complex problems based upon interaction yield conditions [3]. Thirdly, they will provide good initial feasible solutions for neighborhood problems.

In this paper, *a kinematical restricted maximization procedure* is developed by combining the *primal-dual* method of LP [5] with a semi-inverse approach similar to the idea of [1] and then applied to the problem of minimum weight plastic design of multi-story multi-span plane frames subjected to five sets of design loads.

2. Formulation of the Design Problem Fig.1 shows a multi-story multi-span plane frame to be designed by Foulkes' theory [2] and the five sets of design

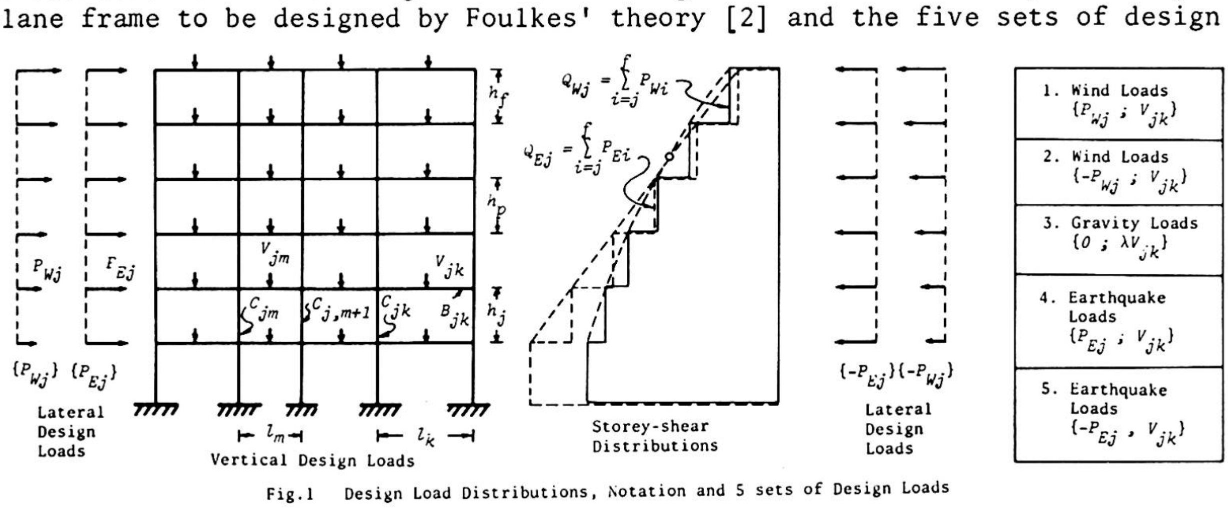


Fig.1 Design Load Distributions, Notation and 5 sets of Design Loads

loads. The fully-plastic moments of (j,k)-beam and (j,k)-column are denoted by $B_{j,k}$ and $C_{j,k}$, respectively. Without loss of practical generality, it may be assumed that the story-shear force distributions defined in Fig.1 be such that

$$\begin{aligned} &Q_{Ej} \geqq Q_{Wj} \quad for \quad j=1,2,\ldots,p, \text{ and} \\ &Q_{Ej} \leqq Q_{Wj} \quad for \quad j=p+1,\ldots,f. \end{aligned} \tag{1}$$

The factor λ for the design gravity loads is assumed to be $\lambda \leqq 2.0$.

The design problem for five sets of design loads is treated in the following three or four steps:

(i) Solve the basic problem for the two sets of co-directional lateral design loads 1 and 4, i.e., for $\{P_{Wj};V_{j,k}\}$ and $\{P_{Ej};V_{j,k}\}$,
(ii) Construct a statically admissible bending moment field for the two sets of design loads 2 and 5, i.e., for $\{-P_{Wj};V_{j,k}\}$ and $\{-P_{Ej};V_{j,k}\}$,
(iii) Construct a statically admissible bending moment field for the design gravity loads 3, i.e., for $\{0;\lambda V_{j,k}\}$,
(iv) If the step (ii) or (iii) is not possible, modify the collapse mechanism locally and find the corresponding modified design.

The basic problem (i) may be stated in terms of the static variables defined in Fig.2(a) as follows:

$$Minimize \quad G = g\{\sum_{k=1}^{s} l_k \sum_{j=1}^{f} B_{jk} + \sum_{j=1}^{f} h_j \sum_{k=1}^{s+1} C_{jk}\}, \quad (g:\ constant) \tag{2}$$

$$\left.\begin{aligned} subject\ to: &\sum_{k=1}^{s+1}(c^B_{Ijk} + c^T_{Ijk}) = h_j Q_{Ij}\ , \quad b^L_{Ijk} + b^R_{Ij,k-1} = c^B_{Ij+1,k} + c^T_{Ijk}\ , \\ &\tfrac{1}{2}(b^L_{Ijk} - b^R_{Ijk}) + \tfrac{1}{4} l_k V_{jk} \leqq B_{jk} \\ &-B_{jk} \leqq b^L_{Ijk} \leqq B_{jk}\ , \ -B_{jk} \leqq b^R_{Ijk} \leqq B_{jk}\ , \ B_{jk} \geqq 0, \\ &-C_{jk} \leqq c^B_{Ijk} \leqq C_{jk}\ , \ -C_{jk} \leqq c^T_{Ijk} \leqq C_{jk}\ , \ C_{jk} \geqq 0, \end{aligned}\right\} \tag{3a-i}$$

where b^L_{Ijk}, b^R_{Ijk}, c^B_{Ijk} and c^T_{Ijk} are free variables. In the expression (2), f and s denote the numbers of stories and spans, respectively. In the constraints (3), the first subscript I denotes the kind of design loads and is to be either E or W. The second and third subscripts refer to the story number from below and member number from left, respectively. For the sake of brevity, the equations of moment equilibrium about interior and exterior joints have been written in one and the same form with the convention that all the undefined quantities with respect to non-existent members shall be disregarded and dropped as null. This convention will also be used hereafter, unless otherwise stated.

3. Kinematical Restricted Maximization Procedure-Semi-Inverse Primal-dual Method.

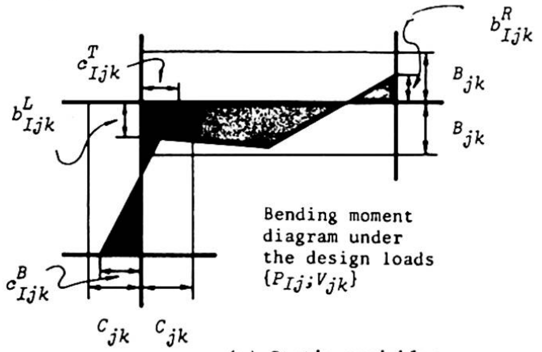
The idea of the proposed approach may be summarized by referring to Fig.3 as follows. A design problem formulated as a linear programming problem [6] of a *mixed* type [5], may often be such that a certain set of constraints may be anticipated to be *inactive* due to the nature of the problem. From the original primal problem

$[PO]$: $Minimize\ \{G(x) \mid x \in S_1 \cap S_2\}$

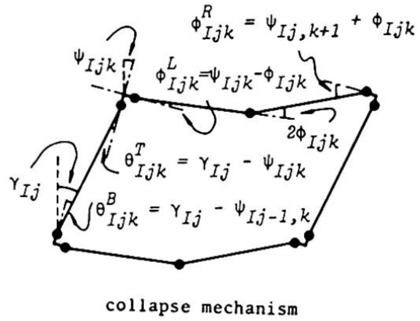
of a large size, a subproblem

$[PS]$: $Minimize\ \{G(x) \mid x \in S_1\}$

may be derived by tentatively disregarding a certain set of constraints which are anticipated to be inactive and which define the set S_2. Then the dual problem to



(a) *Static variables*



(b) *Kinematic variables*

Fig.2 Definitions of the variables

[*PS*], i.e.

[*DS*]: *Maximize* $\{D(\mathrm{u}) \mid \mathrm{u} \in V\}$

must involve a smaller number of dual variables and a greater number of equality constraints. Therefore, if the solution u^o to [*DS*] can be found more easily compared to the solution to the dual problem of [*PO*], then the corresponding solution x^o to [*PS*] may also be readily found simply by solving the set of simultaneous linear equations derived from the duality theorem of *LP*. It remains then to check if $\mathrm{x}^o \in S_2$. The procedure may also be called "a semi-inverse primal-dual method."

4. A Class of General Solutions to the Problem (i) It is now shown that the kinematical maximization procedure is fruitful for rectangular frames due to their regularity in the optimality criteria-based collapse mechanism. Let

$$S_2:\ (b^L_{Ijk},\ b^R_{Ijk}) \geq -B_{jk},\ (c^B_{Ijk},\ c^T_{Ijk}) \geq -C_{jk},\ B_{jk} \geq 0,\ C_{jk} \geq 0, \qquad \text{(4a-d)}$$

Then the dual problem [*DS*] may be written in terms of the kinematic variables defined in Fig.2(b) as follows:

$$\textit{Maximize}\quad D=\Theta\{\sum_{j=1}^{f} h_j(Q_{Wj}\gamma_{Wj}+Q_{Ej}\gamma_{Ej})+\sum_{k=1}^{s}\sum_{j=1}^{f}\tfrac{1}{2}l_k V_{jk}(\phi_{Wjk}+\phi_{Ejk})\} \qquad (5)$$

$$\textit{subject to}\quad \gamma_{Ij} \geq \underset{k}{Max.}\{\psi_{Ijk},\ \psi_{Ij-1,k}\}$$

$$\psi_{Ijk} \geq \phi_{Ijk} \geq 0 \quad (k=1,2,\ldots,s), \qquad \psi_{Ij,s+1} \geq -\phi_{Ijs}$$

$$\{(\psi_{Wjk}+\psi_{Wj,k+1}+2\phi_{Wjk})+(\psi_{Ejk}+\psi_{Ej,k+1}+2\phi_{Ejk})\} = \Theta l_k \qquad \text{(6a-d)}$$

$$\{(2\gamma_{Wj}-\psi_{Wj-1,k}-\psi_{Wjk})+(2\gamma_{Ej}-\psi_{Ej-1,k}-\psi_{Ejk})\} = \Theta h_j$$

The inequalities (6a, b) restrict the directions of plastic hinge rotations and the equalities (6c, d) are the generalized Foulkes conditions defined by Chan [6] and Prager [7]. The latter will be referred to as *FCP* conditions.

The equations (6d) indicate that $\psi_{Wjk}+\psi_{Ejk} \equiv \psi_j$ (independent of k). The problem defined by (5) and (6) may then be simplified to a problem in terms of ψ_{Wjk}, γ_{Wj}, ϕ_{Wjk} and ψ_j only. After some manipulation on the inequalities, γ_{Wj} may be expressed in terms of ψ_{Wjk} and ψ_j only, and then ψ_{Wjk}, in terms of ψ_{Wpk} and ψ_j only. Finally, for those problems in which the load conditions:

$$h_jQ_{Ij}+h_{j+1}Q_{I,j+1} \geq \sum_{k=1}^{s} l_kV_{jk}, \quad \begin{pmatrix} 1 \leq j \leq p-1 & \text{for } I=E, \\ p+1 \leq j \leq f & \text{for } I=W, \end{pmatrix}$$

$$h_pQ_{Ep}+2h_{p+1}Q_{Ep+1}-h_{p+1}Q_{Wp+1} \geq \sum_{k=1}^{s} l_kV_{pk}, \qquad \text{(7a,b)}$$

and the geometrical conditions:

$$l_m \leq l_k \leq 2l_m,\ l_m \leq 2h_1\ (l_m = \underset{k}{Min.}\{l_k\}), \qquad \text{(8a,b)}$$

are satisfied, the problem [*DS*] may be reduced to the following form:

$$\textit{Maximize}\quad D^*=\Theta(-\Delta M_p\xi + \Delta M_{p+1}\eta), \qquad (9)$$

$$\textit{subject to}\quad \xi \equiv \underset{k}{Max.}\{\psi_{Wpk}\},\quad \eta \equiv \underset{k}{Min.}\{\psi_{Wpk}\},\quad (\xi \geq \eta)$$

$$\xi \leq \tfrac{1}{2}h_{p+1}\Theta + Min.\{0,\eta\},\quad Max.\{\tfrac{1}{2}l_m\Theta,\xi\} \leq \tfrac{1}{2}h_{p+1}\Theta+\eta, \qquad \text{(10a-d)}$$

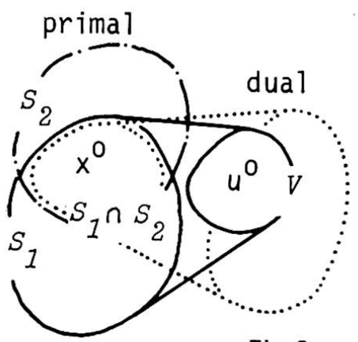


Fig. 3

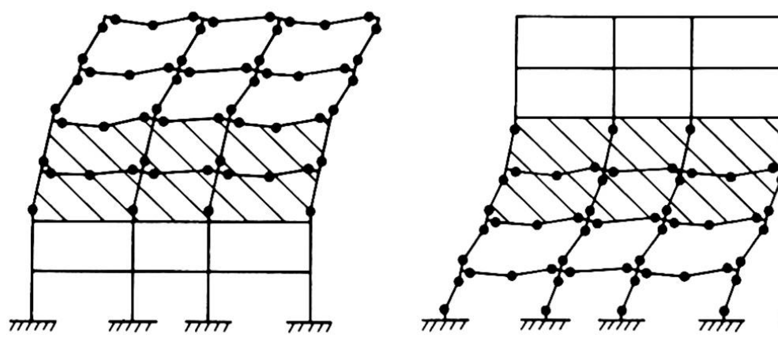


Fig. 4

$$0 \leqq \psi_{Wpk} \leqq \tfrac{1}{2}l_m\Theta,\ (k=1,2,\ldots,\ s),\ a \leqq \psi_{Wp,s+1} \leqq b,$$
$$a \equiv Max.\{-(l_s-l_m)\Theta/2,\ -\Psi_{Wps}\} \leqq 0$$
$$b \equiv Min.\{l_s\Theta/2,\ l_m\Theta-\psi_{Wps}\} \geqq l_m\Theta/2 \qquad \text{(10e-h)}$$

where $\Delta M_p \equiv h_p(Q_{Ep}-Q_{Wp}) \geqq 0$ and $\Delta M_{p+1} \equiv h_{p+1}(Q_{Wp+1}-Q_{Ep+1}) \geqq 0$. The solution to this reduced problem may readily be derived as summarized in Table 1. In those problems where (7) and (8) are satisfied, the generalized Foulkes mechanism defined by the *FCP* conditions can thus be constructed as shown in Fig.4 for Case (B) as an example.

The solution to the problem [*PS*] corresponding to this problem[*DS*] may also be derived straightforwardly. By assuming that some statical restrictions defined and checked later will be satisfied, the resulting bending moment diagram may be understood best by conceiving it as the result of superposition of the constituent elementary moment diagrams (with equal corner values for $k\neq m$) shown in Fig.5. Such a decomposition was first introduced in [1]. Each diagram is referred to as "frame moment diagram." The minimum weight plastic design corresponding to Table 1 may be compactly summarized as Table 2 in terms of "Maximum Story-Shear Force Design" defined by

$$B^*_{jk} \equiv Max.\{B^W_{jk},\ B^E_{jk}\},\qquad C^*_{jk} \equiv Max.\{C^W_{jk},\ C^E_{jk}\} \qquad \text{(11a,b)}$$

where $\{B^W_{jk},\ C^W_{jk}\}$ and $\{B^E_{jk},\ C^E_{jk}\}$ denote the designs only for $\{P_{Wj};\ V_{jk}\}$ and $\{P_{Ej};\ V_{jk}\}$, respectively, derived by means of [1]. B^*_{jk} and C^*_{jk} are given by

$$B^*_{jk} = \tfrac{1}{4}l_kV_{jk}\ (k\neq m);\quad C^*_{jk} = \tfrac{1}{4}(l_{k-1}V'_{j,k-1}+l_kV'_{jk});\ (k\neq m,\ m+1)$$
$$B^*_{jk} = \tfrac{1}{4}(h_jQ_{Ij}+h_{j+1}Q_{I,j+1}-\sum_{k\neq m} l_kV_{jk}),\ \begin{pmatrix}1 \leqq j \leqq p-1 & \text{for}\ I=E \\ p+1 \leqq j \leqq f & \text{for}\ I=W\ \text{and}\end{pmatrix}$$
$$j=p,\ I=E\ \text{for}\ \Delta M_p \geqq \Delta M_{p+1},\ \text{and}\ j=p,\ I=W\ \text{for}\ \Delta M_p \leqq \Delta M_{p+1};$$
$$C^*_{jn} = \tfrac{1}{4}(h_jQ_{Ij}-\sum_{k\neq n-1,n} l_kV'_{jk}),\ \begin{pmatrix}1 \leqq j \leqq p\ \ \text{for}\ I=E, \\ p+1 \leqq j \leqq f\ \ \text{for}\ I=W, \\ n=m,\ m+1\end{pmatrix}$$
$$V'_{jk} \equiv \sum_{i=j}^{f} (-1)^{i-j}V_{ik}\ . \qquad \text{(12a-e)}$$

The yield inequalities in (3) provide restrictions on the design loads in accordance with the classification of the solutions listed in Table 2. These

Table 1 Generalized Foulkes Mechanism

<table>
<tr><th colspan="2"></th><th>γ_{Wj}</th><th>γ_{Ej}</th><th>ψ_{Wjk}</th><th>ψ_{Ejk}</th><th>$\phi_{Wjk}(k\neq m)$</th><th>$\phi_{Ejk}(k\neq m)$</th><th>$\phi_{Wjm},\ \phi_{Ejm}$</th></tr>
<tr><td colspan="2">$j=1$</td><td rowspan="2">0</td><td>$\frac{1}{2}(h_1+\frac{1}{2}l_m)\Theta$</td><td rowspan="2">$0$</td><td rowspan="2">$\frac{1}{2}l_m\Theta$</td><td rowspan="2">$0$</td><td rowspan="2">$\frac{1}{2}(l_k-l_m)\Theta$</td><td rowspan="11">$0$</td></tr>
<tr><td colspan="2">$j=2,\ \ldots,\ p-1$</td><td>$\frac{1}{2}(h_j+l_m)\Theta$</td></tr>
<tr><td rowspan="4">$j=p$</td><td>(A)</td><td>0</td><td>$\frac{1}{2}(h_p+l_m)\Theta$</td><td>$0$</td><td>$\frac{1}{2}l_m\Theta$</td><td>$0$</td><td>$\frac{1}{2}(l_k-l_m)\Theta$</td></tr>
<tr><td>(B)</td><td>$\frac{1}{2}(l_m-h_{p+1})\Theta$</td><td>$\frac{1}{2}(h_p+h_{p+1})\Theta$</td><td>$\frac{1}{2}(l_m-h_{p+1})\Theta$</td><td>$\frac{1}{2}h_{p+1}\Theta$</td><td>*</td><td>*</td></tr>
<tr><td>(C)</td><td>$\frac{1}{2}l_m\Theta$</td><td>$\frac{1}{2}h_p\Theta$</td><td>$\frac{1}{2}l_m\Theta$</td><td>0</td><td>$\frac{1}{2}(l_k-l_m)\Theta$</td><td>$0$</td></tr>
<tr><td>(D)</td><td>$\frac{1}{2}h_p\Theta$</td><td>$\frac{1}{2}l_m\Theta$</td><td>$\frac{1}{2}h_p\Theta$</td><td>$\frac{1}{2}(l_m-h_p)\Theta$</td><td>*</td><td>*</td></tr>
<tr><td rowspan="4">$j=p+1$</td><td>(A)</td><td>$\frac{1}{2}h_{p+1}\Theta$</td><td>$\frac{1}{2}l_m\Theta$</td><td rowspan="5">$\frac{1}{2}l_m\Theta$</td><td rowspan="5">0</td><td rowspan="5">$\frac{1}{2}(l_k-l_m)\Theta$</td><td rowspan="5">$0$</td></tr>
<tr><td>(B)</td><td>$\frac{1}{2}l_m\Theta$</td><td>$\frac{1}{2}h_{p+1}\Theta$</td></tr>
<tr><td>(C)</td><td>$\frac{1}{2}(h_{p+1}+l_m)\Theta$</td><td>$0$</td></tr>
<tr><td>(D)</td><td>$\frac{1}{2}(h_p+h_{p+1})\Theta$</td><td>$\frac{1}{2}(l_m-h_p)\Theta$</td></tr>
<tr><td colspan="2">$j=p+2,\ \ldots,\ f$</td><td>$\frac{1}{2}(h_j+l_m)\Theta$</td><td>$0$</td></tr>
</table>

restrictions may be summarized as shown in Table 3, where

$$M_{Ij} \equiv \frac{1}{4}(h_j Q_{Ij} + h_{j+1} Q_{I,j+1} - \sum_{k=1}^{s} l_k V_{jk}) \tag{13}$$

It may now be concluded that the present solutions (A~D) are the rigorous solutions to the problems in which all the geometrical and loading conditions are satisfied.

5. Design for Five Sets of Design Loads.

It may readily be confirmed that a statically admissible bending moment field for $\{-P_{Ej};\ V_{jk}\}$ and $\{-P_{Wj};\ V_{jk}\}$ can be constructed just by inverting the frame moment diagrams as shown in Fig.6.

For design gravity loads, it is convenient to consider again the decomposed moment diagram with the respectively equal corner values $\lambda l_k V'_{jk}/8$, as shown in Fig.7. The conditions that the bending moment diagram given by superposing the elementary diagrams in Fig.7 be statically admissible in a frame designed by the procedure in Section 4, lead again to further restrictions on the design gravity loads. An examination of these restrictions indicates that there are a number of practically useful design solutions within the range defined by them.

6. Concluding Remarks

It may now be concluded that, for the class of design problems in which all the previous and supplementary conditions are satisfied, the solutions (A~D) are the rigorous minimum weight plastic designs. The present designs have apparently clarified the nature of minimum weight plastic designs. While these designs must be modified for practical use so as to satisfy a number of structural requirements, the present solutions will at least provide a basis

Table 2

Vertical axis: $\Delta M_p - \Delta M_{p+1}$ (from $-\Delta M_{p-1}$ to ΔM_{p+2}); horizontal axis: l_m (marks h_p, h_{p+1}, $Min\{h_p+h_{p+1},\ 2h_1\}$)

(A) $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_{p+1}$; otherwise $B_{jk}=B^*_{jk}$, $C_{jk}=C^*_{jk}$	(B) $C_{p+1,n}=C^*_{p+1,n}+\frac{1}{4}(\Delta M_p-\Delta M_{p+1})$; $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_p$; $B_{p+1,m}=B^*_{p+1,m}-\frac{1}{4}(\Delta M_p-\Delta M_{p+1})$; otherwise $B_{jk}=B^*_{jk}$, $C_{jk}=C^*_{jk}$
(C) $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_p$; otherwise $B_{jk}=B^*_{jk}$, $C_{jk}=C^*_{jk}$	(D) $C_{pn}=C^*_{pn}+\frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$; $B_{pm}=B^*_{pm}-\frac{1}{4}\Delta M_{p+1}$; $B_{p-1,m}=B^*_{p-1,m}-\frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$; otherwise $B_{jk}=B^*_{jk}$, $C_{jk}=C^*_{jk}$

	(A)	(B)	(C)	(D)
$j=1,2,\ldots,p-2$	$M_{Ej}\geq 0$			
$j=p-1$	$M_{Ej}\geq 0$		$M_{Ep-1}\geq\frac{1}{4}(\Delta M_{p+1}-\Delta M_p)$	
$j=p$	$M_{Ep}\geq\frac{1}{4}\Delta M_{p+1}$	$M_{Ep}\geq\frac{1}{4}\Delta M_p$	$M_{Wp}\geq\frac{1}{4}\Delta M_p$	$M_{Wp}\geq\frac{1}{4}\Delta M_{p+1}$
$j=p+1$		$M_{Wp+1}\geq\frac{1}{4}(\Delta M_p-\Delta M_{p+1})$		
$j=p+2,\ldots,f$	$M_{Wj}\geq 0$			
$k\neq m,\ m+1$; $j=1,2,\ldots,f$	$l_{k-1}V'_{j,k-1}+l_k V'_{jk}\geq 0$			
$n=m,\ m+1$; $j=1,\ldots,p-1$	$h_j Q_{Ej}\geq\sum_{k\neq n,n-1} l_k V'_{jk}$			
$n=m,\ m+1$; $j=p$	$h_p Q_{Ep}+\frac{1}{4}(\Delta M_p-\Delta M_{p+1})\geq\sum_{k\neq n,n-1} l_k V'_{pk}$			
$n=m,\ m+1$; $j=p+1$	$h_j Q_{Wj}\geq\sum_{k\neq n,n-1} l_k V'_{jk}$			$h_{p+1}Q_{Wp+1}+\frac{1}{4}(\Delta M_{p+1}-\Delta M_p)\geq\sum_{k\neq n,n-1} l_k V'_{p+1,k}$
$n=m,\ m+1$; $j=p+2,\ldots,f$	$h_j Q_{Wj}\geq\sum_{k\neq n,n-1} l_k V'_{jk}$			

Table 3

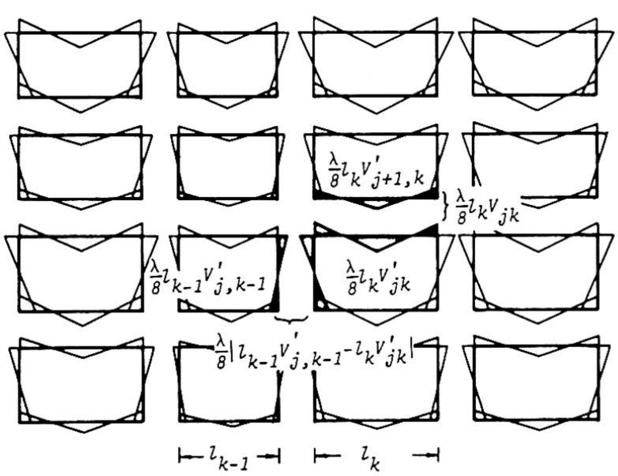


Fig. 7 Decomposed Moment Diagram for Gravity Loads

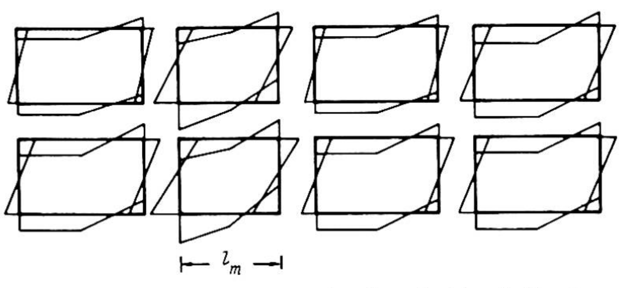


Fig. 5 Frame moments under the wind load (P_{Wj})

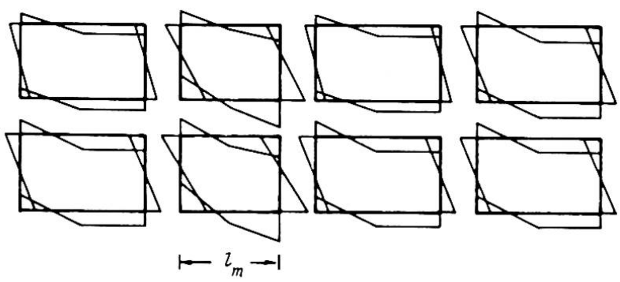


Fig. 6 Frame moments under the wind load ($-P_{Wj}$)

for initial designs useful in such countries where fairly large lateral design loads must be assigned for PLASTIC DESIGN so that frames can withstand against strong winds and strong motion earthquakes. The present solutions may be said to be a class of the most fundamental designs in the sense that a number of useful designs to practical neighborhood problems can be derived by appropriate but mostly local modifications. Three cases:
(a) $h_f P_{Wf} \leq \sum_{k=1}^{g} l_k V_{fk}$, (b) $l_m \geqq 2h_1$ and (c) $\exists l_k \geqq 2l_m$ have been treated in [8].

The present solution and the solutions in [1, 3, 4] indicate that a frame designed by these solutions would collapse in an extremely deteriorated overcomplete mechanism under a designated set of design loads according to the rigid-plastic analysis. It is therefore necessary to confirm the safety of such a frame against possible collapse due to inelastic instability according to a more refined theory of large-deflection elastic-plastic analysis. For this purpose, static and dynamic large-displacement analyses have been carried out on minimum weight frames in [9~11] under alternating lateral loads well beyond their static stability limits and under strong motion earthquake disturbances, respectively.

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[11]Osamu Ohta, Tsuneyoshi Nakamura & Shuzo Ishida,"Collapse Behavior and Imperfection Sensitivity of Minimum Weight Plastic Frames," *Summaries of Technical Papers at 1974 Annual Meeting of Architectural Inst.Japan*, 753-754,1974.

SUMMARY

A kinematical restricted maximization procedure has been developed by combining the primal-dual method of linear programming with a semi-inverse approach. Some general solutions to practical problems of minimum weight plastic design have been derived analytically by applying the proposed method.

RESUME

Une procédure cinématique de maximisation limitée a été développée par combinaison de la méthode primale-duale de la programmation linéaire avec une approche semi-inverse. Quelques solutions générales pour des problèmes pratiques de dimensionnement plastique, conduisant à un poids minimum ont été obtenues analytiquement par application de la méthode proposée.

ZUSAMMENFASSUNG

Ein begrenztes kinematisches Maximierungsverfahren wird bei einer Kombination der "primal-dual"-Methode der linearen Programmierung mit einem "semi-inversen" Verfahren entwickelt. Allgemeine analytische Lösungen praktischer Probleme der plastischen Bemessung auf Minimalgewicht werden durch Anwendung der vorgelegten Methode gefunden.

Optimality Criteria and Dual Methods in Truss Design

Critères d'optimisation et méthodes duales dans le dimensionnement de treillis

Optimierungskriterien und Dualmethoden in der Berechnung von Fachwerken

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1. INTRODUCTION

In the Introductory Report of the 10th Congress of IABSE Gellatly and Dupree[1] describe the optimality criteria approach to the optimum design of large structural systems. In handling large structural systems the direct solution approach by numerical mathematical programming methods is often excessively slow and cumbersome as a result of the large numbers of variables which must be optimized. The optimality criteria approach is intended to overcome the difficulties posed by having large numbers of variables. Gellatly and Dupree consider the optimality approach to the design of structures in which element mass and stiffness are proportional. Such structures include those composed of axial force bars, membrane plates and shear panels. For this class of structures Gellatly and Dupree derive an optimality criterion, their equation (2), for the minimum weight design of a truss subject to a single displacement constraint. They then use this optimality criterion, (2), to develop a recursion relationship, (8), which allows any arbitrary set of member areas to be modified iteratively so as to eventually produce an optimal set of member sizes. The important time-saving feature of this approach is that at each iteration the existing set of member sizes is altered by applying the simple relationship (8) to each area in turn. There is no complicated numerical search involved.

Gellatly and Dupree then continue to describe a large computer program, OPTIM II, in which this optimality criterion and redesign formula is used to design structures with multiple displacement constraints (stiffness requirements) and also individual member size constraints. They point out that neither the optimality criterion itself nor the redesign formula is valid for anything other than a single displacement constraint but, despite this lack of rigour, OPTIM II still obtains remarkably good numerical results very quickly. This is not disputed here; OPTIM II is an efficient program, but its lack of rigour is perplexing and it makes it difficult to interpret and identify those occasional cases in which OPTIM II performs poorly.

The purpose of this paper is to examine a new dual formulation of optimum design problems for this class of structures. In particular the problem of how best to handle multiple constraints is examined and an interpretation of the dual problem is presented which has considerable relevance in the development of improved optimum design algorithms for large structural systems.

2. THE OPTIMUM DESIGN PROBLEM

For simplicity of notation a truss structure composed only of axial force bars is considered, being typical of the general class of structures with member stiffness proportional to member mass. The minimum weight (minimum volume) design problem can be posed as that of finding the set of member areas A_i, $i = 1, \ldots, N$, which

$$\text{Minimize } W = \sum_{i=1}^{N} L_i A_i \tag{1}$$

subject to M independent nodal displacement constraints (Gellatly and Dupree consider only a single generalised stiffness constraint),

$$g_m \equiv \sum_{i=1}^{N} \left(\frac{FUL}{E\delta}\right)_{mi} \frac{1}{A_i} \leqslant 1 \qquad m = 1, \ldots, M \tag{2}$$

and subject also to N member size constraints, one for each member

$$g_{M+i} \equiv \frac{\overline{A_i}}{A_i} \leqslant 1 \qquad i = 1, \ldots, N \tag{3}$$

In constraints (2) F and U are the member actual forces and virtual forces associated with unit displacement in the direction of the nodal constraint. δ_m is the maximum permitted displacement of a node in constraint m, $m = 1, \ldots, M$. E is the elastic modulus, and each of constraints (2) is derived from specific applied loads and virtual force systems. In constraints (3) $\bar{A}_i$ is the minimum permissible size of member i, derived either from maximum member stress limits or from fabricational considerations.

In the above formulation it is assumed that F and U are constants, hence $\bar{A}_i$ is also constant. This assumption is valid for statically determinate trusses. It is strictly invalid for indeterminate trusses, however, F, U and hence $\bar{A}_i$ do not usually alter appreciably as members sizes alter and it is common to assume them constant, obtain an altered set of member sizes in some way, update the values of F, U and $\bar{A}_i$, solve again and continue in this iterative fashion until the process converges to an optimum solution. This iterative solution technique is used by both mathematical programming and optimality criterion devotees, the essential difference between them being only the way in which the altered set of member sizes is obtained. It is assumed here that this iterative method for indeterminate structures is used and so in the above formulation F, U, L, E, δ and $\bar{A}$ are all known constants. Our problem is how best to find the optimal set of member sizes.

Recently the present author[2] has shown that there is a dual formulation of the problem expressed in relationships (1), (2) and (3). Derivation of the dual problem is accomplished by exploiting the fact that the Lagrangian function of the above problem has a saddle point as a stationarity condition. A full proof of the dual formulation is given in reference[2] and here it is merely stated as

$$\left.\begin{aligned}
&\text{Maximise } V = \sum_{i=1}^{N} L_i \left\{\sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m + \frac{\overline{A_i}}{L_i} \lambda_{M+i}\right\}^{\frac{1}{2}} \\
&\text{subject to } \sum_{m=1}^{M+N} \lambda_m = 1 \\
&\qquad \lambda_m \geqslant 0 \qquad\qquad m = 1, \ldots, M + N
\end{aligned}\right\} \tag{4}$$

The solution of (4) is equivalent exactly to the solution of the primal problem, (1), (2) and (3). At the solution point (minimum of W, maximum of V) the following transformation relationships hold, with superscript asterisk denoting optimal values,

$$\left.\begin{aligned} &(\text{Minimum}) \quad W^* \equiv V^{*2} \quad (\text{Maximum}) \\ &A_i^* \equiv V^* \left\{ \sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m^* + \frac{\overline{A_i}}{L_i} \lambda_{M+i}^* \right\}^{\frac{1}{2}} \qquad i = 1, \ldots, N \end{aligned}\right\} \qquad (5)$$

The dual variables in dual problem (4) are the λ_m, $m = 1, \ldots, M + N$ and it will be noted that there is a dual variable λ_m for each of the primal constraints (2) and (3). The dual variables are therefore similar to the unknown Lagrange multipliers of the primal problem. All λ's must be non-negative; any value of $\lambda = 0$ denotes that the primal constraint to which it corresponds is inactive at the optimum. The single constraint in dual problem (4) requires that all λ's sum to unity.

3. PROBLEMS WITH ONLY DISPLACEMENT CONSTRAINTS

Gellatly and Dupree[1] consider only a single displacement constraint and their equations (2) and (8) represent an optimality criterion and a resizing formula for this problem. Their equation (2) contains a single unknown Lagrange multiplier corresponding to the single constraint. This unknown multiplier may be eliminated by substitution into the constraint which must perforce be active; consequently their resizing formula (8) contains no unknown multipliers. A major difficulty is encountered if this method is extended to multiple displacement constraints. In this case there will be M unknown Lagrange multipliers, one for each constraint, and since it is not known *a priori* which of the multiple displacement constraints are active and which are slack at the optimum it is not possible to eliminate the unknown multipliers by substitution. Consequently when a member resizing formula for multiple constraints is developed corresponding to Gellatly and Dupree's equation (8) it contains all the M unknown Lagrange multipliers. In order to use the resizing formula it is necessary to supply values to all the unknown Lagrange multipliers but there is no way of knowing what these values should be. This constitutes the major difficulty of using optimality criteria methods for multiple constraints. In order to get round this difficulty OPTIM II uses the envelope method which resizes each member according to the single constraint resize formula for each displacement constraint and then selects the largest resulting size. This process seems intuitively logical but has no theoretical rigour.

If the dual approach is examined for multiple displacement constraints only, the dual problem becomes

$$\left.\begin{aligned} &\text{Maximise } V = \sum_{i=1}^{N} L_i \left\{ \sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m \right\}^{\frac{1}{2}} \\ &\text{subject to } \sum_{m=1}^{M} \lambda_m = 1 \\ &\qquad\qquad \lambda_m \geq 0 \qquad\qquad m = 1, \ldots, M \end{aligned}\right\} \qquad (6)$$

At the optimum, we have

$$\left.\begin{aligned} &(\text{Minimum}) \quad W^* = V^{*2} \quad (\text{Maximum}) \\ &A_i^* = V^* \left\{ \sum_{m=1}^{M} \left(\frac{FU}{E\delta}\right)_{mi} \lambda_m^* \right\}^{\frac{1}{2}} \qquad i = 1, \ldots, N \end{aligned}\right\} \qquad (7)$$

Problem (6) consists of maximizing V, a non-linear function of the M dual variables λ_m subject only to a single linear equality constraint and non-negativity of the dual variables. This is easily done by classical optimization methods. Once λ_m^*, m = 1, ..., M are known, relationships (7) give the minimum weight and optimal member sizes directly.

Several features of the dual problem can be noted. Firstly the number of dual variables is M, the number of displacement constraints. This means that the dimensionality of the original problem, which had N member size variables, is greatly reduced. Thus a large structure with perhaps 1000 members to be sized and 5 displacement constraints has a dual problem which consists of maximizing a non-linear function V of only 5 variables. In most large structural problems there are usually many more members than displacement constraints so the reduction in dimensionality afforded by the dual problem is of considerable advantage. Secondly, the dual problem itself is of a convenient form for rapid solution. The single linear equality constraint may be eliminated by substitution, converting the problem to one of unconstrained form with non-negativity requirements. First and second derivatives can be easily evaluated which makes solution comparatively simple. Thirdly, the result gives immediate information about which constraints in the primal problem are active and which are slack since a value of $\lambda_m = 0$ corresponds to a slack constraint. Finally the dual approach has the theoretical rigour which is lacking in the envelope method.

A physical interpretation of the primal/dual problems in terms of structural behaviour is illuminating. Consider a structure constrained by M independent displacement constraints, i.e.

$$\left.\begin{array}{ll} \text{Minimize } W & \\ \text{Subject to } g_m \leq 1 & m = 1, \ldots, M \end{array}\right\} \quad (8)$$

If each of the M constraints in (8) is multiplied by a multiplier λ_m, m = 1, ..., M, such that the sum of the λ_m's is unity, and all the constraints are then summed into a single surrogate constraint we have

$$\left.\begin{array}{l} \text{Minimize } W \\ \text{Subject to } \sum_{m=1}^{M} \lambda_m g_m \leq 1 \end{array}\right\} \quad (9)$$

Examination of the dual problems corresponding to (8) and (9) shows them to be identical providing the λ_m's in (9) solve problem (6) optimally. This demonstrates that in responding to multiple constraints the structure apportions its member sizes as if all the independent constraints were surrogated into a single generalised stiffness requirement. The structure therefore responds to a single fictitious surrogated stiffness requirement and, since the λ_m must solve (6), the surrogate stiffness requirement is such that the independent sitffness requirements are combined together in such a way as to maximize their constraining potential.

This physical interpretation may partly help to explain the good results often obtained by the envelope method as used in OPTIM II. The envelope method resizes a a member by applying a single resize formula to each constraint in turn and selects the highest resulting member size. These highest sizes form a resized set. By this means the constraining potential of all the constraints is maximized. This is in the same spirit as the more rigorous dual approach outlined above but is mathematically different and is not rigorous. However, it may be conjectured that the good results obtained by OPTIM II correspond to problems in which the enveloping and surrogation approaches are similar and that the occasional poor performance of

OPTIM II corresponds to problems in which the member sizes obtained by enveloping are very different from those which satisfy the more correct surrogated constraint in (9).

4. PROBLEMS WITH DISPLACEMENT AND MEMBER SIZE CONSTRAINTS

As Gellatly and Dupree demonstrate, a displacement constraint governs the distribution of material throughout the structure. A member stress or size constraint only controls the material in an individual member. Difficulties arise when both types of constraints are present together since the distribution of material required to optimally satisfy a displacement constraint may violate the amount of material required to satisfy one or more of the individual member constraints. There is no optimality criterion of practical use for combined types of constraints. Somewhat *ad hoc* methods are usually used such as active/passive sets of variables as in OPTIM II to handle both types of constraints.

The primal problem concerning us here is that given in (1), (2) and (3) and the corresponding dual problem is given in (4) and (5). On examining the dual problem it at first appears that its dimensionality, (M + N), is greater than that of the primal problem, N. This would negate the advantage which the dual approach has of reducing problem dimensionality. Fortunately, very recent research has shown that the N dual variables corresponding to member size constraints may be effectively eliminated by an iterative process. A brief summary of this now follows.

Consider dual problem (4) for a single displacement constraint (with dual variable λ_0) and a full set of N member size constraints. If we write

$$\overline{W_i} = L_i \overline{A_i} \qquad \overline{\delta_i} = \left(\frac{FUL}{\overline{AE}}\right)_i$$

$$\overline{W} = \sum_{i=1}^{N} \overline{W_i} \qquad \overline{\delta} = \sum_{i=1}^{N} \overline{\delta_i}$$

and if δ is the maximum permissible nodal displacement, dual problem (4) is

$$\left.\begin{aligned} &\text{Maximize } V = \sum_{i=1}^{N} \sqrt{\overline{W_i}}\, \left\{\frac{\overline{\delta_i}}{\delta}\lambda_0 + \lambda_i\right\}^{\frac{1}{2}} \\ &\text{Subject to } \sum_{i=o}^{N} \lambda_i = 1 \\ &\qquad \lambda_i \geq 0 \qquad\qquad i = 0, \ldots, N \end{aligned}\right\} \qquad (10)$$

Necessary conditions for a constrained maximum of V with respect to the N member size dual variables only are that

$$\frac{\partial V}{\partial \lambda_i} = 0 \qquad\qquad i = 1, \ldots, N$$

This leads to

$$\lambda_i^* = \frac{\overline{W_i}}{\overline{W}}\left(1 + \frac{\lambda_0}{\delta}\left[\overline{\delta} - \delta - \frac{\overline{W}}{\overline{W_i}}\overline{\delta_i}\right]\right) \qquad\qquad i = 1, \ldots, N \qquad (11)$$

Substituting (11) into V of (10) gives

$$V = \sqrt{\overline{W}}\,\{1 + \lambda_0 \left[\frac{\overline{\delta} - \delta}{\delta}\right]\} \tag{12}$$

If $\overline{\delta} < \delta$ this denotes that member sizes evaluated from the member size constraints alone will satisfy the displacement constraint and hence λ_0 will be zero. We are interested in the case where $\overline{\delta} > \delta$ and the displacement constraint must be active. In this case V as given in (12) is maximized by as large a value of λ_0 as is possible. However, λ_0 may not increase to a value such as to drive any of the λ_i^*, i = 1, ..., N in (11) below zero. The highest possible value of λ_0 is therefore that value which first puts any λ_i^* equal to zero, i.e.

$$\lambda_0 = \min_{i=1,\dots,N} \left\langle \left(1 - \frac{\overline{\delta}}{\delta} + \frac{\overline{W}}{\overline{W_i}}\,\frac{\overline{\delta_i}}{\delta}\right) \right\rangle \tag{13}$$

This value of λ_0 drives one of the λ_i^* to zero. Let the variable driven to zero be $\lambda_N^* = 0$. This is now eliminated as a slack member size constraint.

A new dual problem may now be formed with λ_N eliminated. This replaces problem (10) and is

$$\left.\begin{aligned}
&\text{Maximize } V = \sum_{i=1}^{N-1} \sqrt{\overline{W}_i}\,\{\frac{\overline{\delta_i}}{\delta}\lambda_0 + \lambda_i\}^{\frac{1}{2}} + \sqrt{\overline{W}_N}\,\{\frac{\overline{\delta_N}}{\delta}\lambda_0\}^{\frac{1}{2}} \\
&\text{Subject to } \sum_{i=o}^{N-1} \lambda_i = 1 \\
&\qquad \lambda_i \geq 0 \qquad\qquad i = 0, \dots, N = 1
\end{aligned}\right\} \tag{14}$$

Problem (14) is treated in a similar way to problem (10). Relationships similar to (11) are established for the λ_i^*, this time for i = 1, ,.., N - 1. An expression for V similar to (12) is found and a new value of λ_0 is determined as (13). If the new value of λ_0 is greater than its previous value another of the λ_i^* is eliminated, another problem similar to (14) but with (N − 2) values of λ_i is set up and the process is continued in this iterative fashion until the value of λ_0 reduces. The previous iteration's results for all the λ's are then optimal. Relationships (5) then give the minimum weight and optimal member sizes.

The iterative procedure described above forms into a very simple algorithm since the relationships of the types of (11), (12) and (13) are very concise in nature. Using this iterative dual approach the interactions of member size constraints and a displacement constraint may be optimized very rapidly, the dimensionality of the method being essentially unity. An advantage of the method is that it starts essentially with a fully-stressed design (all member size dual variables active and $\lambda_0 = 0$). The activity level of the displacement constraint, λ_0, is then progressively increased, knocking out member size constraints as they become slack. In many practical design situations a first requirement is to examine the fully-stressed design and check it against possible displacement limitations. If the displacements are excessive the fully-stressed design needs to be altered in some way so as to optimally satisfy displacement limitations. This is precisely how the dual approach outlined above tackles the problem and it is therefore well suited to implementation in practical optimum design programs.

The treatment above is limited to the combination of a single displacement constraint and member size constraints. If multiple displacement constraints are present the iteration algorithm is more complex and has not yet been fully investigated. However, it has already been shown in this paper that multiple displacement constraints behave as a single surrogated constraint. This suggests a possible solution algorithm in which the multiple constraints are first solved separately and the single surrogate constraint formed and then the above algorithm used to handle the interactions of the surrogate constraint and the member size constraints. This remains to be further investigated.

5. CONCLUSIONS

This paper has examined a dual approach to the optimum design of structures whose elements have stiffness proportional to mass. It has shown that a study of duality gives insight and rationale for some of the successful, non-rigorous approaches to truss design such as the optimality criterion approach used in OPTIM II. It would have been more satisfying to give numerical results confirming the speed and efficiency of the dual algorithms suggested in this paper but space limitations preclude this. Nevertheless it can be stated that the dual approach does provide a means of very rapidly solving optimum design problems for large structural systems. The reduction in dimensionality and the ease with which the dual problems may be manipulated and solved makes the approach a very serious competitor to the much-used, less rigorous optimality criteria methods. From a practical structural engineering point of view it should be stressed that although duality theory and the associated algebra may seem unnecessarily complicated and abstract, the algorithms which may be developed from it are rigorous and are very simple to operate, giving practically useful results very rapidly. Furthermore the dual-based algorithms often tend to be similar to those suggested by engineering intuition. This is very satisfying and a firmer theoretical basis for intuitive design approaches adds considerable strength to them.

As the present author has commented in the Introductory report to the 10th IABSE Congress[3] a major advantage of a study of dual methods is that it sheds new light on well-known problems and enables the nature of the problems to be understood more deeply. Sometimes, as in the case here, this extra insight allows new solution algorithms to be developed. The ultimate usefulness of these algorithms remains to be fully investigated in a continuing program of research.

6. REFERENCES

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SUMMARY

The paper examines a new dual approach to the optimum design of trusses with multiple displacement and member size constraints. Comparison is made with optimality criteria approaches to the same problem. Reductions in problem dimensionality and simple solution algorithms arise from casting the problem into dual space, which also gives insight into some ad hoc, intuitive artifices often employed in the solution of these problems.

RESUME

Une nouvelle méthode duale est présentée pour le dimensionnement optimal de treillis, soumis à des contraintes de déplacements multiples et de types de profils. Une comparaison est faite avec la méthode des critères d'optimisation. Des réductions de la dimension des problèmes ainsi que des algorithmes simples pour leur résolution sont obtenus en situant le problème dans l'espace dual, ce qui permet également d'analyser quelques artifices de calcul souvent utilisés dans la solution de tels problèmes.

ZUSAMMENFASSUNG

Der Bericht behandelt eine neue Dualmethode für die Optimierung von Fachwerken mit mehrfachen Formänderungs- und Formgebungsrestriktionen. Die Ergebnisse werden mit der Methode der Optimalitätskriterien verglichen. Eine Abminderung der Komplexität und einfache Lösungsalgorithmen resultieren aus der Problemprojektion in einem Dualraum, was auch Einblick in gewisse intuitive Verfahren gewährt, die bei der Lösung solcher Probleme oft angewendet werden.

Die Bedeutung des Kraft- und Weggrössenverfahrens für die Optimierung von Tragwerken nach der Lagrange'schen Multiplikatorenmethode

The role of the Force- and Displacement-Method for the Optimization of Structures with the Lagrangian-Multiplier-Technique

Rôle de la méthode des forces et des déformations dans l'optimisation des structures selon la méthode de Lagrange

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1. Problemstellung

Im Konstruktiven Ingenieurbau stehen heute eine Reihe leistungsfähiger Berechnungsverfahren zur Verfügung. Das Dimensionieren von Tragwerken erfolgt dagegen durch den Ingenieur, wobei Können und Erfahrung eine wesentliche Rolle spielen. Kann man eine Gewichts- oder Kostenfunktion definieren, so läßt sich dieses Problem als Optimierungsaufgabe formulieren, die als Folge der Bemessungskriterien i.a. nichtlinear und nichtkonvex ist. Aus der Vielzahl der Lösungsverfahren zur Bestimmung eines lokalen Minimums [1] wird hier das Verfahren der Optimalitätskriterien betrachtet, das eine problemorientierte Variante der Lagrange'schen Multiplikatorenmethode darstellt.

Dem Optimierungsmodell liegt ein durch n Elemente diskretisiertes Tragwerk zugrunde. Es wird vorausgesetzt, daß für jedes Element i die Elementflexibilität $\underline{f}_i$ +) umgekehrt proportional von einer Querschnittsvariablen (Entwurfsvariable) $\alpha_i > 0$ abhängt und daß sich das Gewicht des Tragwerkes als lineare Funktion (Zielfunktion) dieser Entwurfsvariablen darstellen läßt:

$$W = \sum_{i=1}^{n} w_i = \sum_{i=1}^{n} \overline{w}_i \, \alpha_i \qquad (1)$$

Als Nebenbedingungen werden Spannungs- und Verformungsrestriktionen berücksichtigt, wobei σ_{il}^{o} und δ_{jl}^{o} die zulässige Spannung des Elementes i bzw. die zulässige Verformung in Richtung des Freiheitsgrades j infolge Lastfall l bedeutet. Zusätzlich kann eine Einschrankung der Variablen durch untere und obere Schranken α_i^u bzw. α_i^o vorgegeben werden. Damit ergibt sich folgende Optimierungsaufgabe:

$$\text{Minimiere} \quad W = \sum_{i=1}^{n} \overline{w}_i \, \alpha_i$$

unter Berücksichtigung der Restriktionen

+) Matrizen und Spaltenvektoren werden durch Unterstreichen gekennzeichnet, ein hochgestelltes T bedeutet die Transponierte.

$$\sigma_{il} - \sigma_{il}^{o} \leqq 0 \qquad (i=1,\ldots,n\ ;\ l=1,\ldots,p)\ , \qquad (2)$$

$$\delta_{jl} - \delta_{jl}^{o} \leqq 0 \qquad (j=1,\ldots,q\ ;\ l=1,\ldots,p)\ , \qquad (3)$$

$$\alpha_{i}^{u} - \alpha_{i} \leqq 0 \qquad (i=1,\ldots,n)\ , \qquad (4)$$

$$\alpha_{i} - \alpha_{i}^{o} \leqq 0 \qquad (i=1,\ldots,n)\ . \qquad (5)$$

Es bedeutet q die Anzahl der Freiheitsgrade und p die Anzahl der Lastfälle. Die Spannungen σ und die Verformungen δ sind nichtlineare Funktionen der Entwurfsvarieblen $\underline{\alpha}$, so daß die Restriktionen einen nichtkonvexen Lösungsbereich beschreiben. Da die Problematik bei einem Lastfall bzw. mehreren Lastfällen dieselbe ist, wird im folgenden aus Gründen der Übersichtlichkeit auf den Belastungsindex l verzichtet.

2. Notwendige und hinreichende Optimalitätsbedingungen

Die Herleitung notwendiger Extremalbedingungen der nichtlinearen Optimierungsaufgabe erfolgt mit der verallgemeinerten Lagrange'schen Multiplikatorenmethode [2]. Da sämtliche Variablen $\underline{\alpha}$ nichtnegativ definiert und alle Restriktionen als Ungleichungen gegeben sind, sind diese Bedingungen hinreichend für ein lokales Minimum der Zielfunktion [3]. Bezeichnet man mit $G_j \leq 0$ die allgemeine Form der Restriktionen (2) und (3), so lautet die Lagrange'sche Funktion:

$$J = W + \sum_{j=1}^{m} \lambda_j G_j + \sum_{i=1}^{n} \mu_i (\alpha_i^u - \alpha_i) + \sum_{i=1}^{n} \eta_i (\alpha_i - \alpha_i^o) \ . \qquad (6)$$

Die Lagrange'schen Parameter λ_j, μ_i und η_i sind festgelegt durch:

$$\lambda_j \geqq 0 \ , \quad \text{für} \quad G_j \overset{=}{<} 0 \qquad (j=1,\ldots,m) \qquad (7)$$

$$\mu_i \geqq 0 \ , \quad \text{für} \quad \alpha_i \overset{=}{>} \alpha_i^u \qquad (i=1,\ldots,n) \qquad (8)$$

$$\eta_i \geqq 0 \ , \quad \text{für} \quad \alpha_i \overset{=}{<} \alpha_i^o \qquad (i=1,\ldots,n) \qquad (9)$$

Als notwendige und hinreichende Bedingung für einen stationären Wert von W müssen die partiellen Ableitungen von J nach den Variablen $\underline{\alpha}$ verschwinden. Mit $\partial(\ldots)/\partial\alpha_k = (\ldots)_{,k}$ erhält man:

$$W_{,k} + \sum_{j=1}^{m} G_{j,k} - \mu_k + \eta_k = 0 \qquad (k=1,\ldots,n) \qquad (10)$$

Mit (8) und (9) folgt:

$$-\sum_{j=1}^{m} \lambda_j G_{j,k} \begin{cases} \geqq W_{,k} \\ = W_{,k} \\ \leqq W_{,k} \end{cases} , \quad \text{für} \begin{cases} \alpha_k = \alpha_k^o \\ \alpha_k^u < \alpha_k < \alpha_k^o \\ \alpha_k = \alpha_k^u \end{cases} \qquad (11)$$

Für alle "passiven " Restriktionen $G_j < 0$ ist nach (7) der Lagrange'sche Parameter λ_j gleich Null, so daß in der Optimalitätsbedingung (11) nur die "aktiven" Restriktionen $G_j = 0$ berücksichtigt zu werden brauchen.

3. Rekursionsformeln zur Bestimmung der optimalen Konstruktion

3.1 Aktive Verformungsrestriktionen

Einzelne Verformungsgrößen können mit Hilfe des Prinzips der virtuellen Kräfte berechnet werden. Es gilt:

$$\delta_j = \sum_{i=1}^{n} e_{ij} = \sum_{i=1}^{n} \underline{S}_i^T \underline{f}_i \underline{\tilde{S}}_i \qquad (j=1,\dots,q') \quad , \tag{12}$$

wobei e_{ij} die virtuelle Verzerrungsenergie, S_i die Schnittgrößen infolge der Belastung, $\tilde{S}_i$ die Schnittgrößen infolge der virtuellen Einheitsbelastung in Richtung der gesuchten Verformungsgröße des Elementes i und q' die Anzahl der aktiven Verformungsrestriktionen darstellt. Als partielle Ableitung nach den Variablen α_k $(k=1,\dots,n)$ erhält man mit $e_{kj} = \bar{e}_{kj}/\alpha_k$:

$$G_{j,k} = -\bar{e}_{kj} / \alpha_k^2 \quad . \tag{13}$$

Bezeichnet $k \in N1$ eine "aktive" Variable α_k mit dem Wert $\alpha_k^u < \alpha_k < \alpha_k^o$ und $k \in N2$ eine "passive" Variable mit $\alpha_k = \alpha_k^u$ oder $\alpha_k = \alpha_k^o$, so muß für alle aktiven Variablen $k \in N1$ das Gleichheitszeichen in der Optimalitätsbedingung (11) erfüllt sein. Mit $W_{,k} = \bar{w}_k$ und (13) folgt:

$$\sum_{j=1}^{q'} \lambda_j \bar{e}_{kj} / \alpha_k^2 = \bar{w}_k \qquad (\forall k \in N1) \quad . \tag{14}$$

Diese Gleichung stellt i.a. ein hochgradig nichtlineares Gleichungssystem mit den Unbekannten λ_j $(j=1,\dots,q')$ und α_k $(k=1,\dots,n)$ dar, das nur iterativ gelöst werden kann. Ist nur eine einzige Verformungsrestriktion zu berücksichtigen, d.h.

$$\delta_j^o = \sum_{k \in N1} \bar{e}_{kj} / \alpha_k + \sum_{k \in N2} e_{kj} \quad , \tag{15}$$

so läßt sich der Lagrange'sche Parameter λ_j eliminieren. Die Gleichungen (14) aufgelöst nach α_k $(k \in N1)$ und in (15) eingesetzt, liefert:

$$\lambda_j = \left(\frac{1}{\delta^*} \sum_{k \in N1} \sqrt{\bar{e}_{kj} \bar{w}_k} \right)^2 \quad \text{mit } \delta^* = \delta^o - \sum_{k \in N2} e_{kj} \quad . \tag{16}$$

Bei mehreren aktiven Verformungsrestriktionen ist eine Bestimmung von λ_j $(j=1,\dots,q')$ aus (14) nur dann möglich, wenn $\bar{e}_{kj} / \alpha_k^2$ als invariant betrachtet werden. In diesem Fall stellt (14) ein überbestimmtes lineares Gleichungssystem in $\underline{\lambda}$ dar:

$$\underline{G}\,\underline{\lambda} = \underline{E} \tag{17}$$

mit

$$\underline{G} = \left[\bar{e}_{kj} / \bar{w}_k \alpha_k^2 \right] \tag{18}$$

und $\underline{E} = \{1,\dots,1\}$ für alle $k \in N1$ und $j=1,\dots,q'$. Mit Hilfe der ersten Gauß'schen Transformation kann eine Lösung für $\underline{\lambda}$ gefunden werden. Es gilt:

$$\underline{\lambda} = \left[\underline{G}^T \underline{G} \right]^{-1} \underline{G}^T \underline{E} \quad . \tag{19}$$

In Bezug auf die ursprüngliche Gleichung (17) stellt $\underline{\lambda}$ die beste Lösung im Sinne der kleinsten Quadrate dar. Mit den bekannten $\underline{\lambda}$-Werten und der Annahme invarianter Größen $\bar{e}_{kj}$ (bei stat. best. Systemen) entkoppelt sich das Gleichungssystem (14), so daß die aktiven Variablen α_k $(k \in N1)$ bestimmt werden können:

$$\alpha_k = \left(\sum_{j=1}^{q'} \lambda_j \bar{e}_{kj} / \bar{w}_k \right)^{1/2} \tag{20}$$

Bei stat. unbest. Systemen sind die Größen $\bar{e}_{kj}$ komplizierte Funktionen von $\underline{\alpha}$. Da sich eine Änderung von α_k in erster Linie auf die Schnittgrößen des

Elementes k auswirkt, kann (20) iterativ angewendet werden, d.h.

$$\alpha_k^{\nu+1} = \left(\sum_{j=1}^{q'} \lambda_j^{\nu} \bar{e}_{kj}^{\nu} / \bar{w}_k \right)^{1/2} , \qquad (21)$$

wobei ν den Iterationsschritt kennzeichnet und λ_j^{ν} $(j=1,\dots,q')$ für $q'=1$ aus (16) bzw. für $q' \geq 2$ aus (19) mit den Werten $\bar{e}_{kj}^{\nu}$ und α_k^{ν} berechnet wird. Da die passiven Variablen α_k $(k \in N2)$ i.a. nicht im voraus bekannt sind, muß ihre Bestimmung ebenfalls iterativ erfolgen. Dabei können die Schranken $\underline{\alpha}^u$ und $\underline{\alpha}^o$ durch die Bedingungsgleichungen

$$\alpha_k^{\nu+1} = \begin{cases} \alpha_k^o \\ \alpha_k^{\nu+1} \\ \alpha_k^u \end{cases} \quad \text{für} \quad \begin{cases} \alpha^{\nu+1} \geqq \alpha_k^o \\ \alpha_k^u < \alpha_k^{\nu+1} < \alpha_k^o \\ \alpha_k^{\nu+1} \leqq \alpha_k^u \end{cases} \qquad (22)$$

berücksichtigt werden. Alle Variablen, für die α_k^u bzw. α_k^o maßgebend ist, werden in der nächsten Iteration zu den passiven gezählt.

3.2 Aktive Spannungsrestriktionen

Sind ausschließlich Spannungsbeschränkungen vorgeschrieben, so kann die Bestimmung der Variablen $\underline{\alpha}$ nach der bekannten "stress-ratio"- Methode [4] erfolgen, in der jedes Element entsprechend seiner spannungsmäßigen Auslastung dimensioniert wird. Es gilt:

$$\alpha_k^{\nu+1} = \alpha_k^{\nu} \left| \sigma_k^{\nu} / \sigma_k^o \right| \qquad (23)$$

wobei σ_k^{ν} die maßgebende Spannung des Elementes k im ν-ten Iterationsschritt bedeutet. Als Ergebnis erhält man eine sogenannte "voll-beanspruchte" Konstruktion, die in jedem Element die zulässige Spannung ausnutzt, wenn nicht der durch α_k^u festgelegte minimale Querschnitt maßgebend ist.

Bei aktiven Verformungsrestriktionen können Spannungsbeschränkungen berücksichtigt werden, wenn man in jeder Iteration die nach (23) berechneten $\underline{\alpha}$-Werte in der Bestimmungsgleichung (22) als zusätzliche untere Schranken auffaßt.

3.3 Konvergenz des Verfahrens

Die Anwendung der Gleichungen (16), (19), (21) bis (23) verlangt nach jeder Iteration eine vollständige Berechnung der Konstruktion. Um jeweils eine zulässige Lösung zu erhalten, werden sämtliche Variablen $\underline{\alpha}^{\nu}$ mit einem globalen Skalierungsfaktor multipliziert, so daß keine der Restriktionen (2) und (3) verletzt und mindestens eine identisch erfüllt wird. Danach erfolgt die Bestimmung der aktiven Verformungsrestriktionen, wobei alle Verformungen, die im Verlauf des Iterationsprozesses einmal ihren zulässigen Wert erreicht haben, weiterhin zu den aktiven gezählt werden. Ergibt sich jedoch nach (19) ein negativer λ-Wert so muß die entsprechende Restriktion aufgrund der Nichtnegativitätsbedingung (7) wieder eliminiert werden. Erst wenn alle aktiven Verformungen bekannt sind, ist mit einer schnellen Konvergenz zu rechnen. Das Konvergenzverhalten kann durch eine Begrenzung der Schrittweite in aufeinanderfolgenden Iterationen beeinflußt werden. Mit

$$\alpha_k^{\nu+1} = \alpha_k^{\nu} \qquad (k=1,\dots,n)$$

ist die optimale Konstruktion gefunden, für die das Gewicht ein (lokales) Minimum annimmt.

4. Die Bedeutung des Kraft- und Weggrößenverfahrens

Bisher wurde nur das Iterationsverfahren zur Lösung der Optimierungsaufgabe betrachtet. Über die Lagrange'schen Parameter $\underline{\lambda}$ bei mehreren aktiven Restriktionen wurde im Sinne der kleinsten Quadrate verfügt. Im Vergleich mit anderen Verfahren [4] ergibt sich hierdurch ein stabiles Konvergenzverhalten bei nur wenigen Iterationsschritten. Die wiederholte Berechnung des Tragwerkes nach der Finiten-Elementmethode erfordert bei den vorliegenden Problemen einen erheblichen Rechenaufwand und verdient damit besondere Beachtung. Ohne auf die Möglichkeiten der Ableitung von Elementmatrizen [5] einzugehen, werden hier nur die Lösungsverfahren betrachtet. Diese Verfahren folgen direkt aus den klassischen Minimalprinzipien elastischer Tragwerke.

Das Prinzip vom Minimum der Potentiellen Energie

$$\text{Min}\left\{ \frac{1}{2}\underline{\delta}^T \underline{K}\,\underline{\delta} - \underline{P}^T \underline{\delta} \right\} , \tag{24}$$

mit der positiv definiten Gesamtsteifigkeitsmatrix $\underline{K}$, den Lasten $\underline{P}$ und den Verschiebungen $\underline{\delta}$, liefert als notwendige und hinreichende Bedingung die Grundgleichung der Verschiebungemethode:

$$\underline{K}\,\underline{\delta} = \underline{P} . \tag{25}$$

Das Prinzip vom Minimum der Komplementärenergie

$$\text{Min}\left\{ \frac{1}{2} \underline{S}^T \underline{f}\,\underline{S} \;\middle|\; \underline{N}\,\underline{S} = \underline{P} \right\} , \tag{26}$$

mit der Hyperdiagonalmatrix $\underline{f}$ der Elementflexibilitätsmatrizen, den verallgemeinerten Spannungen $\underline{S}$ und der Gleichgewichtsmatrix $\underline{N}$ ergibt die Grundgleichungen der Kraftmethode:

$$\begin{aligned} &\underline{N}\,\underline{S} = \underline{P} && \text{(Gleichgewicht)} , \\ &\underline{B}_x^T \underline{f}\,\underline{S} = 0 && \text{(Verträglichkeit)} . \end{aligned} \tag{27}$$

$\underline{B}_x^T$ ist der Kern der Gleichgewichtsmatrix $(\underline{N}\,\underline{B}_x^T = 0)$.

Den geringsten Aufwand für die einmalige Berechnung eines Tragwerkes erfordert im allgemeinen die Verschiebungsmethode: Der einfache Aufbau, die positive Definitheit und Bandstruktur der q x q Matrix $\underline{K}$ erleichtert die Berechnung. Bei einer mehrmaligen Berechnung des Tragwerkes mit variabler Flexibilität $\underline{f}$ zeigt jedoch die Kraftmethode gewisse Vorteile: Die q Gleichgewichtsgleichungen (27) müssen nur einmalig gelöst werden, die Verträglichkeitsbedingungen lassen sich einfacher darstellen und mit geringerem Aufwand für jede Wiederbemessung lösen. Als Lösung erhält man die n Schnittgrößen $\underline{S}_i$ zur Iteration nach (12). Mit dem in [6] näher beschriebenen Lösungsverfahren kann zudem die Bandstruktur der Gleichgewichtsgleichungen gewahrt werden. Ein genauer Vergleich des numerischen Aufwandes beider Methoden führte zu dem Ergebnis, daß mit steigender Zahl der Wiederbemessungen der Aufwand A_F der Kraftmethode abnimmt. Das Verhältnis des Aufwandes A_D der Verschiebungsmethode zur Kraftmethode nimmt jedoch bei wachsendem n/q ab. In den für die Praxis wichtigen Stabtragwerken ist jedoch i.a. $n/q < 2$. Für ein System mit 1000 Freiheitsgraden der Verschiebung und einem speziellen Elementtyp (s) ergibt sich die in Bild 1 dargestellte Abhängigkeit [6].

Umfangreiche numerische Untersuchungen [7] an den aus der Literatur bekannten optimalen Tragwerken bestätigen in allen Fällen die Überlegenheit der Kraftmethode.

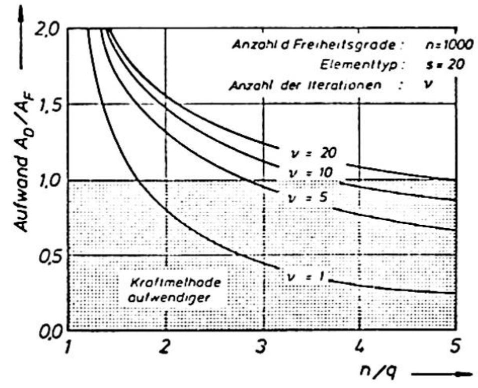

Bild 1: Vergleich der Kraft- und Verschiebungsmethode

Lastfall	Knoten (P_z=−1000 lbs)
1	1
2	1-4,7-13,19-28,37
3	1-37
4	1,4-7,13-19,28-37

Tabelle 1: Belastungsangaben

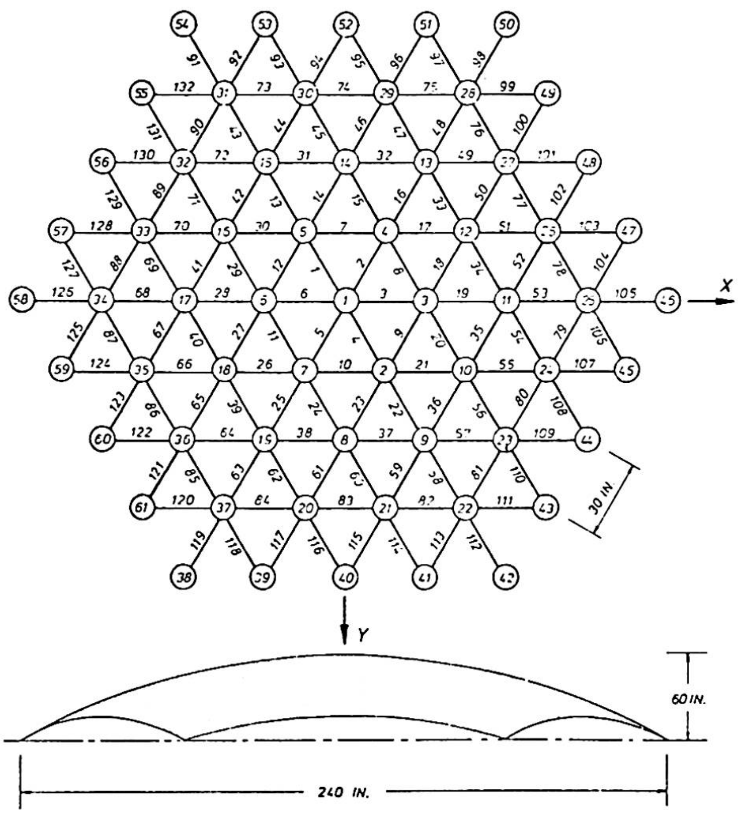

Bild 2: Fachwerkkuppel

5. Numerische Ergebnisse

Die Zuverlässigkeit des Optimierungsverfahrens soll hier an einem ausgewählten Beispiel gezeigt werden. Die in Bild 2 dargestellte Fachwerkkuppel, die in den Knoten 38 - 61 unverschieblich gelagert ist, wird durch vier Lastfälle beansprucht. Die genauen Belastungsangaben sind in Tabelle 1 zusammengestellt. Als Material wird Aluminium mit einem Elastizitätsmodul von $E = 10^7$ psi und dem spezifischen Gewicht von $\rho = 0.1$ lbs/in^3 verwendet. Für alle Stäbe beträgt der minimale Querschnitt 0.1 in^2, wobei die zulässige Spannung von ± 25000 psi nicht überschritten werden darf. Die Verschiebungen sämtlicher Freiheitsgrade in z-Richtung werden auf ± 0.1 in. begrenzt. Alle Entwurfsbedingungen sind mit denen aus [8] identisch.

Ausgehend von einer zulässigen Konstruktion mit querschnittsgleichen Stäben (W_1=358.85 lbs) wird die optimale Kuppel nach 15 Iterationen und einem Gewicht von 161.63 lbs gefunden, das um 10.7% geringer ist als in [8]. Während zu Beginn der Optimierung nur die Verschiebung von Knoten 1 (LF 1) den maximal erlaubten Wert von -0.1 in. erreicht, sind von der 13. Iteration an 41 Verformungsrestriktionen zu berücksichtigen, die jeweils durch einen der 4 Lastfälle aktiviert wurden. Spannungen waren in keiner Phase des Iterationsprozesses maßgebend. Bild 3 zeigt das stabile Konvergenzverhalten, wobei insgesamt eine Gewichtsreduktion von 55% erreicht wird. Die Querschnittsflächen der optimalen Kuppel, die symmetrisch zu den beiden Achsen 38-50 und 44-56 ausgebildet

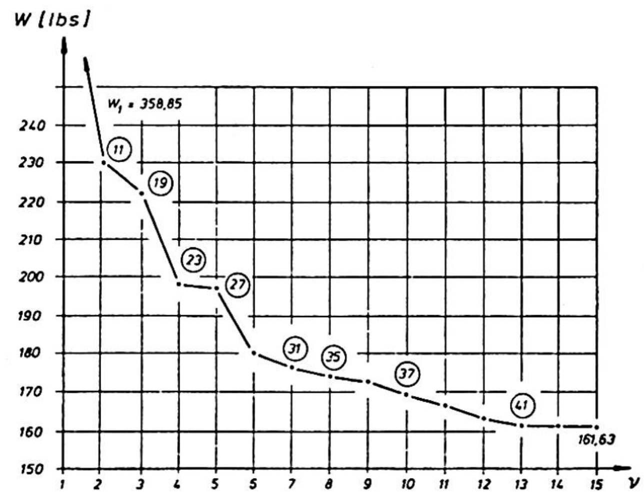

Bild 3: Iterationsverlauf

ist, sind in Tabelle 2 zusammengestellt. Bei n/q=1.19 konnte die Kraftmethode äußerst wirtschaftlich eingesetzt werden. Die Rechenzeit (TR 440) betrug nur 182 sec.

Stab	Fläche	Stab	Fläche	Stab	Fläche	Stab	Fläche
4	1.0176	36	0.4831	62	0.3177	111	0.1003
5	1.1732	37	0.3051	63	0.6572	112	0.2403
9	0.9720	38	0.3514	80	0.3062	113	0.3088
10	0.8322	56	0.3207	81	0.2128	114	0.1429
21	0.2990	57	0.1904	82	0.1003	115	0.5000
22	0.3395	58	0.3378	83	0.1003	116	0.1003
23	0.5773	59	0.3431	84	0.3347	117	0.4381
24	0.4148	60	0.29 6	109	0.1003	118	0.3312
25	0.6776	61	0.5494	110	0.4961	119	0.1003

Tabelle 2: Optimale Querschnittsflächen (in^2) eines Quadranten

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ZUSAMMENFASSUNG

Es wird eine spezielle Anwendung der Lagrange'schen Multiplikatorenmethode, die als Verfahren der Optimalitätskriterien bekannt wurde, dargestellt. Eine lineare Transformation der Lagrange-Parameter führte zu einer schnellen und gleichmässigen Konvergenz.

SUMMARY

A special application of the Lagrangian-Multiplier-Technique, known as the optimality-criterion-method, is presented. A simple linear transformation of the Lagrange parameters leads to fast and uniform convergence.

RESUME

Une application spéciale de la technique des multiplicateurs de Lagrange, dite méthode des critères d'optimisation est présentée. Une transformation linéaire entraîne une convergence rapide et uniforme.

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IIb

Optimisation des systèmes et des dimensions pour des comportements structuraux linéaires et non-linéaires

Optimierung der Systeme und der Abmessungen bei linearem und nicht-linearem Verhalten des Tragwerkes

System and Geometrical Optimization for Linear and Non-Linear Structural Behaviour

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Über das Leistungsvermögen von Tragwerken am Beispiel von Balken, Druckbogen und Zugbogen

Capacity Range of Structures, such as Beams, Compression Arches and Tension Arches

Capacité de résistance de structures telles que poutres, arcs de compression et arcs de tension

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1. Einführung

Balken, Druckbogen und Zugbogen sind die Grundformen aller Tragwerke zur Bewältigung von Spannweiten. Die eine Spannweite bestimmenden Größen und ihr Zusammenwirken, die Bandbreite technisch möglicher Spannweiten, lassen sich denn auch an diesen Grundformen am besten studieren. Dies um so mehr als die Gesetzmäßigkeiten verhältnismäßig leicht analytisch faßbar sind.

Ziel des Beitrags sind Spannweitenfunktionen für alle drei Grundformen bei allgemeinen Baustoffgesetzen und ggf. Gleichgewicht am verformten System, wenn nötig mit nichtlinearen Geometriebeziehungen, auf deterministischer Basis und für statische Belastung.

Die Spannweitenfunktionen bilden wichtige Grundlagen für jeden Entwurf und jede Tragwerkentscheidung und sind Hilfen bei der Optimierung.

2. Die Spannweitenfunktion

Die Spannweite ist Ausdruck des Leistungsvermögens. Sie ist bei einem bestimmten Versagenszustand eine Funktion des Systems (S), der Form (F) und der Baustoffe (M) des Tragwerks sowie der Fremdlast (L), die getragen werden muß:

$$l = f \text{ (System, Form, Baustoff, Fremdlast)} \qquad (1).$$

"Fremdlast" ist für das Tragwerk alles, was nicht Teil seiner tragenden Form (= aktives Gewicht g_a) ist, wie etwa das Gewicht von Pfetten (= passives Gewicht g_p), die quer zu einem Balken gespannt sind und die gesamte Verkehrslast p.

Die Spannweitenfunktion (1) läßt sich unter bestimmten Voraussetzungen als Produkt dreier Kenngrößen K schreiben:

$$l = K_{S+F_{längs}} \cdot K_{M+F_{quer}} \cdot K_L = l_{Gr} \cdot K_L \qquad (2a),$$

nämlich dann, wenn 1. das System sich statisch bestimmt verhält, 2. das Gleichgewicht am unverformten System angeschrieben werden kann und 3. Fremdlast $g_p + p$ und aktives Gewicht g_a affin sind. Es beschreiben:

$K_{S+F_{längs}}$	das System und die Verteilung der Tragwerkmasse in seiner Längsrichtung,
$K_{M+F_{quer}}$	die Baustoffe und die Verteilung der Tragwerkmasse in Systemquerrichtung,
K_L	die Fremdlast.

So aufgeschlüsselt sind die sehr unterschiedlichen Einflüsse, die l bestimmen, am leichtesten durchschaubar.

Bei Lastaffinität allein ist: $l = K_{S+F+M} \cdot K_L = l_{Gr} \cdot K_L$ (2b).

Die Einflüsse aus System, Form und Baustoff lassen sich dann nicht mehr trennen.

In (2) ist: $K_L = \frac{1}{1 + (g_p + p)/g_a}$, $0 \leq K_L \leq 1$ (3).

Die Grenzspannweite l_{Gr} ist demnach die Spannweite bei verschwindender Fremdlast ($K_L = 1$). Sie kann nicht mehr übertroffen werden: Das Leistungsvermögen des Tragwerks ist erschöpft.

Den Untersuchungen liegen der Einfachheit halber Spannweitenfunktionen nach (2) zugrunde. Die so gewonnenen Aussagen bleiben qualitativ gültig, auch wenn Fremdlast und tragendes aktives Gewicht nicht affin sind.

3. Die Tragwerkformen

3.1 Balken

Grenzfälle von Balkensystemen sind der "einfache Balken" und der "Kragbalken". Mit ihnen ist der gesamte Leistungsspielraum von Balkensystemen faßbar. Der einfache Balken begrenzt das Leistungsvermögen nach unten, der Kragbalken nach oben. Seilverspannte Balken werden nicht betrachtet. Sie besitzen bei engen Seilabständen hohes Leistungsvermögen und sind dann dem Kragbalken mit dem Idealquerschnitt $m_U = 1$ (s. Bild 2) vergleichbar.

Der Einfluß der Baustoffe und der Querschnittform ist bei beiden Systemen gleich:

$$K_{M+F_{quer}} = \frac{\beta_R}{\gamma} \, \frac{m_U}{\nu} \qquad (4).$$

Dabei bedeuten:

β_R Rechenfestigkeit des Bezugsbaustoffs

$\gamma = a\varrho$ Berechnungsgewicht des Balkenmaterials im Beschleunigungsfeld a (Erde $a = 9{,}81\ m/s^2$)

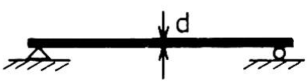
β_R/γ Reißlänge bzw. Zerdrückhöhe des Balkenmaterials bei zugfestem bzw. druckfestem Bezugsbaustoff

m_U bezogenes Bruchmoment $M_U/F\, d\, \beta_R$, als Maß der Beanspruchbarkeit des Querschnitts (Fläche F, Höhe d, Breite b) mit dem Größtmoment

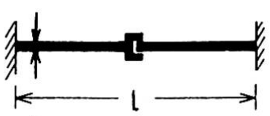
ν Gesamtsicherheitsbeiwert.

Wenn für das Tragvermögen ausnahmsweise der Gebrauchszustand maßgebend ist, muß in (4) m_U/ν durch m des Gebrauchszustands ersetzt werden.

Die Bandbreite des Faktors $K_{S+F_{längs}}$ ist dagegen sehr verschieden:



$8\,\frac{d}{l} \leq \quad \leq 9{,}9\,\frac{d}{l}$ (5)



$8\,\frac{d}{l} \leq K_{S+F_{längs}} \leq$ ∞ : ideal; Bild 1: real (6)

$2\,\frac{d}{l} \leq \quad \leq$ ∞ : ideal; Bild 1: real (1/4 der Werte) (7)

Die Werte auf der linken Seite gehören zu Balken mit konstantem Querschnitt, die auf der rechten zu - in jedem Querschnitt - vollbeanspruchten mit konstanter Höhe und idealem Zweipunktquerschnitt (quasi Fachwerkbalken). Im einen Fall ist die Tragwerkmasse demnach überhaupt nicht auf den Momentenverlauf abgestimmt, im anderen dagegen vollkommen.

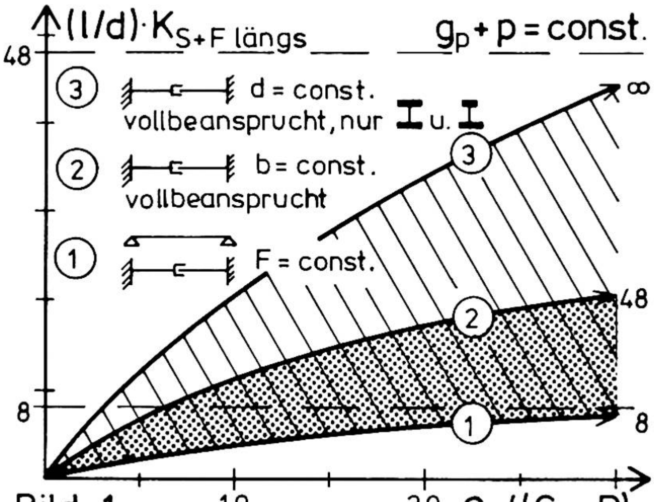


Bild 1

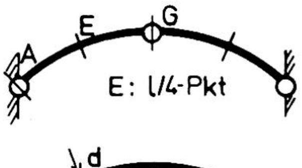
Der ideale Wert ∞ besagt nicht, daß l auch bei realen Kragbalken ∞ groß oder auch nur sehr groß werden kann. Durch nicht affine Fremdlast und einen im Bereich der Kragbalkenspitze technologisch bedingten Mindestbalkenquerschnitt sinkt das Leistungsvermögen außerordentlich ab: der in (6) und (7) angegebene ∞ große Leistungsspielraum schrumpft z.B. allein durch eine konstante Fremdlast auf den in Bild 1 schraffierten endlichen Bereich zusammen. Der baupraktisch nutzbare Spielraum ist noch kleiner, vor allem wenn d ≠ const. ist (im Bild gerastert) oder unterschiedliche Lastfälle zu berücksichtigen sind.

3.2 Druckbogen

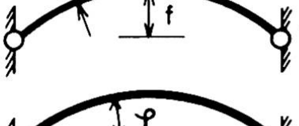
Die nach oben gekrümmte Bogenform ist keine Form minimaler potentieller Energie. Ein Druckbogen hat deshalb den Drang, nach unten durchzuschlagen, sein Tragvermögen geht spätestens mit dem Einsetzen des Durchschlags verloren. Obwohl Durchschlagvorgänge nur mit einer geometrisch nichtlinearen Theorie faßbar sind, genügt für die numerische Traglastrechnung im Schlankheitsbereich, den die technischen Baubestimmungen erlauben, die geometrisch linearisierte Theorie. Bei den baupraktisch allein bedeutsamen Pfeilverhältnissen $f/l \geq 0{,}1$ kann außerdem die Achsdehnung unberücksichtigt bleiben.

Das Leistungsvermögen ist am kleinsten, wenn der Durchschlagvorgang ohne Gleichgewichtsverzweigung abläuft. Dazu gehören Lastkombinationen, die die jeweils kritische Ausweichform durch gleichsinnige Störmomente begünstigen: antimetrische Momente beim 2-Gelenk-Bogen, beim gelenklosen Bogen und beim steilen 3-Gelenk-Bogen, symmetrische dagegen, wenn dieser flach ist (etwa $f/l < 0{,}3$). Die kritische Fremdlast muß demnach zwei Anteile enthalten: einen voraussetzungsgemäß zu g_a affinen - durch K_L erfaßten - und einen anderen, - durch β gekennzeichneten - der die Störmomente erzeugt ($\beta = q_{anti}\,K_L/g_{a,G}$ bei antimetrischer Störlast q_{anti}, $\beta = Q_G K_L/g_{a,G}\,l$ bei symmetrischer Störlast Q_G im Bogenscheitel).

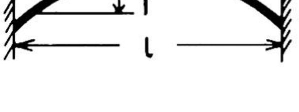
Für den als Stützlinie für g_a geformten Kettenlinienbogen (F = const.) sind die Kenngrößen für System, Form und Baustoff K_{S+F+M}:



$$\frac{1-\cos\rho_A}{\cos\rho_A}\left(\frac{l}{f}\right)\frac{\beta_R}{\gamma}\cos\rho_E\,\frac{n_{\beta,Ecr}}{\nu}\,\frac{1}{\left[1+\frac{\beta}{4}\,\frac{1-\cos\rho_A}{\cos\rho_A}\left(\frac{l}{f}\right)^2\right]\left(1+\frac{v_G}{f}\cos^2\rho_E\right)} \qquad (8)$$



$$\frac{1-\cos\rho_A}{\cos\rho_A}\left(\frac{l}{f}\right)\frac{\beta_R}{\gamma}\cos\rho_E\,\frac{n_{\beta,Ecr}}{\nu} \qquad (9)$$



$$\frac{1-\cos\rho_A}{\cos\rho_A}\left(\frac{l}{f}\right)\frac{\beta_R}{\gamma}\cos\rho_E\left[\frac{n_{\beta,E}}{\nu} \mp \frac{2m_A}{\nu}\left(\frac{d}{l}\right)\sin\rho_E\right]_{cr} \qquad (10).$$

Die bezogenen kritischen Schnittgrößen $n = N/F\beta_R$ und m enthalten implizit die Einflüsse aus (S), (F), (M) und (L). Bis auf das Glied mit v_G, der lotrechten Verschiebung des Scheitelgelenks beim 3-Gelenk-Bogen, stimmen (8)(9)(10) formal mit den Ausdrücken der Theorie 1. Ordnung überein. Ab etwa $f/l > 0{,}3$ gilt (9) auch für den 3-Gelenk-Bogen.

Für flache Kettenlinienbogen ist $\frac{1-\cos\rho_A}{\cos\rho_A}\,\frac{l}{f} \approx 8\,\frac{f}{l}$ (11).

Damit kann bis etwa $f/l < 0{,}3$ gerechnet werden.

Das Leistungsvermögen ist um so kleiner, je größer die Störmomente sind und je schlanker der Bogen ist. Es wird dann auch mehr und mehr f/l-unabhängig. Nur bei sehr kleinen Störmomenten werden in etwa die klassischen Extremstellen für max. l erreicht (z.B. $f/l \approx 0{,}3$ beim Kettenlinienbogen). Der Leistungsabbau kann in allgemeiner Form nur qualitativ angeschrieben werden:

$$n_{\beta,cr} \geq n_{\beta,UII} < n_{\beta,UI} < n_{U,(m=0)} \qquad (12).$$

Bei n_U ist wegen $m = 0$ das Leistungsvermögen des Querschnitts ausgenutzt, durch die Störmomente nimmt es ab auf $n_{\beta,UI}$, durch den Einfluß der Bogenverformungen auf $n_{\beta,UII}$; bei "Stabilitätsversagen" geht das Tragvermögen bereits im Innern des n-m-Interaktionsdiagramms verloren, $n_{\beta,cr}$ ist dann $> n_{\beta,UII}$. Numerische Berechnung ohne besonderen Aufwand nach [1] möglich, dort und in [2] Beispiele zu (12).

Großes Leistungsvermögen setzt gedrungene Bogen voraus. Querschnitte, die dem idealen 2-Punkt-Querschnitt nahekommen, bringen Leistungssteigerung vor allem bei großem l/d, f/l und großen Störmomenten. Der gelenklose Bogen ist am leistungsstärksten. Ausweichen senkrecht zur Bogenebene bedeutet zusätzlichen Leistungsabbau.

3.3 <u>Zugbogen</u>

Ein biegesteifer Zugbogen vermag, dem Druckbogen ähnlich, das Leistungsvermögen des Querschnitts nicht auszunutzen:

$$n_{\beta,cr} = n_{\beta,UII} \begin{matrix} > n_{\beta,UI} \\ < n_{U,(m=0)} \end{matrix} \qquad (13),$$

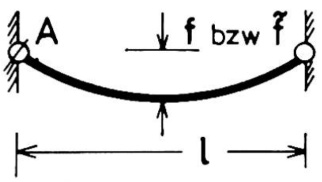
wenn auch bei ihm die Systemverformungen ($n_{\beta,UII} > n_{\beta,UI}$) leistungssteigernd wirken.

Ein Zugbogen muß aber nicht biegesteif sein: Die hängende Bogenform ermöglicht als Form minimaler potentieller Energie den biegeweichen Bogen mit voller Querschnittausnutzung

$$n_{\beta,cr} = n_{U,(m=0)} \qquad (14).$$

Er wird dadurch zum leistungsfähigsten System.

Baupraktisch bedeutsam ist allein der flache Kettenlinienbogen mit



$$K_{M+F_{quer}} = \frac{\beta_R}{\gamma}\,\frac{n_{UA}}{\nu} \qquad (15)$$

$$K_{S+F_{längs}} = \frac{8}{\sqrt{1 + 16(\tilde{f}/l)^2}}\,\frac{\tilde{f}}{l} \approx 8\,\frac{\tilde{f}}{l} \qquad (16).$$

Je nach Bogenbaustoff kann statt n_{UA}/ν auch der Wert des Gebrauchszustandes $n_A \equiv n_{(m = 0)}$ maßgebend sein. Der Zirkumflex kennzeichnet das Pfeilverhältnis des verformten Bogens

$$\tilde{f}/l \simeq f/l \sqrt{1 + 3/8 \cdot (l/f)^2 \cdot (\tilde{b} - b)/l} \qquad (17)$$

mit der gedehnten Bogenlänge $\tilde{b}$.

Von allen Tragwerkformen für Baukonstruktionen dürfen beim biegeweichen Bogen als einziger die Geometriebeziehungen nicht von vornherein linearisiert werden. Dem entspricht (17). Die lineare Beziehung geht um so eher verloren, je flacher der Bogen ist.

Der biegeweiche Bogen ist kinematisch verschieblich, weil seine Achse stets Seillinie der jeweiligen Belastung sein muß. Kritisch sind antimetrische Störungen zusammen mit hoher Entlastung. Sie können mit wachsendem f/l Anlaß großer Verformungen sein, ein zu leichter oder ein in anderer Weise nicht ausreichend stabilisierter Bogen kann nach oben durchschlagen. Dieses Durchschlagproblem, das in [2] behandelt ist, beeinträchtigt das Leistungsvermögen nicht.

4. Die Baustoffe

Die Leistungskenngrößen K enthalten den Baustoffeinfluß in allgemeingültiger Form als Produkt

$$\beta_R/\gamma \cdot n_U \text{ bzw. } \beta_R/\gamma \cdot m_U \qquad (18)$$

Die Spannungsdehnungslinien stecken dabei in n und m, ebenso die Querschnittform und der kritische Dehnungszustand.

Das Leistungsvermögen wächst mit der Reißlänge und der Zerdrückhöhe. Hochfeste Stähle und hochfeste Betone und Leichtbetone kennzeichnen die Entwicklung, mit der Tendenz, auch im Betonbau zu Werten zu kommen, die denen von Baustahl vergleichbar sind.

Für n_U und m_U lassen sich von der Spannungsdehnungslinie unabhängige obere Grenzwerte angeben:

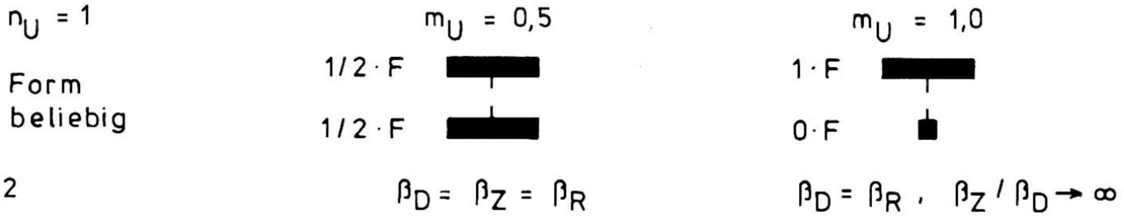


Bild 2

bei homogenem Material. $n_U = 1$ ist im biegeweichen Zugbogen realisierbar. $m_U = 0{,}5$ und $1{,}0$ lassen sich als die Beanspruchbarkeiten der Querschnitte von Fachwerkbalken deuten, deren Diagonalengewicht verschwindend klein ist. Tatsächlich brauchen alle biegebeanspruchten baupraktischen Querschnitte gewisse Zuggurtmassen und, vor allem im Vollwandbereich, Stegmassen, die das Leistungsvermögen verringern. Für sie sind deshalb $m_U = 0{,}5$ und $1{,}0$ unerreichbare Grenzwerte: 0,5 für die Querschnitte des Stahlbaus, 1,0 für die des Spannbetonbaus.

Für Betontragwerke seien noch einige weitere Angaben gemacht:

4.1 Balken

Das Leistungsvermögen der Balkenquerschnitte wird durch die Tragfähigkeit der Biegedruckzone begrenzt (Grenzstauchung ε_{bU}). Voll nutzbar wird es durch eine entsprechend hohe Bewehrung der Biegezugzone, wobei die Bewehrungsgrenze normalerweise aus dem Wunsch folgt, ein Versagen der Druckzone zu vermeiden, bevor die Zugbewehrung fließt (Bruchvorankündigung durch $\varepsilon_e \geq 3$ ‰).

Im übrigen wird hohes Leistungsvermögen durch geschicktes Formen der Querschnitte erreicht. Wie groß dabei der Spielraum ist, zeigen die Grenzformen "Rechteckquerschnitt" mit $m_U \simeq 0{,}25$ und "idealer Zweipunktquerschnitt" mit $m_U = 1{,}0$ bei unbewehrter Druckzone. Die Bandbreite der realen, baupraktischen Querschnitte ist der in Zuggurt und Steg allein schon technologisch bedingten Betonflächen wegen beträchtlich schmaler. Die bei großen Spannweiten bisher gebauten Formen vollwandiger Balken besitzen etwa

$$\begin{aligned} 0{,}35 \leq \; & m_U && \leq 0{,}60 \\ 0{,}40 \leq \; & \bar{\mu} = \mu \beta_S / \beta_R && \leq 0{,}65 \end{aligned} \qquad (19).$$

Je höher m_U ist, um so weniger ist die Beanspruchbarkeit von der σ - ε -Linie des Betons abhängig.

Nur mit Hilfe der Vorspannung gelingt es, dem Idealquerschnitt mit $m_U = 1{,}0$ nahezukommen, denn nur durch Vorwegnehmen der Stahldehnung werden hochfeste Stähle ausnutzbar, so daß sich große und größte Zugkräfte in verhältnismäßig kleinen Betonquerschnitten unterbringen lassen. Die damit erzielbare Einsparung an Querschnittfläche wächst mit der Spannweite. Der Vorspanngrad selbst beeinflußt i.a. nur das Verhalten im Gebrauchszustand, nicht aber das Leistungsvermögen. Auch eine "Druckspannbewehrung" zur Zugvorspannung der Druckzone erhöht das Leistungsvermögen nur durch den Bewehrungsgehalt der Druckzone. Wenn die Gebrauchsfähigkeit dies zuläßt, soll auch bei Vorspannung nicht mehr Bewehrung eingelegt werden, als der Bruchzustand erfordert mit einem möglichst hohen Anteil an Spannstahl.

4.2 Druckbogen

Im Druckbogen sind zweipunktnahe Querschnittformen der einfachen Rechteckform nicht so selbstverständlich weit überlegen wie im Balken, weil die ihnen eigene überragende Steifigkeit verlorengeht, sobald einer der Gurte reißt. Die Tragfähigkeit fällt dann jäh ab, auf Werte, die sich von denen des Rechteckquerschnitts meist nurmehr unwesentlich unterscheiden. Hohlquerschnitte sind deshalb nur dann entscheidend leistungsfähiger, wenn sie im gesamten Beanspruchungsbereich ungerissen bleiben. Dazu bedarf es vielfach gedrungener Bogen, vor allem bei merklichen Störmomenten und mit wachsendem f/l. Auch eine Vorspannung kann manchmal zweckmäßig sein.

Mit dem Bewehrungsgehalt ist das Leistungsvermögen nur im Zugbruchbereich entscheidend zu beeinflussen. Die Wirkung wächst mit den Störmomenten und wird durch die Schlankheit beschleunigt. Doch ist selbst bei großen Störmomenten eine bewehrungsproportionale Leistungssteigerung nicht erreichbar. Nahezu ohne Wirkung bleibt der Bewehrungsgehalt bei Stabilitätsversagen, zu dem sehr kleine bis kleine Störmomente gehören. Dann kommt es vor allem auf die σ - ε -Linie des Betons an.

Die Bandbreite der $n_{\beta,cr}$ des Zweigelenkbogens ist in [2] untersucht.

4.3 Zugbogen

Im biegeweichen Zugbogen hat der Beton, anders als in den mit Biegung arbeitenden Systemen, keine wesentliche Tragfunktion, diese übernehmen die Spannglieder. Der Beton bildet vor allem Raumabschluß oder Fahrbahn, formstabilisierendes Element (Schale, Platte, Gewicht) und Korrosionsschutz der Bewehrung.

Wenn Spannglieder der Festigkeit β_Z die Bewehrung bilden, ist

$$n_{UA} = \mu_z \; \beta_Z / \beta_R \qquad (20).$$

Der Bewehrungsgehalt μ_z hat nur technologische Grenzen: Die Spannglieder sollen des einfachen Korrosionsschutzes wegen im Betonquerschnitt Platz finden. Das ergibt

$$\text{etwa } \mu_z \leq 0{,}15 \qquad (21).$$

Bei Balken und Druckbogen setzt das Tragvermögen der Biegedruckzone dem Bewehrungsgehalt weit niedrigere technische Grenzen: $\mu_z = \beta_R / \beta_S$ oder $\bar{\mu}_z = 1$ als oberste Schranke beim idealen Zweipunktquerschnitt mit $m_U = 1$ und etwa $\bar{\mu}_z \leq 0{,}65$ oder $\mu_z \leq 0{,}015$ bei den baupraktischen Vollwandquerschnitten (19). Der biegeweiche Zugbogen kann demnach etwa 10mal so stark bewehrt werden wie Balken oder Druckbogen. Das, zusammen mit einem hohen β_Z, begründet sein überlegenes Leistungsvermögen.

Die nutzbare Stahlfestigkeit β_Z hängt allein vom plastischen Verformungsvermögen des Bogens ab. Sein Gleichgewicht verlangt ein Spannungsgefälle von den Kämpfern zur Bogenmitte. Deshalb ist das plastische Verformungsvermögen nur mit Stählen nutzbar, die einen Verfestigungsbereich besitzen. Das ist bei allen Spannstählen mehr oder weniger ausgeprägt der Fall. Da sich der Bogen nicht beliebig weit in den Verfestigungsbereich hinein verformen darf, wird β_Z durch das Erreichen kritischer Spannstahldehnungen begrenzt, etwa

$$\text{crit. } \varepsilon_z \leq (1{,}0 \text{ bis } 1{,}5)\, 10^{-2} + \varepsilon_z^{(o)} \qquad (22),$$

mit der Spannbettdehnung $\varepsilon_z^{(o)}$ [3]. Bei Bogen bis etwa $f/l \leq 0{,}1$ wird dadurch β_Z so groß, daß die im Gebrauchszustand zulässige Stahlspannung zul σ_z mit $n_A = \mu_z$ zul σ_z / β_R (23) das Leistungsvermögen bestimmt. - Bei Stählen mit idealelastisch-idealplastischem bzw. sprödem Verhalten wäre $\beta_Z = \beta_S$ bzw. β_Z zu setzen.

5. Die Tragwerkmasse

Die das aktive Gewicht g_a bildende Tragwerkmasse ist dann am wirksamsten eingesetzt, wenn sie

- an jeder Tragwerkstelle und
- in jeder Querschnittfaser voll ausgenutzt ist und
- selbst möglichst wenig Beanspruchung erzeugt.

Damit ist hohes Leistungsvermögen gegeben, nicht aber unbedingt auch ein optimales Tragwerk vom Aufwand und Nutzen her gesehen. Je weniger das Leistungsvermögen gefordert wird, um so mehr darf und wird man von diesen Kriterien abweichen.

Das Abstimmen von Tragwerkmasse und Momentenverlauf lohnt sich demnach am meisten beim Kragbalken, der dadurch viel leistungsfähiger als der einfache Balken wird. Dieser reagiert darauf viel weniger empfindlich, weshalb bei ihm der mögliche Leistungsgewinn nur ein ziemlich grobes Abstimmen rechtfertigt (5) (6). Begründet ist dies in der unterschiedlichen Völligkeit des Momentenbildes beider Systeme: Der Kragbalken braucht, im Gegensatz zum einfachen Balken, die Tragwerkmasse dort, wo sie nur mit kleinem Hebelarm momentenwirksam ist. Ein Tragwerk aus aneinandergereihten, richtig geformten Kragbalken ist deshalb auch leistungsfähiger als ein solches mit Einhängebalken oder aus Durchlaufbalken. Da der einfache Balken auch "Ersatzbalken" der Bogen ist, lohnen auch diese das Abstimmen der Tragwerkmasse auf den Beanspruchungsverlauf nur mit einem ähnlich eng begrenzten Leistungszuwachs. Der Zweigelenkdruckbogen nach (9) kann dadurch wenig mehr als 10 % weiter gespannt werden. Beim biegeweichen Zugbogen scheidet diese Möglichkeit, Leistung zu gewinnen, fast ganz aus.

Nicht ausgenutzte Tragwerkmasse kann sich sehr unterschiedlich bemerkbar machen: solange sie die Grenzspannweite l_{Gr} unbeeinflußt läßt, bedeutet sie eine Leistungsreserve und wirkt wie eine erhöhte Fremdlast, sobald durch sie aber l_{Gr} kleiner wird, wirkt sie leistungsmindernd. Das typische Beispiel für eine solche Leistungsminderung ist der Kragbalken mit F = const.

Bei jedem Tragwerk dürfen bestimmte Mindestabmessungen nicht unterschritten werden, die untere Grenze der Tragwerkmasse ist deshalb technologisch bedingt. Auch das sind nicht ausgenutzte Tragwerkmassen und leistungsmäßig dementsprechend zu behandeln.

6. Die Fremdlast

Beide Anteile der Fremdlast, die nutzungsbedingte Verkehrslast p und das konstruktionsbedingte passive Gewicht g_p beeinflussen das Leistungsvermögen gleich nachteilig durch $K_L < 1,0$. Vor allem bei hoher Leistungsforderung muß deshalb g_p so klein wie möglich gehalten werden. g_p ist nicht immer nur Gewicht, auch die formstabilisierende Vorspannung in Seilwerken und Seilnetzen zählt dazu. Flächentragwerke nutzen die Baumasse vielfältig, sie haben daher meist ein verhältnismäßig kleines g_p, Stabtragwerke mit ihren eindimensionalen Traggliedern dagegen ein großes.

Eine zur Tragwerkmasse nicht affine Fremdlast ist leistungsmäßig über ihre beanspruchungswirksamen Hebelarme zu beurteilen. Sind sie größer als die der Tragwerkmasse, wirkt die Nichtaffinität leistungsmindernd. Nur beim Kragbalken mit einer auf die Beanspruchung abgestimmten Tragwerkmasse ist die Annahme einer Affinität keine gute erste Näherung, weil bei ihm eine konstante Fremdlast sehr leistungsmindernd ist.

7. Das Maßstabgesetz

Die Spannweitenfunktion (1) beschreibt l als absolute Größe; mit γ / β_R multipliziert enthält sie nur mehr relative Größen:

$$l \cdot \gamma / \beta_R = \gamma / \beta_R \cdot f\left(\text{Verhältniswerte für (S),(F),(M),(L)}\right) \qquad (1a).$$

Das ist das Maßstabgesetz des Leistungsvermögens. Beispiel:

$$l \frac{\gamma}{\beta_R} = 8 \frac{m_U}{\nu} \frac{d}{l} \frac{1}{1 + (g_p + p)/g_a} .$$

Die linke Seite sagt nun aus, wie weit die Reißlänge oder Zerdrückhöhe des Bezugsbaustoffs als Spannweite nutzbar ist, - über $\gamma = a\varrho$ ist der Einfluß allgemeiner Schwerkraftfelder enthalten.

Wenn die Beanspruchbarkeit ausgenutzt und damit wie die Reißlänge und Zerdrückhöhe ein Festwert ist, müssen die Maßstabsfaktoren: λ für die Spannweite, $\lambda_{d/l}$ für das Bauhöhenverhältnis und λ_{K_L} für das Lastverhältnis die Bedingung

$$\lambda = \lambda_{d/l} \cdot \lambda_{K_L} \quad \text{oder} \quad \bar{\lambda} = \lambda / \lambda_{d/l} = \lambda_{K_L} \tag{24}$$

erfüllen. Statt λ_{K_L} interessiert λ_{g_a}, der Maßstabsfaktor für die Tragwerkmasse, der mit ihm verknüpft ist. Bei konstant bleibender Fremdlast ist dieser

$$\lambda_{g_a} = \bar{\lambda}\frac{1 - K_L}{1 - \bar{\lambda}K_L} \lesseqgtr \lambda^2 \text{ bei } K_L \lesseqgtr \frac{1}{\bar{\lambda}} \; \frac{\lambda^2 - \bar{\lambda}}{\lambda^2 - 1} \tag{25}$$

$$\lambda_{g_a} \gtrless 0\text{: Leistungsvermögen} \begin{matrix}\text{besteht} \\ \text{versagt.}\end{matrix}$$

Bei $\lambda_{g_a} = \lambda^2$ und $\lambda_{d/l} = 1$ ist das gesamte Tragwerk affin größer geworden. K_L ist auf das Ausgangstragwerk bezogen.

Die Auswertung zeigt: Nur bei kleinen Spannweiten ist es möglich, ein Tragwerk, das sich bei einer Bauaufgabe bewährt hat, durch bloß affines Vergrößern einer größeren Aufgabe anzupassen. Bei großen Spannweiten muß stets und vor allem auch das Bauhöhen- oder Pfeilverhältnis vergrößert werden.

Das bedeutet: Große Tragwerke müssen nicht nur massiger sein als kleine, System, Form und Baustoffe sind schließlich nicht mehr frei wählbar, sondern werden eine Funktion der absoluten Größe.

(25) ist für Balken ermittelt. Die damit gewonnenen Aussagen gelten qualitativ auch für Bogen.

8. Das wirtschaftliche Leistungsvermögen

Das technische Leistungsvermögen endet mit der Grenzspannweite l_{Gr}. Tatsächlich wird ein Tragwerk aber lange vorher bedeutungslos, weil seine Wirtschaftlichkeit verlorengeht.

Aus (2) (3) folgt das aktive Gewicht, das bei gegebener Fremdlast aufzuwenden ist, um eine gegebene Spannweite zu bewältigen:

$$g_a = \frac{1}{l_{Gr}/l - 1} \quad (g_p + p) \geq \text{technolog. } g_a \tag{26}$$

Die Tragwerkmasse, beschrieben durch g_a, wächst demnach hyperbolisch mit abnehmendem Verhältnis l_{Gr}/l oder je mehr das technische Leistungsvermögen ausgeschöpft wird. Sie wird schließlich unwirtschaftlich groß, bei $l_{Gr}/l = 1$ unendlich groß, auch wenn die Fremdlast noch so klein ist.

Ziel des Entwerfens muß es demnach sein, System und Baustoffe so zu wählen, das System so zu formen und das passive Gewicht so zu beeinflussen, daß der Abstand $l_{Gr} - l$ groß genug bleibt, um g_a vernünftig klein zu halten. Wird für ein bestimmtes Tragwerk g_a unwirtschaftlich groß, muß ein leistungsfähigeres mit größerer Grenzspannweite gewählt werden. Ausreichendes Leistungsvermögen ist dabei im gesamten Spannweitenbereich nötig.

Die Tragwerkmasse zeigt zwar, daß die wahren Leistungsgrenzen wirtschaftlich bedingt sind, doch ist das im Leichtbau sinnvolle Prinzip des minimalen Gewichts kein allgemein brauchbares Kriterium für niedrige Herstellkosten oder gar für ein wirtschaftliches Bauwerk. Dazu sind die Stoff- und Verarbeitungskosten der einzelnen Baustoffe viel zu unterschiedlich. Wenn z.B. im Stahlbau $g_a/(g_p + p) \leq 0{,}5$ die wirtschaftliche Grenze wäre, müßte sie im Betonbau um ein Vielfaches höher sein. Außerdem ist der Aufwand für die Stützkonstruktionen einzubeziehen, der vom einfachen Balken über den Kragbalken und Druckbogen bis zum erdverankerten Zugbogen größer und größer wird. Die Wirtschaftlichkeit eines Bauwerks ist deshalb - wenn überhaupt - nur im Einzelfall und nur als Ganzes zutreffend zu beurteilen.

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ZUSAMMENFASSUNG

Balken, Druckbogen und Zugbogen sind die Grundformen aller zur Bewältigung von Spannweiten geeigneten Tragwerke. Die für sie im gesamten Leistungsbereich massgebenden Spannweitenfunktionen werden angegeben und die diese bestimmenden Kenngrössen untersucht und diskutiert. Nichtlinearitäten der Baustoffe und - soweit erforderlich - auch der Geometrie werden berücksichtigt. Der Einfluss unterschiedlicher Baustoffgesetze und der Vorspannung wird studiert. Die Grenzen der Wirtschaftlichkeit und ihre Kriterien werden aufgezeigt.

SUMMARY

Beams, compression arches and tension arches are the fundamentals of all structures suitable to cope with spans. The standard span functions for the whole capacity range are specified and their characteristic values examined and discussed. Nonlinearities of building materials and - as far as necessary - of the geometry are considered. The effects of different laws of building material and of prestressing are studied. Limits of economy and their criteria are shown.

RESUME

Des poutres, des arcs de compression et des arcs de traction constituent les formes fondamentales de toutes les structures franchissant une certaine portée. Les fonctions de portées déterminantes sont indiquées, leurs valeurs caractéristiques sont examinées et commentées. Des non-linéarités des matériaux de construction et - si nécessaire - de la géométrie sont considérées. L'influence de différentes lois relatives aux matériaux de construction ainsi que de la précontrainte sont étudiées. Les limites économiques et leurs critères sont donnés.

A Basic Parameter for Optimum Design of Arch and Suspension Bridges

Un paramètre fondamental pour le calcul optimal de ponts suspendus et en arc

Ein Grundparameter für die Optimierung von Bogen- und Hängebrücken

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1. Introduction

The purpose of this paper is to propose a basic parameter effective to the optimum designs of arch and suspension bridges. Since the dynamic factors (e. g., eigenvalues and eigenvectors) and the static factors (e.g., influence lines for deflection and bending moment) of an arch (or suspension) bridge are subjected to this parameter only, designated by F , we are able to determine the F value which satisfies the structural optimization of the bridge, which means that one constraint can be made for the design variables of the bridge. For the optimum design of an arch (or suspension) bridge, its geometry and the cross sectional areas of the elements such as the arch and the stiffening girder will be the design variables. These design variables are usually found by mathematical and numerical search methods. Although these search methods are applicable to a variety of problems, they require repeating similar calculation changing the values of the design variables until the optimum conditions are satisfied. So, it will save much computer cost to give the one constraint for the design variables.

There are many analogous points between a suspension bridge and an arch bridge, and they may be said to be essentially of the same type of structure from the view-point that they have girders stiffened with parabolic members (= cable and arch) respectively. So, both structures can be analyzed by a common theory (2).

In general, the cross sections of the elements such as the arch and the stiffening girder are variable. For these elements, the average values should be used as approximate values. The errors due to the approximation seem to be small judging from numerical examples.

2. Theory

In this paper, the bridges are assumed to satisfy the following conditions:
(i) The stiffening girder is of uniform cross section and simply supported at both ends.
(ii) The cross section of the arch (or cable) is constant and its mass is transferred to the stiffening girder.
(iii) The flexural rigidity of the arch can be transferred approximately to the stiffening girder.
(iv) The arch (or cable) configuration is given by a parabolic function.
(v) The arch (or cable) and stiffening girder are connected with an infinite number of hangers whose elongations are completely neglected.

When the arch and stiffening girder shown in Fig. 1 is forcibly deformed by the amount given by

$$w = \sum_n a_n \sin\frac{n\pi x}{l} \qquad (1)$$

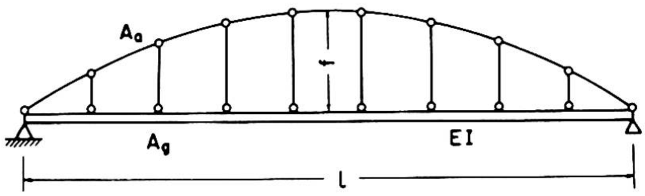


Fig. 1

where l : span, the horizontal thrust ΔH of the arch is found from the compatibility condition:

$$\Delta H = \frac{16fEB}{\pi l^2}\sum_n \frac{a_n}{n} \qquad \text{for } n = 1,3,5,... \qquad (2)$$

$$= 0 \qquad \text{for } n = 2,4,6,... \qquad (3)$$

where $$B = \frac{A_a}{\frac{A_a}{A_g} + 1 + 8\left(\frac{f}{l}\right)^2 + 19.2\left(\frac{f}{l}\right)^4} \qquad (4)$$

A_a(A_g) : cross sectional area of arch (girder). From this, we see that the arch resists symmetric deformation only and does not resist asymmetric deformation. In other words, for asymmetric deformation the arch bridge is reduced to a simple girder.

The amplitude of the simple girder loaded with a periodical uniform load $p_g \sin\omega t$ (in Fig. 2) is given by

$$w = \frac{4p_g}{\pi\rho}\sum_n \frac{1}{n(\omega_{gn}^2-\omega^2)}\sin\left(\frac{n\pi x}{l}\right) \qquad (5)$$

where

$$\omega_{gn} = \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho}}$$

(= n-th natural frequency of the girder) and ρ : mass per unit length of the girder.

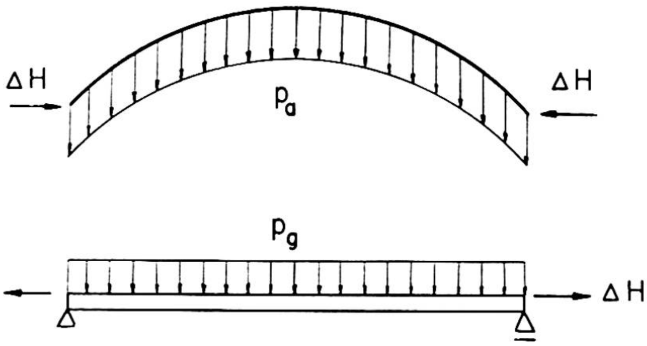


Fig. 2

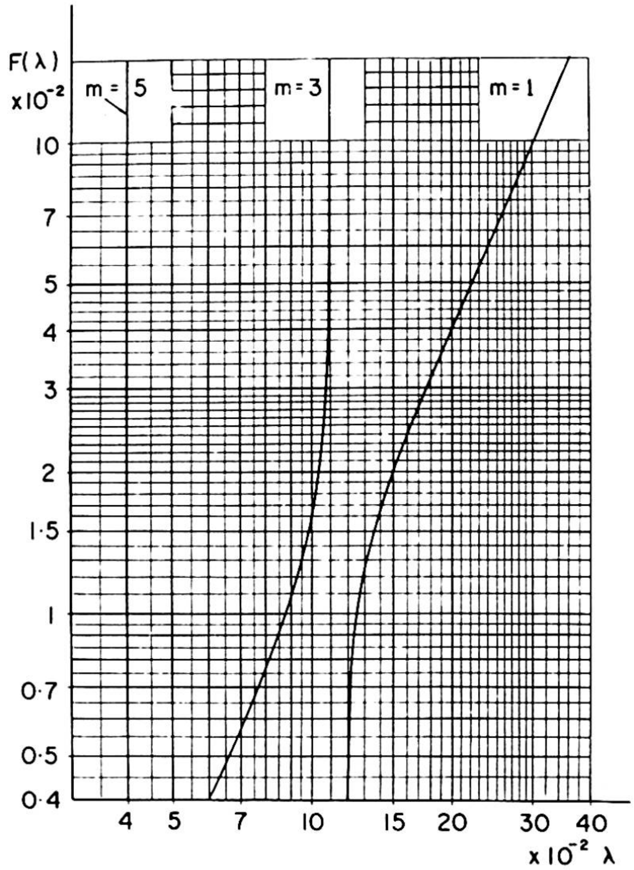
When the arch bridge is forced to vibrate at the amplitude represented by Eq. (5), the thrust ΔH caused in the arch is computed directly from Eqs. (2) and (5), i.e.,

$$\Delta H = \frac{64fEB}{\pi^2 \rho l^2} \sum_n \frac{1}{n^2(\omega_{gn}^2 - \omega^2)} p_g \tag{6}$$

When the arch is isolated from the girder, retaining its deformation, a uniform load p_a must be placed on the arch to let it satisfy the equilibrium condition of force and moment, and its magnitude is determined from,(3)

$$p_a = \frac{8f}{l^2} \Delta H = \frac{512Ef^2B}{\pi \rho l^4} \sum_n \frac{1}{n^2(\omega_{gn}^2 - \omega^2)} p_g \tag{7}$$

Fig. 3



Let us superpose the arch and girder to restore the arch bridge. The arch bridge constructed in this way is subjected to a uniform load with the magnitude

$$p_0 = p_a + p_g \tag{8}$$

Using the condition that the applied force must be zero for free vibration, i.e.,

$$p_a + p_g = 0 \tag{9}$$

we arrive at the following frequency equation:

$$1 + \frac{512Ef^2B}{\pi^2 \rho l^4} \sum \frac{1}{n^2(\omega_{gn}^2 - \omega^2)} = 0$$

$$n = 1,3,5,\ldots, \tag{10}$$

which can be expressible in the following nondimensional form,(1)

$$F(\lambda) = \frac{\pi^6 I}{512 f^2 B} = \sum_n \frac{1}{n^6(1-n^4\lambda^2)} - 1.0014 \tag{11}$$

where

$$\lambda = \frac{\omega_{g1}}{\omega}, \quad \omega_{g1} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho}} \quad \ldots (12)$$

The left hand side, i.e., F -- value is a non-dimensional value to be determined from the dimensions of the arch bridge. The relation between F and λ is shown in Fig. 3. The m-th natural mode $\phi_m(x)$ is computed by substituting the m-th natural frequency ω_m, obtained from Eq. (11), into Eq. (5). That is,

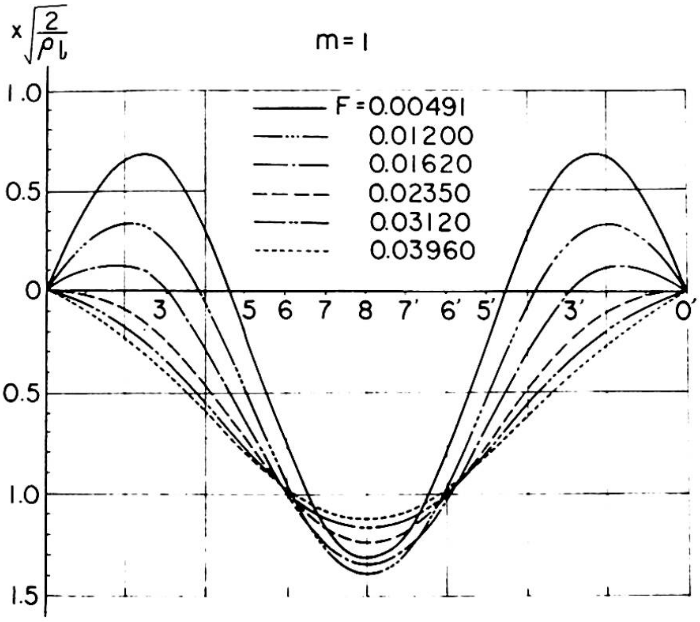


Fig. 4

$$\phi_m(x) = \sum_n b_{mn} \sin\frac{n\pi x}{l}, \quad \left(b_{mn} = \frac{1}{n(\omega_{gn}^2 - \omega_m^2)} \right) \tag{13}$$

For the normalized mode $\Phi_m(x)$, we have

$$\Phi_m(x) = C_m \sum_n b_{mn} \sin\left(\frac{n\pi x}{l}\right), \quad C_m^2 = \left(\frac{2}{\rho l}\right)\left(\sum b_{mn}^2\right)^{-1} \tag{14}$$

The first normalized mode $\Phi_{m=1}(x)$ is shown in Fig. 4 for some F-values. Once the m-th natural frequencies ω_m and the normalized modes $\Phi_m(x)$ have been found, the dynamic and static responses are easily determined.

The static deflection w_s at x due to the force P_0 applied at x_j is found from

$$w_s = \sum_m \frac{\Phi_m(x)\,\Phi_m(x_j)}{\omega_m^2} P_0 \tag{15}$$

and the bending moment M^B is calculated from

$$M^B = -EI\frac{d^2 w_s}{dx^2} \tag{16}$$

Note that these responses are subjected to the non-dimensional parameter F. For example, the influence lines for deflection at $l/4$ and $l/2$ points are shown in Figs. (5) and (6).

Fig. 5

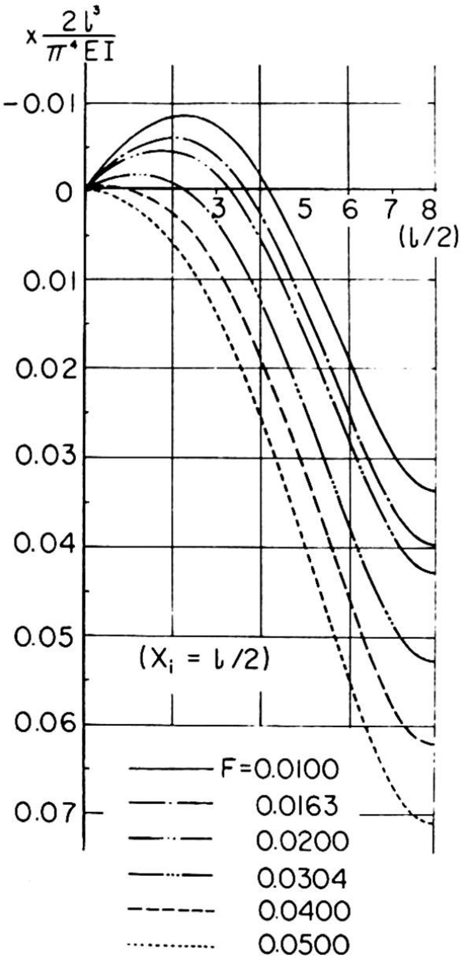


Fig. 6

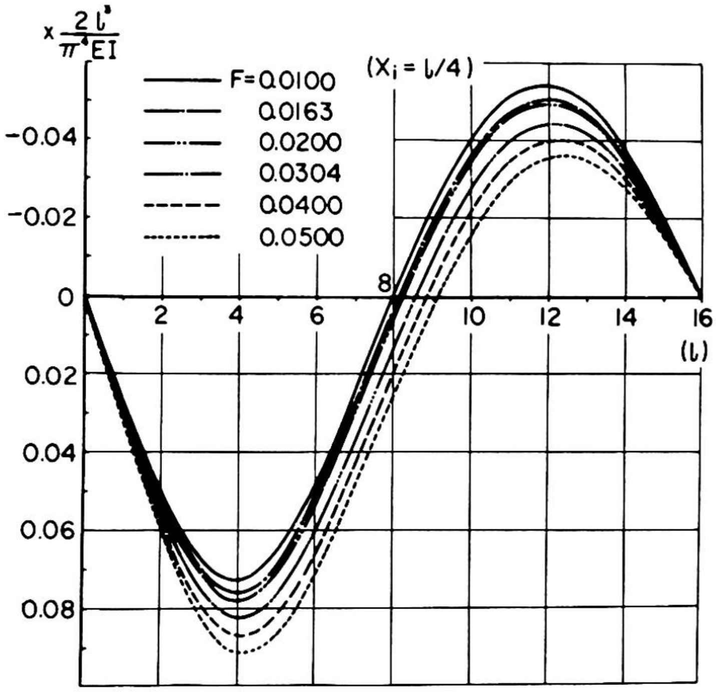
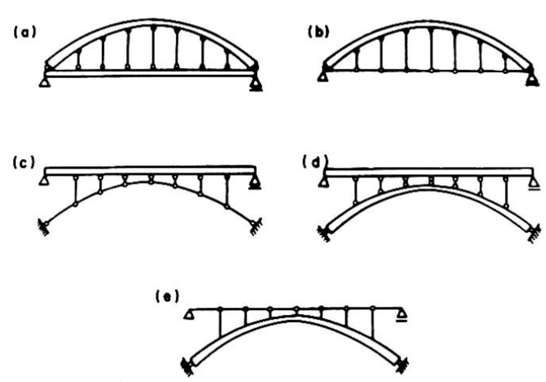
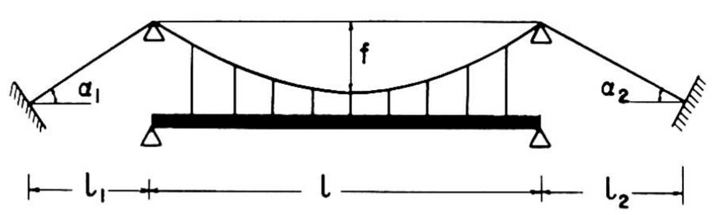


Fig. 7



The aforementioned equations can be used for the arch bridges shown in Fig. 7 by changing the cross sectional areas and flexural ridigities of arches and stiffening girders. For the system (e) in Fig. 7, the flexural ridigity I_g of the girder is zero and the cross sectional area A_g of the girder is infinity.

Fig. 8



The above equations derived for arch bridges can be applied to suspension bridges. For the suspension bridge shown in Fig. 8, the B in Eq. (4) is

$$B = \frac{A_c}{1 + 8\left(\frac{f}{l}\right)^2 + 19.2\left(\frac{f}{l}\right)^4 + \frac{l_1}{l}\sec^3\alpha_1 + \frac{l_2}{l}\sec^3\alpha_2} \tag{17}$$

where A_c : cross sectional area of the cable.

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SUMMARY

This paper proposes a basic parameter effective to the optimum design of arch and suspension bridges. The dynamic factors (for example, eigenvalue problem) and static factors (for example, stress and deformation) of these bridges are subjected to this parameter only, which means that one constraint can be made for some design variables. So, numerical calculation will easily be done on the basis of this parameter. Several diagrams are shown.

RESUME

Ce mémoire propose un paramètre fondamental qui est efficace pour le calcul optimal de ponts suspendus et en arc. Les facteurs dynamiques (par exemple le problème des valeurs principales) et les facteurs statiques (par exemple la contrainte et la déformation) de ces ponts ne dépendent que de ce paramètre. Le nombre de variables peut alors être réduit et les calculs numériques effectués facilement. Quelques diagrammes sont présentés.

ZUSAMMENFASSUNG

In dieser Mitteilung wird ein für die Optimierung von Bogen- und Hängebrücken geeigneter Grundparameter vorgeschlagen, der dynamische Faktoren (z.B. Eigenwertprobleme) und statische Faktoren (z.B. Spannung und Deformation) dieser Brücken berücksichtigen kann. Dies bedeutet, dass die Zahl der Entwurfsvariablen reduziert und die Berechnung vereinfacht werden kann. Diagramme für die praktische Anwendung werden angegeben.

Planning of Floor System at Long-Span Suspension Bridges

Conception du système de platelage pour des ponts suspendus de longue portée

Deckensysteme für weitgespannte Hängebrücken

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1. Introduction

When a long-span suspension bridge is planned, the selection of its floor system as well as its suspended structure has great influence on its safety and economy, and its erection and maintenance. When a floor system is planned at a long-span suspension bridge provided with stiffening truss girders, many kinds of floor systems can be proposed as discussed later in this paper. At the present study, structural features of various floor systems are examined and compared with one another on such condition as fabrication, erection, maintenance, economy, etc..

Through discussions the relationship of planning of the floor system with construction methods will be evaluated in detail for a design example of bridge in Japan.

2. Suspended Stiffening Structures and Floor System

In the planning of a long-span suspension bridge two type of suspended stiffening structures are considered: one is a truss type structure and another is a box girder type one. Since the former is more conventional than the latter in Japan, a truss type stiffening structure with a floor system combined with an open grating floor, as shown in Fig. 1.

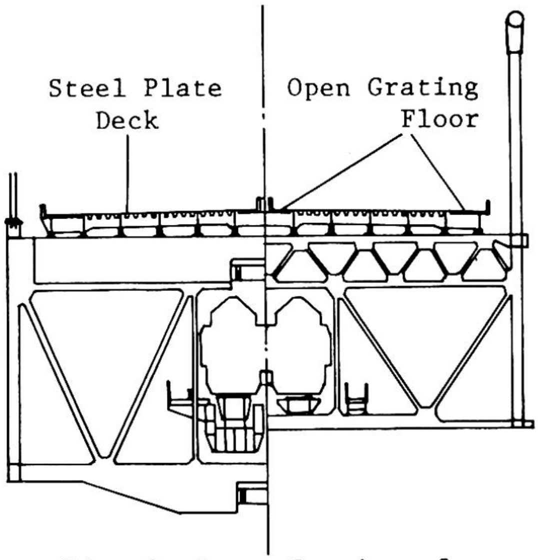

Fig. 1 Cross Section of Suspension Bridge

Many kinds of construction methods for the floor system can be proposed as discussed later in this paper. Now, the comparative study was carried out on a heavy weight floor system (closed steel grating floor) with a light weight one (steel plate deck) in steel amount and cost at their construction time, at an illustrated suspension bridge, which has a length of 1630 m consisting of a main span of 870 m and two side spans of each 380 m, and has a width of 30 m. The result of this comparision is given in the Table 1, which shows that the bridge with the light weight floor system has the advantage of the heavy weight one in steel amount and cost. Since there is an opinion that the floor system had better be heavier judged from the aerodynamic stability of a long-span suspension bridge, the relative merits for aerodynamic stability between heavy and light weight floor systems have to be discussed separately.

Table 1 Comparision for steel construction of super-structure at suspension bridge

Steel Works	Bridge with Closed Steel Grating Floor			Bridge with Steel Plate Deck		
	Weight (ton)	Unit Price (10^3 yen)	Sum of Money (10^6 yen)	Weight (ton)	Unit Price (10^3 yen)	Sum of Money (10^6 yen)
Floor System	11 420	350	3 997	11 930	400	4 772
Stiffening Structure	26 750	400	10 700	26 250	400	10 500
Cable	20 840	600	12 504	18 580	600	11 148
Tower	10 930	400	4 372	10 230	400	4 112
Anchorage	5 660	300	1 698	4 980	300	1 494
Total	75 600		33 271	61 970		32 026

3. Outline of Each Floor System

In planning of a floor system for a long-span suspension bridge, its laod-carrying capacity, durability, aerodynamic stability, deformation adaptability, easy and fast erection, easy maintenance, overall cost saving and so on, have to be examined. Several floor systems including new construction methods which have been developed by authors, will be discussed as follows:

(1) Floor system with reinforced concrete slab: A conventional reinforced concrete slab deck is considered to be generally cheapest one among various floor decks at present day in Japan. On the other hand, site works of forming and reinforcing at high elevation of a bridge are not always suitable for safe and fast erection.

(2) Floor system with closed steel grating Floor[1],[2]: This type of floor, as shown in Fig. 2, was adopted in Verrazano Narrows

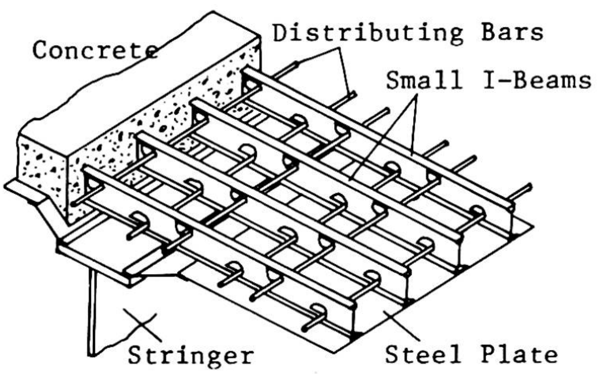

Fig. 2 Detail of Grating Floor System

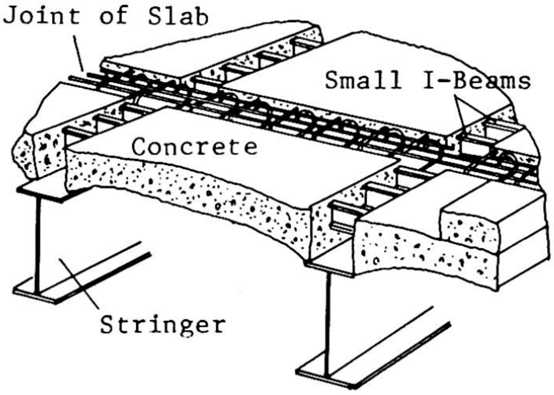

Fig. 3 Detail of Precast Concrete Steel Grating Floor

(UAS), Kanmon Bridge(Japan) and so on.

(3) Floor system with precast concrete steel grating floor: This floor is illustrated in Fig. 3, and its slab concrete is precast at a shop and after it is connected to steel stringer, concrete is cast between slab and slab, and also between slab and stringer.

(4) Floor system with prefabricated steel deck plate sandwiching concrete: This deck proposed by authors[3], consists of two steel plates and concrete sandwiched between them. These plates are connected with stud bolts, and stud shear connectors are welded to both of the plates making a steel-concrete composite deck. Photo. 1 shows shop assembly of this deck before filling up concrete. Fig. 4 and 5 show jointing methods of this deck.

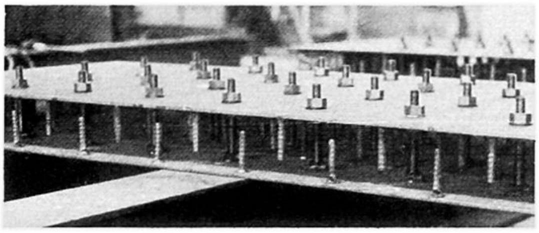

Photo. 1 Assembly of deck

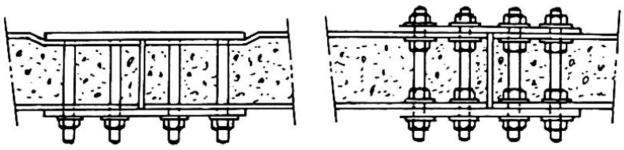

Fig. 4 Jointing of Deck Plates

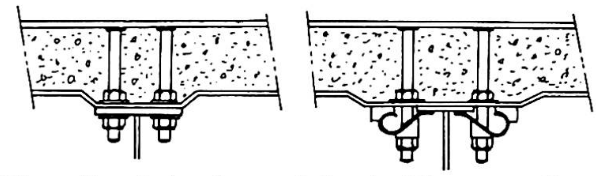

Fig. 5 Jointing of Deck Plate to Beam

(5) Floor system for prefabricated composite girder: This composite girder, proposed by the authors[4] as shown in Fig. 6, consists of an inverted steel T-beam without an upper flange and a steel grating floor frame, which is directly attached at a shop. After the prefabricated floor deck is connected to main cross beam of stiffening trusses, the slab concrete is cast at the site.

(6) Floor system with orthotropic steel plate deck: A typical steel deck panel which is well known is shown in Fig. 7.

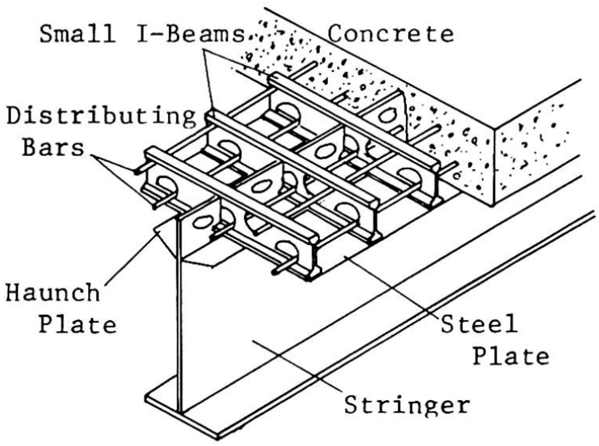

Fig. 6 Detail of Prefabricated Composite Girder

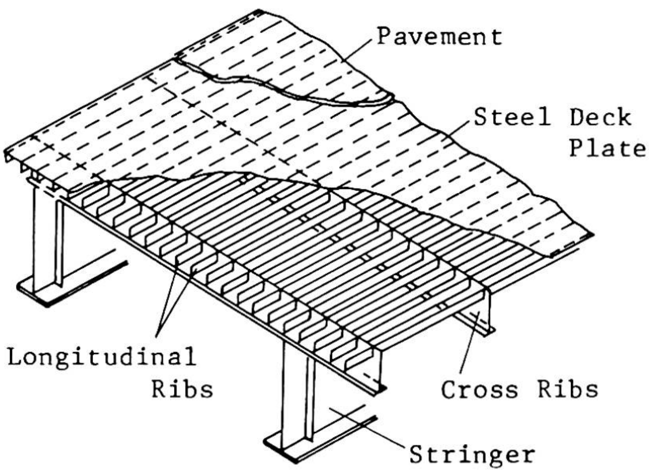

Fig. 7 Ditail of Orthotropic Steel Plate Deck

(7) Hollow steel plate deck: This deck developed by the authors has such a cross section as shown in Photo. 2, and the welded steel deck consists of two face plates and core plates which are installed diagonally as shown Photo. 2. To apply this deck to a floor system at a suspension bridge, it is set on main

cross beams of trusses directly without stringer.

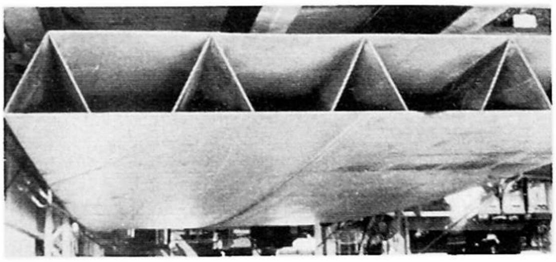

Photo. 2 Hollow Steel Plate Deck

4. Comparision of Floor Systems in Terms of Weight and Cost

In order to evaluate which floor system will be the most suitable for a long-span suspension bridge, the design of each floor system outlined above was carried out under the same design requirements that each floor system has a span length of 12 m and a width of 11 m, and carries a live load of 20 tons truck specified at the Specification for Design of Highway Bridges, Japan Road Association, 1974. As the result of the design, dimension and construction cost of each floor system were obtained, and then unit weight and unit cost per square meters of a floor area could be calculated as shown in Table 2. The value of unit weight and unit cost show that the heaviest reinforced concrete slab is cheapest in cost while the lightest steel plate deck and hollow steel plate deck are high-priced. Therefore, it might be not only very difficult, but also risky to make decision only by these two conditions, because for a long-span suspension bridge the third condition expressed in terms of a kind of function or performance of the floor system has to be examined.

5. Function Condition and Decision Matrix

As function conditions, fabrication, erection, construction time, wind-resistance, paving, maintenance and overall economy may be considered for long-span suspension bridges. Each of the function conditions are defined as follows:

(1) Fabrication condition: the nature of fabrication works to evaluate easiness or hardness of steel works at a shop and time requirement for fabrication.
(2) Erection condition: the nature of erection works to evaluate easiness or hardness of field works and safety for operation at the site.
(3) Construction time: the time nature of erection works to evaluate a construction period.
(4) Wind-resistance: the condition of resistance against wind depending upon the height of a floor system and some other requirements.
(5) Paving: the nature of paving works depending upon the smoothness floor surface.
(6) Maintenance: the nature of maintenance works to be evaluated by painting on steel surface of a floor system, etc..
(7) Overall economy: an effect of the weight of a floor system on an overall construction cost of the whole bridge, because as seen in Table 1, the weight of the floor system of a suspension bridge may have great influence on the overall construction cost of the bridge.

While the weight and cost of a floor system is deterministic and certain, these function or performance conditions are uncertain and not deterministic. Therefore, it will be reasonable to evaluate a degree of those conditions by "excellent", "good", "ordinary" and "undesirable", to which marks may be given, respectively, with 4 points, 3 points, 2 points and one point for trial. Furthermore, a so-called emphasis coefficient k, may be proposed to evaluate

Table 2 Comparision of Floor Systems

Conditions \ Floor Systems		Reinforced Concrete Floor	Closed Steel Grating Floor	Precast Concrete Steel Grating Floor	Prefabricated Steel Deck Sandwiching Concrete Floor	Prefabricated Composite Floor	Steel Plate Deck	Hollow Steel Plate Deck
Unit Weight of Floor System $\left(\frac{kg}{m^2}\right)$		530	460	490	380	470	220	220
	in Ranking	7	4	6	3	5	1	1
Unit Cost of Floor System $\left(\frac{yen}{m^2}\right)$		50 000	60 000	65 000	70 000	65 000	85 000	75 000
	in Ranking	1	2	3	5	3	7	6
Fabrication	F_1	4	3	2	1	1	1	2
	$k_1 = 2$	8	6	4	2	2	2	4
Erection	F_2	1	3	3	2	3	4	4
	$k_2 = 3$	3	9	9	6	9	12	12
Construction Time	F_3	1	2	3	3	3	4	4
	$k_3 = 3$	3	6	9	9	9	12	12
Wind-Resistance	F_4	3	3	3	3	3	3	4
	$k_4 = 2$	6	6	6	6	6	6	8
Paving	F_5	3	3	3	2	3	2	2
	$k_5 = 2$	6	6	6	4	6	4	4
Maintenance	F_6	3	2	3	2	2	2	2
	$k_6 = 2$	6	4	6	4	4	4	4
Overall Economy	F_7	1	2	2	3	2	4	4
	$k_7 = 3$	3	6	6	9	6	12	12
Total	ΣF_i	16	18	19	16	17	20	22
	$\Sigma k_i F_i$	35	43	46	40	42	52	56
Mean Value: $\Sigma F_i / 7$	in Point	2.29	2.57	2.71	2.29	2.43	2.86	3.14
	in Ranking	6	4	3	6	5	2	1
Mean Value: $\frac{\Sigma k_i F_i}{\Sigma k_i}$	in Point	2.06	2.53	2.71	2.35	2.47	3.06	3.29
	in Ranking	7	4	3	6	5	2	1

relative importance among the function condition or to emphsize relatively a specific condition. Here, the value of k is taken tentatively two or three, because it is very difficult to give deterministic numbers verified by numerical statistical data.

As shown in Table 2, each floor system depending on construction mehtods and each function condition with its emphasis coefficient will make a decision matrix and its outcome will express functional nature or performance evaluated by marks. In Table 2,

F_i = the i-th function condition with i=1 to 7,

k_i = the i-th emphasis coefficient with i=1 to 7.

The decision-making for function or performance will be made by either $\Sigma F_i /7$ or $\Sigma k_i F_i /\Sigma K_i$, where

$\Sigma F_i /7$ = a mean value for $k_i = 1$

$\Sigma k_i F_i /\Sigma k_i$ = a weight mean value.

The final decision has to be made in the overall result for weight, cost and function of each floor system, depending on the importance of these three factors because there is no common objective function among the factors for the most optimum floor system.

6. Conclusion

The following decision-making in planning will be concluded from Table 2 as an example:

(1) The most conventional reinforced concrete floor system is cheaper in construction cost, but is heavier in weight and undesirable in performance or function.

(2) Steel plate deck or hollow steel plate deck is more expensive in construction cost, but is lighter in weight and more desirable in performance or function, especially in erection and overall economy.

(3) The emphasis coefficient has to be determined more precisely, objectively by various field conditions at the site of bridge erection and subjectively by designer's judgement. With well-selected values of the emphasis coefficient, more weighted evaluation for the nature of function or performance could be made.

(4) When the suitability of a floor system cannot be judged from deterministic ranking alone based on its comparative designs, the relative evaluation of the floor system on its performance or function which is generally uncertain, will be of great help to approach to its optimum construction method.

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3] Y. Maeda and S. Matsui, Symposium on Mass-produced Steel Structures of IABSE (PRAHA), 1971.

4] Y. Maeda, T. Suruga and H. Yamada, Symposium on Mass-prodused Steel Structure of IABSE (PRAHA), 1971.

SUMMARY

The present study is intended to plan properly the floor system which will be optimum for a long-span suspension bridge with stiffening truss. Various construction methods for the floor system are examined in construction cost and weight by comparative designs, and also in its performance or function by a decision matrix.

RESUME

Le but de cette étude est de concevoir de façon optimale le système de platelage d'un pont suspendu de longue portée, dont le tablier est une poutre à treillis. Plusieurs types de platelage sont considérés, du point de vue méthode de construction, coût, poids, performances, utilisation; une matrice de décision est proposée.

ZUSAMMENFASSUNG

Zweck dieses Berichtes ist es, das Deckensystem weitgespannter Hängebrücken mit Fachwerkaussteifung zu optimalisieren. Verschiedene Deckensysteme werden vom Standpunkt der Ausführung, der Kosten, des Gewichts und der Nutzung anhand einer Entscheidungsmatrix überprüft.

IIb

Stochastic Optimization Methods in Collapse Load Analysis

Méthodes d'optimisation stochastique dans le calcul de la charge de rupture

Stochastische Optimierungsmethoden für Bruchlastberechnungen

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1. Introduction

The deterministic optimization of statically indeterminate reinforced concrete or steel structures of non-linear behaviour has been worked out in detail e.g. [1, 2, 3]. In contrast to this in the field of the stochastic frame optimization a great number of problems are left insolved.

It is well known [4] that the failure probability of statically indeterminate structures is lower than that of statically determinate ones. This is due to the fact that in the semiprobabilistic design used almost all over the world, the failure probability is associated with one critical cross section /elementary beam length/ only. In reality, the failure of a statically indeterminate structure is not characterized with the failure of one, but of several critical sections /elementary beam lengths/. Obviously, the probability of the simultaneous failure of several critical sections /elementary beam lengths/ is lower than the failure probability of one critical section /elementary beam length/ alone.

In this contribution the increase of the plastic collapse load of a given probability is investigated for statically indeterminate linear plane structures on the basis of the investigations carried out at the Hungarian Institute for Building Science [5, 6, 7].

2. The structural model

The model of the structures investigated is characterized with the following conditions:

/a/ the plane structure is formed of linear bars;

/b/ only one-parametric concentrated static loads are taken into account, with the restriction, that constant moment length cannot appear;

/c/ the influence of shear and normal forces and longitudinal deformations is neglected;

/d/ the collapse mechanism is determined by plastic hinges due to bending only;

/e/ rigid-plastic material behaviour is assumed, i.e. the rotations are concentrated in the plastic hinges and the bars between the plastic hinges are rigid;

/f/ the critical elementary bar lengths /hereinafter referred to as critical sections/ at which, in case of concentrated loads, plastic hinges can be formed are the discontinuity points of the functions or the first derivatives of the bending moments or those of the plastic moment capacities;

/g/ all the quantities influencing collapse load are assumed deterministic but the bending moment capacity is assumed random variable with infinitely divisible distribution function [8] .

As the consequence of conditions /c/ and /d/ the stability problem is not investigated.

Condition /b/ regarding the lack of constant bending moment lengths means that the position of the critical sections is deterministic. If constant bending moment lengths exist, the position of the critical sections should be a random variable and together with the moment capacity can be characterized with an extremal distribution function only.

In accordance with condition /g/ the distribution function among others could be the normal or gamma-type distribution.

3. Formulation and solution of the problem

The problem is solved by the kinematic approach of the plastic analysis to determine the smallest load factor in case of which a collapse mechanism can be formed. For the solution the so called Combinations of Mechanisms method was used in which from a set of independent elementary mechanisms the real collapse mechanism with the smallest load factor is determined from the linear combination of these elementary mechanisms. This method which is well known for the deterministic model [9, 1, 2] was developed for the stochastic model. A related economic problem was independently solved in [10] .

The problem for both models can be formulated as one of mathematical programming, where the objective function is the λ load factor

$$\lambda = \underline{\theta}^* \underline{M} \rightarrow \min \qquad /1/$$

and the constraints are the following system of linear equations

$$\underline{\theta}^* = \underline{t}^* \underline{\underline{\theta}}_f \qquad /2/$$

$$\underline{t}^* \underline{e} = 1 \qquad /3/$$

where $\underline{\theta}$ is the vector of the inelastic rotations at s critical sections;

$\underline{\underline{\theta}}_f$ is the matrix of the inelastic rotations of the set of m independent elementary mechanisms and m = s-n, where n is the degree of statical indeterminacy;

$\underline{e}$ is the vector of external work, done by loads during the formation of elementary mechanisms;

$\underline{t}$ is the vector of constants of the linear combinations forming critical collapse mechanism.

The vector of the inelastic rotations was divided according to [1, 2] as

$$\underline{\theta} = \underline{\theta}^+ - \underline{\theta}^- \qquad /4/$$

and the method was completed with the justification of the uniqueness condition for /4/ in [6,7] as

$$\underline{\theta}^+ \odot \underline{\theta}^- = \underline{0} \qquad /5/$$

where the symbol $\odot$ is the so called logical product. The justification showed for both the deterministic and the stochastic model that the uniqueness condition /5/ is always fulfilled automatically for the extrema of the objective function. Consequently, this non linear condition can be neglected and the remaining constraints are linear. The vector $\underline{t}$ can be written in the form

$$\underline{t} = \underline{t}' - \underline{t}'' \qquad /6/$$

where $\underline{t}'$ is the new variable vector which in case of subsequent $\underline{t}''$ will always be non-negative,

$\underline{t}''$ is a constant vector.

Having /4/ and /6/ the objective function can be written in the following form

$$\lambda = \underset{(2s+m)}{\underline{M}^*} \cdot \underset{(2s+m)}{\underline{x}} \rightarrow \min \qquad /7/$$

and the constraints will be replaced by the following system of linear equations

$$\underset{(2s+m,\, s+1)}{\underline{\underline{A}}} \; \underset{(2s+m)}{\underline{x}} = \underset{(s+1)}{\underline{b}} \qquad /8/$$

where $\underline{M}^* = \left[\underline{M}^{+*}, \underline{M}^{-*}, \underline{0}^*\right]$. $\underline{x}^* = \left[\underline{\Theta}^{+*}, \underline{\Theta}^{-*}, \underline{t}'^*\right]$

$$\underline{\underline{A}} = \begin{bmatrix} \underline{\underline{I}} & -\underline{\underline{I}} & -\underline{\underline{\Theta}}^*_f \\ \underline{0}^* & \underline{0}^* & \underline{e}^* \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} -\underline{\underline{\Theta}}^*_f \cdot \underline{t}'' \\ 1+\underline{e}^* \, \underline{t}'' \end{bmatrix}$$

$\underline{M}^+$ and $\underline{M}^-$ are vectors of the positive and negative plastic moment capacities at the critical sections and

$\underline{\underline{I}}$ and $-\underline{\underline{I}}$ are identity matrices of appropriate signs.

The plastic moment capacities for the deterministic model are fixed values, but for the stochastic model they are random variables of known distribution function. The combination of these plastic moment capacities results in the collapse load factor, which, consequently, will also be a random variable of the same type of distribution function.

Any point of the distribution function of the collapse load factor i.e. the collapse load of a given probability can be determined as follows.

It is well known [8] that any linear combination of random variables with infinitely divisible distribution function will be of the same type of distribution function. The mean value, the standard deviation etc. of the resulting distribution can be expressed knowing the mean values, standard deviation etc. of the initial distribution and the combination coefficients as:

$$\bar{\lambda} = (\underline{\Theta}^+)^* \underline{\bar{M}}^+ + (\underline{\Theta}^-)^* \underline{\bar{M}}^- \qquad /9/$$

$$D^2(\lambda) = \left\{\underline{(\Theta^+)^2}\right\}^* \cdot \underline{q}^+ + \left\{\underline{(\Theta^-)^2}\right\}^* \cdot \underline{q}^- \qquad /10/$$

where $\underline{q}^+$ and $\underline{q}^-$ are the variances of the respective plastic moment capacities.

Assume according to [11] that the failure probability of a structure will be $p_o=8,2.10^{-5}$. Knowing the distribution function of λ determine that value of λ_s, depending on vectors $\underline{\theta}^+$ and $\underline{\theta}^-$ for which the probability of occurence of the smallest λ will be less than the given p_o. If u_o will be the quantile p_o of the standardized distribution function, then this λ_s value will be

$$\lambda_S = D(\lambda)\, u_o + \bar{\lambda} \qquad /11/$$

Using the previous expressions the value of λ_S can be given as function of rotation vectors as

$$\lambda_S = u_o\sqrt{\underline{x}^* \underline{\underline{Q}}\, \underline{x}} + \underline{\bar{M}}^* \underline{x} \qquad /12/$$

where $\underline{\underline{Q}} = \langle \underline{q}^+, \underline{q}^- \rangle$ is a diagonal matrix, formed of vectors $\underline{q}^+$ and $\underline{q}^-$.

The minimum of this objective function, which in this way is deterministic, will be the collapse load of the given probability according to the stochastic model.

For the deterministic model the objective function is linear and for its solution the simplex method is appropriate. However, for the stochastic model, the objective function is concave as was shown in [6] . This type of problem, with linear constraint can be solved by the cutting plane method [12] well suitable for computer applications [13] .

4. Practical application of the method

The effectiveness of the more exact stochastic model was checked on some practical examples of different parameters.

The deterministic and stochastic models can be compared by prescribing similar failure probabilities for critical sections using the deterministic model $/p_i/$ and for the whole structure using the stochastic model $/p_o/$ and determining how much the load bearing capacity computed according to the deterministic model will be exceeded by the one computed according to the stochastic model.

It was proved [7] that for this condition the deterministic load bearing capacity will be a lower bound solution of the stochastic load bearing capacity. In [6, 7] two simple upper bound solutions were also given.

Simple one span, one storey frames were analysed in case of 7 loading schemes, consisting of vertical and horizontal concentrated loads. The possible distributed loads were modelled by a system consisting of an odd number of concentrated loads.

The distribution function of the plastic moment capacities of the critical sections was assumed to be of normal distribution.

The span $/l/$ to height $/h/$ ratio was assumed as $l/h=2,4,1/2$.

The assumed ratios of the plastic moment capacities of the girder $/M_l/$ and the column $/M_h/$ are shown in the Table 1.

Table 1

Plastic moment capacity type		1	2	3
M_l/M_h	+	3/2	1	3
	-	1	2/3	3/2

Signs + and - indicate moments, producing tension at the inner and outer side, respectively, of the bars. The coefficient of variation of the plastic moment capacities was assumed as r=0.015, 0.05, 0.15 and 0.25. Of course for the latter and small failure probabilities the assumed normal distribution gives a considerable error. The convergence of the solution was very slow in case of high coefficients of variations, too.

Altogether 3o frames were investigated using both the deterministic and the stochastic model.

The results of the calculation for the frame shown in Fig.1 are given in Table 2.

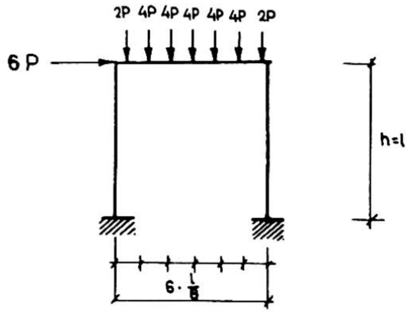

Fig.1

Table 2

parameters / number of example	l/h	plastic moment capacity type	$\frac{\lambda_{so}}{\lambda_{do}}$	$\frac{r_\lambda}{r}$	p_i
1o	2	1	1.1o4 1.o51	o.553 o.782	$1.9.10^{-2}$ $1.6.10^{-3}$
13	4	1	1.o78 1.o26	o.663 o.887	$6.1.10^{-3}$ $4.1.10^{-4}$
16	1/2	1	1.116 1.o61	o.5oo o.738	$3.o.10^{-2}$ $2.7.10^{-3}$
22	2	3	1.o82 1.o32	o.648 o.862	$7.4.10^{-3}$ $5.8.10^{-4}$

where λ_{so} and λ_{do} are the collapse load factors for the stochastic and for the deterministic model, respectively,

r_λ is the coefficient of variation of the collapse load factor for the frame,

r is the coefficient of variation of the plastic moment capacity at the critical sections,

p_i is failure probability of the plastic moment capacity at the critical sections, assuming the failure probability of the whole frame $p_o = 8{,}2 \cdot 10^{-5}$.

The two values in each box in Table 2 correspond to the lower and upper bound values after iterations consuming prefixed computer time.

5. Discussion of the results

/a/ From the results it became clear, that a substantial difference is observed between the load bearing capacity of the deterministic and the stochastic structural models. This difference is given in Table 3.

Table 3

r	0.015	0.05	0.15
$\lambda_{so} / \lambda_{do}$	2-3 %	3-12 %	22 %

/b/ The different analyses according to the deterministic and stochastic models give not only different collapse load factors, but in some cases different failure mechanisms too, as is shown in Fig.2.

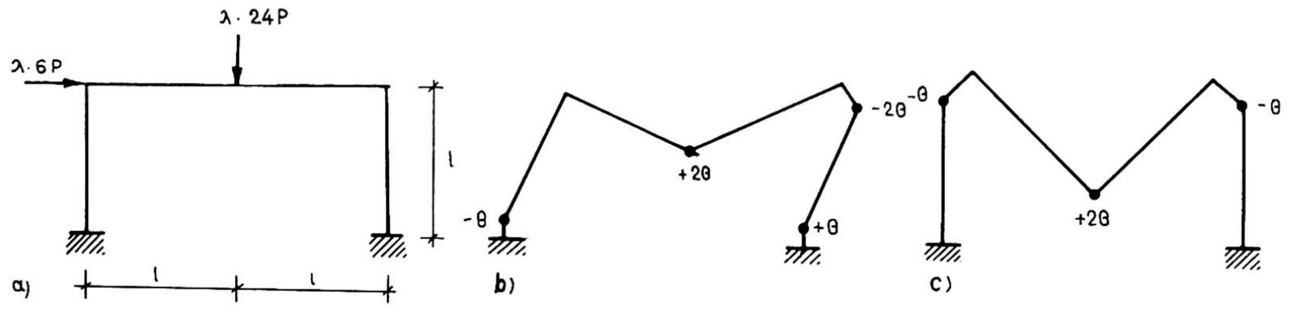

Fig.2

a - the frame scheme; b - failure mechanism according to the deterministic model; c - failure mechanism according to the stochastic model.

/c/ The coefficients of variation of the collapse load factor of the frame for the stochastic model are much lower than for the deterministic model, as can be seen in Table 2. The ratio of r_λ / r was between 0,5 and 0,78.

/d/ There is another way of comparison of the results obtained according to the two models. This is the determination of the failure probabilities of the plastic moment capacities at the critical sections p_i at a given failure probability of the whole frame $p_o = 8{,}2 \cdot 10^{-5}$ according to the stochastic model. These values of p_i in case of examples of good convergence were in the range of 10^{-3} - 2.10^{-2}, which is much higher than in case of the deterministic model, where in each critical section $p_i = 8{,}2.10^{-5}$ should be maintained.

6. Conclusions

The stochastic structural model for statically indeterminate plane structures formed from linear bars gives considerably higher load bearing capacity, lower coefficient of variation, higher failure probability in each critical section, than the deterministic structural model. In some cases the failure mechanisms can also be different for stochastic and deterministic models.

It is planned to investigate distributions more realistic than the normal one taking the elastic-plastic material behaviour and the randomness of the critical section position into account. Examples of more complicated structural schemes are planned to be analysed by applying computational methods of better convergence.

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SUMMARY

The increase of the plastic collapse load of a given probability is investigated for statically indeterminate linear plane structures, assuming the plastic moment capacities at the critical section to be random variables of infinitely divisible distribution. The Combinations of Mechanisms method was developed for the stochastic structural model. The mathematical and computational problems were solved and 30 simple frame examples were investigated. The results showed higher plastic collapse load, lower coefficient of variation and higher possible critical section failure probabilities for the stochastic model as compared to the deterministic one.

RESUME

L'augmentation de la charge plastique de rupture pour une probabilité donnée est examinée pour des systèmes de barres hyperstatiques en plan, sous la condition que les capacités de moment plastique sont des variables probables d'une distribution infiniment divisible. La "combinaison des mécanismes" est développée pour le cas du modèle stochastique. Les problèmes mathématiques et d'ordinateur sont résolus et 30 portiques simples examinés. Les résultats ont montré pour le modèle stochastique une charge de rupture plastique élevée, un moindre coefficient de variation et une plus grande probabilité de rupture possible comparé au modèle déterministique.

ZUSAMMENFASSUNG

Die Erhöhung der plastischen Bruchlast gegebener Wahrscheinlichkeit wurde bei statisch unbestimmten ebenen Stabwerken unter der Bedingung geprüft, dass die plastische Momenten-Tragfähigkeit in den kritischen Querschnitten eine unbegrenzt dividierbare Zufallsvariante ist. Die Methode der "Kombination der Mechanismen" wurde im Fall eines stochastischen Konstruktionsmodells weiterentwickelt. Mathematische und rechnungstechnische Fragen wurden gelöst und das Zahlenmaterial von 30 einfachen Rahmen geprüft. Die Ergebnisse zeigen eine höhere plastische Bruchlast, kleinere Variationskoeffiziente und grössere mögliche Wahrscheinlichkeit der Zerstörung im Falle des stochastischen Modells gegenüber dem deterministischen Modell.

IIb

Structural System Optimization Based on Suboptimizing Method of Member Elements

L'optimisation du système structural basée sur la suboptimisation d'éléments

Optimierung der Tragstrukturen auf Grund der Suboptimierungsmethode der Teilelemente

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1. INTRODUCTION

The complexity and difficulty arised in the optimization procedure of a practical structural system are caused mainly by the various characteristic and numerous design variables and constraints involved in a structural system. The methodological expansion on the treatment of such design variables and constraints has been expected for the efficient optimum design method of the structural systems. This paper presents practical optimization methods intended to solve the problems based on suboptimization of structural elements.

In the optimum design methods presented herein, suboptimization of the structural elements are performed first for the range of possible loadings and design variables, then suboptimized relationships between an intensive design variable and design constraints, objective function etc. are introduced. Using these relationships logical reductions in the number of design variables and constraints, and introduction of material selection variables may be possible. Objective function is also simplified, and geometrical and discrete variables can be treated easily. The optimum solutions are found by sequential linear programming algorithm and graphical approach. Examples of cost minimization problems of highway girders and minimum weight design of trusses are presented. Using the methods direct optimum design diagrams for highway girders have been established.

2. OPTIMUM DESIGN USING SUBOPTIMIZATION OF STRUCTURAL ELEMENTS AND SLP METHOD

2.1 Girder Problems

Problem Formation - The cost minimization problems of constant-depth highway welded plate girders are solved by SLP method using suboptimization of girder elements. The design variables are assumed as cross sectional dimensions, length, ℓ, and steel type, M, to be used for each girder segment. Design criteria imposed in the steel girder section are constraints on allowable stresses, plate thicknesses for stability of the girder and minimum rigidities of vertical and horizontal stiffeners which are taken from "Specifications for Steel Highway Bridges". (Ref. 13) Discrete constraints on commercial availability of plate thicknesses are also considered.

Total cost of the girder, TCOST, is assumed to consist of material cost, CM, fabrication cost, CFF×(1+FF), and welding cost, CWM + CWF×(1+FF), which are evaluated with reference to "Tables of Prime Costs for Steel Highway Bridges".(Ref. 14)

$$TCOST = \sum_{i=1}^{NM} COST_i \times \ell_i = \sum_{i=1}^{NM} [\ CM_i + CFF_i \times (1+FF) + CWM_i + CWF_i \times (1+FF)\ ] \times \ell_i \qquad (1)$$

in which FF = factor of indirect fabrication cost, CWM = cost for welding materials, CWF = welding cost.

Suboptimization of Girder Elements - In the girder problems, behavior variables are determined by the arrangement of moment of inertia, I, and length, ℓ, of each girder segment and usually dimensions of a girder section are determined by applied maximum bending moment. For this reason suboptimization of the girder sections are performed first for various combinations of steel types, M, web heights, WH, and bending moments, BM, by taking into account all of the design variables and constraints.

The mixed-discrete nonlinear optimization problems of the girder sections may be solved quite effectively by a modified branch and bound algorithm and SLP method, where the order to branch and bound of discrete variables is preassigned according to their importance for the design of girder section, and only two adjacent discrete values to the continuous optimum solution are examined for their optimality. Macro flow chart of the algorithm is shown in Fig. 1. The results of suboptimization of girder elements are arranged in terms of moment of inertia and I-RBM, I-COST, I-SDIM, RBM-GW relationships for each steel type and web height are introduced, where RBM = maximum resisting bending moment, COST = minimum cost per unit length, SDIM = optimum sectional dimensions, GW = girder weight per unit length. I-RBM and I-COST relationships shown in Fig. 2 may be expressed as

$$RBM(I) = a \cdot I + b, \quad COST(I) = c \cdot I + d \quad (2)$$

The coefficients a, b, c and d are all constants for the particular range of I, M and WH. Since flange plate thicknesses are increased discretely as applied bending moment increases, unit price of the steel plate and size of the fillet welding are changed also discretely and I-COST relationships are varied discontinuously at such points. On the contrary I-RBM relationships are varied linearly and may be expressed by several linear equations accurately.

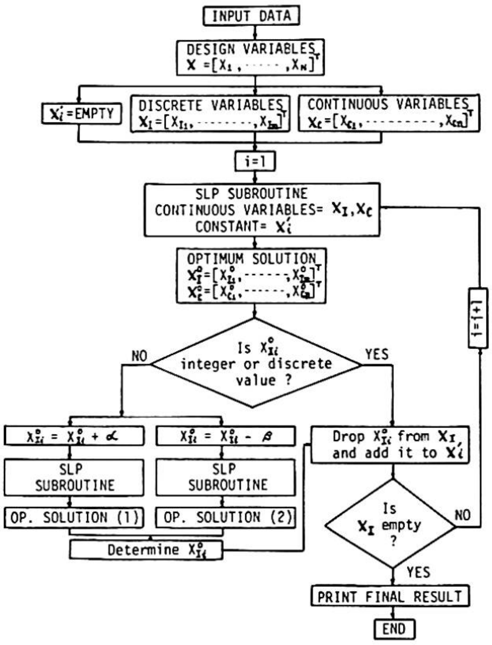

Fig. 1 Macro Flow Chart of Modified Branch and Bound Method with SLP Subroutine

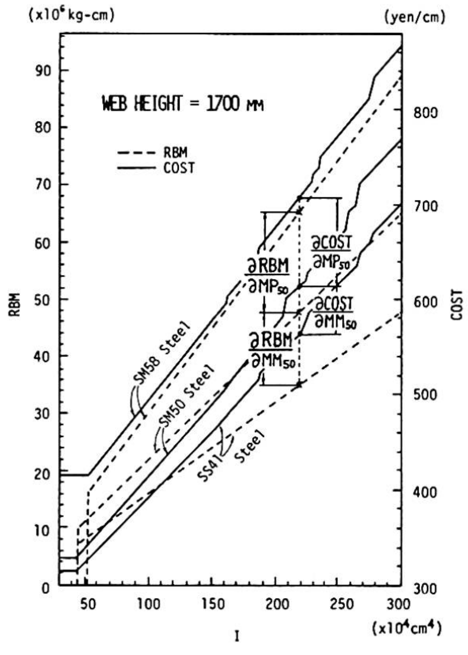

Fig. 2 I-RBM, I-COST Relationships for Girders with Web Height = 1700 mm

Simplification of Problem and Introduction of Material Selection Variables - I-RBM relationships introduced by this method express the allowable upper limit of resisting bending moments of the girder sections to satisfy all of the constraints. Minimum costs of the girder sections with I, WH and M may be evaluated directly from related I-COST relationships. Therefore by using these relationships I of each girder element may be considered as a new design variable instead of all of the sectional dimensions if web height is preassigned as a design parameter and BM ≤ RBM relationship comes to a new intensive constraint in place of all of the restrictions. This reduction in the number of design variables and constraints to be considered simultaneously gives significant advantages to solve complex structural optimization problems, such as simplification of the problem formulation and evaluation of the sensitivities, reduction of the core size and computation time, improvement of the convergency to the optimum solution. Furthermore the differences of values between two material types at a value of I in the I-RBM and I-COST relationships may be considered as the partial derivatives with respect to the design variable for selecting optimum steel type to be used for each girder element. (Fig. 2) The material selection variables M are introduced based on this concept, which consist of MP and MM. The former are provided for selection of the stronger steel type and the latter are for the weaker. MP and MM are treated as independent continuous variables same as I and ℓ.

Optimization by SLP Method - The girder is analysed by the displacement method and the behavior variables and their partial derivatives with respect to I, ℓ, M are evaluated by using the influence line analysis. Partial derivatives of RBM and COST with respect to I, ℓ, M are also evaluated from related I-RBM and I-COST relationships. The nonlinear optimum design problem is approximated with a linear programming problem by the first order terms of Taylor series expansion and an improved solution is determined by Simplex algorithm. Adaptive move limit constraints on the changes of design variables are also added to ensure the convergency to optimum solution. Since the material selection variables are assumed here as continuous variables, which are modified to the nearest discrete steel types at every iteration of analysis. If a solution comes closely to the optimum solution, all steel types are fixed as most profitable and material selection variables are eliminated from a set of the design variables. Then the problem is reanalysed until optimum solution is obtained. The optimum sectional dimensions for each girder element may be decided directly from the related I-SDIM relationships.

Examples - The method has been applied to many cases of simple span, 2 ~ 3-span continuous constant-depth highway girders and three examples are presented in Table 1 in which the solutions are compared with the results by graphical approach described later. In Table 1 BW = bridge width, SL = span length, WH = web height, P_ℓ = a concentrated live load, q_ℓ = uniformly distributed live load, q_d = distributed dead load which differs with each girder segment, but averaged value in the girder is shown in the table.

Approximate convergency to the optimum solution including material selection is quite well by using move limit constraints, but computation time and number of iteration of reanalysis required for the optimum solution are increased so much as number of design variables and constraints increases as seen in Table 1. Comparisons of several solutions with different initial inputs should be made for confirmation of the global optimum solution.

Table 1 Optimum Solutions by SLP Method and Graphical Method

	Seg. No.	SLP Method L (cm)	SLP Method I (cm⁴)	M*	Graphical Method L (cm)	Graphical Method I (cm⁴)	M*	Design Condition
1 - SPAN**	1	296.7	1389224	41	293.7	1376687	41	BW= 8.00 m
	2	701.2	1507252	58	710.0	1520473	58	SL= 30.0 m
	3	1500.0	2113595	58	1500.0	2113532	58	WH= 200 cm
	Min.TCOST		1643675(YEN)			1643622 (YEN)		P_ℓ= 17.990 t
	CPU TIME		150~200(sec)			10 ~ 16 (sec)		q_ℓ= 1.259 t/m
	No.of Iter.		15 ~ 20			3 ~ 5		q_d= 2.310 t/m
2-SPAN***	1	323.1	779103	50	285.0	697606	50	BW= 8.00 m
	2	725.7	1430407	50	618.8	1288148	50	SL= 30.0 m
	3	1997.0	1695642	50	1962.0	1712473	50	WH= 170 cm
	4	2683.0	1085665	50	2702.2	1136303	50	p_ℓ= 17.955 t/m
	5	3000.0(s)	1429441	58	3000.0(s)	1431270	58	q_ℓ= 1.257 t/m
	Min.TCOST		2891515(YEN)			2893060 (YEN)		q_d= 2.300 t/m
	CPU TIME		60~100(sec)			3 ~ 4 (sec)		
	No.of Iter.		20 ~ 25			5 ~ 8		
3 - SPAN***	1	248.8	888658	41	233.8	846592	41	BW= 8.00 m
	2	559.9	1250520	50	546.5	1238921	50	BL= 90.0 m
	3	1850.0	1700003	50	1805.6	1723537	50	WH= 190 cm
	4	2486.0	1217784	50	2501.5	1217830	50	Span Ratio =
	5	2812.5(s)	2180159	50	2812.5(s)	2128666	50	1 : 1.2 : 1
	6	3153.0	2180159	50	3153.5	2128666	50	$p_{\ell 1}$= 18.042 t
	7	3841.0	1159787	50	3898.8	1112415	50	$p_{\ell 2}$= 17.747 t
	8	4500.0	1486529	50	4500.0	1529952	50	$q_{\ell 1}$= 1.263 t/m
	Min.TCOST		4241036(YEN)			4224079 (YEN)		$q_{\ell 2}$= 1.242 t/m
	CPU TIME		300~450(sec)			10 ~ 15 (sec)		q_{d1}= 2.030 t/m
	No.of Iter.		25 ~ 35			8 ~ 12		q_{d2}= 2.031 t/m

** Calculated by FACOM 230-28
*** Calculated by HITAC 8800/8700
(s) indicats intermediate support

* 41 = SS41 (JIS) Steel
50 = SM50 (JIS) Steel
58 = SM58 (JIS) Steel

2.2 TRUSS PROBLEMS

The truss problems are solved as weight minimization problems and cross sectional dimensions of the member and coordinates of the panel points are considered as design variables. The steel is fixed as SS41 (JIS) only.

Suboptimization of member elements - In the truss problems, suboptimization of the compression and tension members for many combinations of applied loads and member lengths are treated first, then sectional area A - maximum allowable stress σ_a, A - optimum sectional dimensions, SDIM, relationships for various member lengths are introduced. A-σ_a Relationships at any member length may closely be approximated as

$$\sigma a = \{a \cdot (A-b)\}^{1/n} + C \qquad \text{or} \qquad \sigma a = d \cdot A + e \tag{3}$$

in which a, b, c, d, e and n are all constants related to the member length and A. A-σ_a relationships express the allowable upper limits of the stresses of members with A which are guaranteed to satisfy all of the constraints prescribed to the member design.

By using A-σ_a relationships all design variables and constraints imposed in the member design can be replaced only by A and $\sigma \leq \sigma_a$ relationships respectively, moreover the derivatives of σ_a with respect to the geometry variables X_g can be evaluated simply as

$$\frac{\partial \sigma_{ai}}{\partial X_{gj}} = \frac{\Delta \sigma_{ai}}{\Delta X_{gj}} \tag{4}$$

in which $\Delta\sigma_{ai}$ represents the change in σ_a at i-th member due to the change in i-th member length. The problem is approximated as a linear programming problem and reanalyzed until optimum solution is obtained.

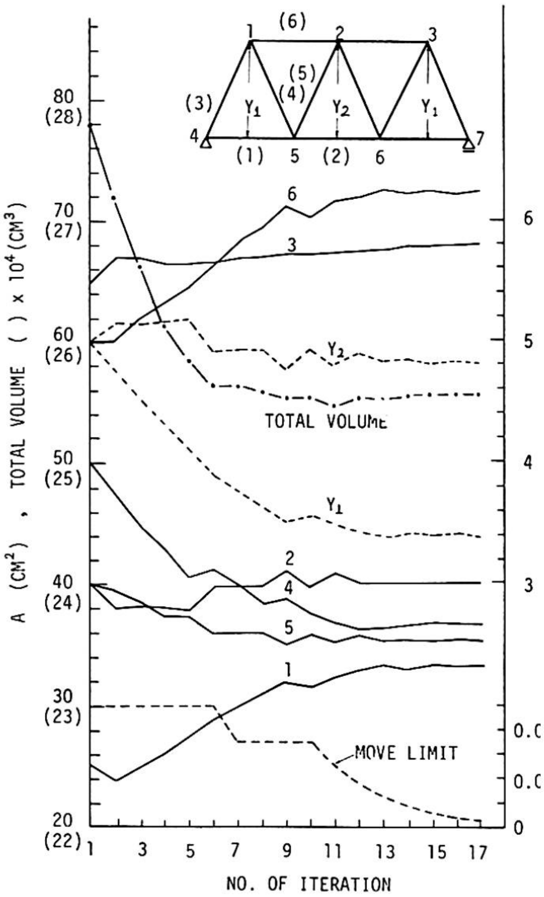

Fig. 3 11-Bar Truss, Moving Loads, $\sigma \leq \sigma_a$ Constraints

<u>Examples</u> - An example of eleven bar truss subjected to the moving loads P=50 ton, q_ℓ=4 ton/m, and the dead load q_d=2 ton/m is shown in Fig. 3. The panel length is fixed as 5 m. Sectional areas of member 1 to 6 and coordinates of panel point 1 and 2, Y_1, Y_2, are assumed as the design variables and only $\sigma \leq \sigma_a$ constraints of the members are taken into account. The initial Y_1 and Y_2 are assumed as 500 cm, however they are reduced finally to 340 cm and 483 cm respectively. Furthermore, members 1, 2, 3, 4 and 6 are fully stressed, while sectional area of member 5 is determined by the maximum slenderness ratio requirement. The minimum total volume obtained is 25.56 × 10^4cm^3 and maximum live loads displacement is 1.17 cm at panel point 5.

In the case maximum live loads displacements of the panel points are limited to 1.0 cm, the optimum solution is found such that the sectional areas A_i are 29.58, 45.45, 66.71, 45.13, 39.30, 62.90(cm^2) respectively and Y_1 = 428 cm, Y_2 = 549 cm with the total volume 27.53 × 10^4cm^3. The total volume increases 7.7% more than previous solution and only member 3 and 6 are fully stressed.

<u>Topological Member Arrangement</u> - If the constraints on lower limits of member sections are not imposed, sectional areas of unnecessary members come to 0 cm^2. Then optimum topological member arrangement of truss may also be determined. Several simple examples on this problem are shown in Ref. 2).

3. GRAPHICAL OPTIMIZATION OF HIGHWAY GIRDERS BASED ON SUBOPTIMIZATION OF GIRDER ELEMENTS

SLP method has been used successfully on a wide range of large and complex structural optimization problems, however in the optimization procedures partial derivatives of the behavior variables and objective function with respect to the design variables should be evaluated at every iteration of reanalysis. Therefore as depicted in the previous girder examples computation time is so much increased as number of design variables increases and more efficient methods to solve the large optimization problems are expected. Graphical optimization method, an approximate approach based on suboptimization of girder elements, has been developed for solving such problems and applied to the cost minimization girder problems.

<u>Design Procedure by Graphical Method</u> - In the graphical approach, a minimum cost diagram related to the initial girder arrangement is developed first by using maximum bending moment diagram of the girder and I-RBM, I-COST relationships. Then improvement of I, ℓ and M of each girder segment is performed by investigation of the change in minimum cost at the adjoining two segments due to a change of segment length, $\Delta\ell$. In case of Fig. 4, the change of minimum cost

of the girder, $\Delta TCOST_i$, due to a change of $\Delta \ell_i$ can be evaluated as

$$\Delta TCOST_i = \Delta COST_i \cdot \ell_i - \Delta COST_{i+1} \cdot \Delta \ell_i \quad (5)$$

If $\Delta TCOST_i$ is positive, $\Delta \ell_i$ may proceed to $-\Delta \ell_i$ direction. The improvement due to $\Delta \ell_i$ may be finished when $\Delta TCOST_i$ converges to zero them next improvement on ℓ_{i+1} is performed. After the improvement of all segments is accomplished, the girder is reanalyzed with new I, ℓ and M and the procedure is repeated until a converged solution will be obtained. Three highway girder examples are given in Table 1.

In this approach, attention is paid only to the change of objective function in order to improve the design variables of a girder segment, and effects to the over all behavior variables caused by changes of the design variables are evaluated by reanalysis of the girder. In this sense graphical method is more approximate approach than SLP method, but convergency to the global optimum solution by this method is quite well as seen in Table 1. Computation times required for optimum solution are reduced notably as 3 ~ 5 sec. and 10 ~ 15 sec. on HITAC 8800/8700 for 2 and 3-span continuous girder problems respectively, which are 1/12 ~ 1/30 cpu. time compared with SLP method. Larger reduction in cpu. time is made as number of variables and constraints increases.

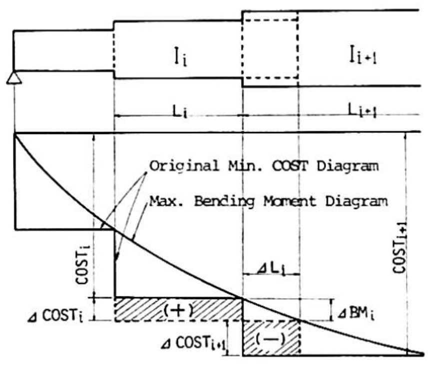

Fig. 4 The Change of TCOST due to ΔL_i

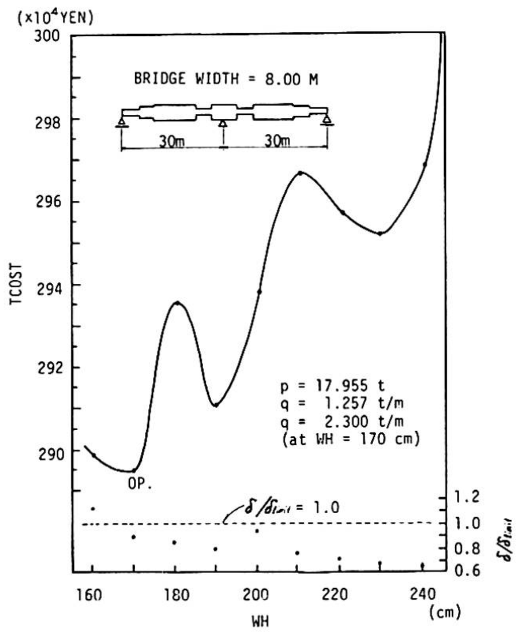

Fig. 5 WH-TCOST, δ/δ_{Limit} Relationships for 2-Span Continuous Girder (SL = 30 m, BW = 8.00 m)

Optimum Web Height – To decide the optimum web height at each span length, optimum solutions for several web heights should be compared with each other. Fig. 5 shows an example for 2-span continuous girder with span length 30 m. As seen clearly in the figure, several local minimum solutions exist on web heights and the girder with WH=170 cm gives absolutely minimum cost in this example. For this reason, web height should be treated as a parametric variable in cost minimization highway girder problems.

Optimum Design Diagrams for Highway Girder Bridges – For the purpose of direct optimum design or planning of 1~ 3-span constant-depth highway welded plate girders, various optimum design diagrams and tables such as span length – minimum total cost, optimum WH, I, ℓ, M, and I – SDIM relationships for the girders with nonuniform cross sections, and bending moment – minimum cost, optimum WH, I, M, GW diagrams for the girders with uniform cross sections have been established by using the graphical method, and they will be published soon.[6,7,8]

The optimum design diagrams mentioned above may be utilized as one of the suboptimized structural size design programs in a general purpose system program for highway bridges.

4. CONCLUSIONS

Practical structural optimization methods based on suboptimization of structural elements, SLP and graphical method are presented.

An element size optimization for minimum cost is formulated as a mixed-discrete nonlinear programming problem, and a modified branch and bound algorithm with SLP can be solved the problem effectively. Cpu. time was 1.0 sec. on FACOM 230-75 required for an optimum solution of the girder section.

By using the relationships obtained from suboptimization of structural elements, structural optimization problems may be simplified and be solved effectively. Moreover material selection variables and graphical optimization algorithm have been developed on the basis of this design concepts.

SLP method may be utilized successfully on a wide range of large and complex structural optimization problems and its approximate convergency to the optimum solution is quite well, however computation time and number of iteration of re-analysis increases so much as design variables and constraints increases.

Graphical optimization method is a practical and efficient design method for the cost minimization problems of highway girders. Formation of the computer program is simple, and excellent convergency to the global optimum solution and existence of several local minima on web height have been confirmed. Design diagrams prepared for direct cost minimum design or planning of highway girders have been established by this method. The design diagrams may be utilized as one of the suboptimized structural size design data in a general purpose system program for highway bridges.

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SUMMARY - The optimum design concepts based on suboptimization of structural elements are presented. Large scale and complex structural cost minimization problems may be simplified, and treatments of various types of design variables and constraints such as sizing, material selection, geometry, continuous, discrete come to ease by this concept . SLP method and graphical optimization method are used effectively to find the minimum cost solutions of highway girder and truss examples.

RESUME - Les concepts de l'optimisation basés sur la suboptimisation d'éléments structuraux sont présentés. Cette suboptimisation permet de simplifier des problèmes de minimisation de coût de structures complexes de grande dimension; elle facilite le traitement de variables de projet, de contraintes de types variés telles que dimensionnement, sélection de matériaux, géométrie, continu, discret,... La méthode "SLP" et la méthode d'optimisation graphique s'emploient pour trouver efficacement des solutions permettant de construire, au coût minimum, des ponts et des charpentes.

ZUSAMMENFASSUNG - Das Konzept des optimierten Entwurfes aufgrund der Suboptimierung struktureller Elemente wird dargestellt. Durch dieses Konzept lassen sich die Probleme der Kostenminimierung vereinfachen sowie die Behandlung verschiedener Arten von Entwurfsvariablen und Randbedingungen, wie z.B. Abmessungen, Materialwahl, Geometrie, stetige und unstetige Formen, überdies erleichtern. Die SLP-Methode und die Methode graphischer Optimierung werden verwendet, um die effektiven Minimalkosten eines Brückenträgers une eines Fachwerks zu erhalten.

Optimierungsprobleme beim Projektieren von Stahlbetonbrücken

Optimization Problems in the Design of Concrete Bridges

Problèmes d'optimisation dans les projets de ponts en béton armé

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Die Aufgabe der Automatisierung des Projektierungsablaufs im Stahlbetonbrückenbau kann als Aufgabe der mathematischen Programmierung betrachtet werden. Es soll der Vektor (eingeordneter Brückenparameter-Satz) ermittelt werden, der dem gegebenen System von Einschränkungen entspreche und eine Funktion des Zweckes minimisierte.

Der Optimisierungsvorgang umfasst den Projektierungsablauf die Varianteneinschätzung und die Auswahl von optimalen Lösungen.

Eine der wichtigsten und aufwendigsten Stufen, die den grössten Teil der Maschinenzeit in Anspruch nimmt ist die Berechnung unter Berücksichtigung der Raumwirkung der Konstruktion, der Einflüsse der plastischen Verformungen, der dynamischen Einwirkungen der Belastungen.

Da bei der Auswahl der optimalen Lösungen eine grosse Anzahl von Varianten zu untersuchen und zu analysieren war, waren ausführliche (aufwendige) Berechnungsverfahren unter Anwendung von EDV auch in der Stufe des Skizzenprojektierens schwer zu verwirklichen sind. Man muss wenig aufwendige Berechnungsverfahren mit genügender Genauigkeit schaffen.

Allgemeine Verfahren für vereinfachte Berechnungen, die auf grobannähernder Idealisierung des Berechnungsschemas gegründet sind, führen meist zu wesentlichen Fehlern, was mit sich irrationelle Verteilung des Materials in der Konstruktion bringt.

Es wird eine prinzipiell neue Auffassung der Ausarbaitung neuer vereifachten. Berechnungsverfahren empfohlen, welches auf der mathematischen Verarbeitung des gewonnenen Resultats von den in den EDV durchgeführten strengen räumlichen Berechnungen basiert [1] .

Gegenwärtig sind Algorithmus und Programm (SPIKA) für einen vollen Zyklus der räumlichen Berechnung der Plattenbalkenkonstruktionen ausgearbeitet, die die konstruktion Einflussflächen für verschiedene Spannungen und Verschiebungen, ihre Belastungen an den ungünstigsten Stellen, die Ermittlung des Extremums der rechnerischen und massgebenden Werte für Spannungen und Verschiebungen einschliessen.

Das Programm SPIKA für räumliche Berechnung von Plattenkonstruktionen ist mehrmals beim Projektieren von Brücken und anderen Bauwerken verwendet.

Indem man umfangreiche bei der räumlichen Berechnung dev Brückenüberbauten gewonnene Ergebnisse ausnutzt, kann man einfache mathematiche Modelle zusammenstellen, welche auch Abhängigkeiten zwischen Form, Anordnung, Grosse der Bauteile und verformtem – gespanntem Zustand der Konstruktion unter ständiger, ungünstiger Verkehrslas sowie anderen rechnerischen Belastungen widerspiegelt. Zur Herstellung solcher mathematischen Modelle ist die Anlage der Regressionsanalyse verwendet.

Die Verfahren der Regressionsanalyse sind auf der Aufwendung einer grossen Anzahl von gespeicherten statischen Angaben begründet, die aus Versuch, langzeitiger Beobachtung des Verhaltens der tatsächlichen Konstruktion oder aus übrigen Quellen erhalten sind. In gegebenen Falle ist die Information als Ergebnis mehrmals durchgeführter räumlicher Berechnungen gespeichert.

Das mathematische Modell des räumlichen Verhaltens der Konstruktion vom vorgegebenen Schema stellt eine Formel dar, wo die gesuchte Extremspannung oder - verschiebung als von den geometrischen Hauptparametern der Konstruktion und von den physisch - mechanischen Eigenschaften des Materials und der Belastung abhängige Funktion dargestellt ist.

Die Extremspannung oder - verschiebung in einem Bauteil der Plattenbalkenkonstruktion einer frei gelagerter Brücke kann als Funktion

$$P = f(\ell, G, B_i, D_i, K, M, H, C_B, x, y) \qquad (1)$$

ausgedrückt werden;

wobei:

ℓ - Spannweite,

G - Durchfahrtsprofil,

B_i - geometrische Parameter der Träger (i = 1, 2, ..., k),

D_i - geometrische Parameter von Platten,

K - Anzahl von Trägern,

M - physisch-mechanische Kennwerte vom Material,

H - Belastungsangaben,

C_B - Information über Anordnung des Brückenüberbaues,

x, y - Koordinaten des Überbauguerschnitts.

Die Formeln wie (1) lassen den Einfluss von mehreren Parametern auf den gespannten-verformten Zustand der Konstruktion analysieren. Praktisch ist es zweckmässiger für gestellte Aufgaben nur einen Teil von Parametern der Funktion (1) zu berücksichtigen, die anderen werden festgestellt.

Bei der Konstruktion der mathematischen Modelle sind für die EDV bestimmte Programme der Regressionsanalyse verwendet. Mit diesen Programmen kann man ein polynomiales Modell gegebenen Grades Zusammenstellen:

$$P = \beta_0 + \sum_{1 \leq i \leq n} \beta_i x_i + \sum_{1 \leq i \leq j \leq n} \beta_{ij} x_i x_j + \ldots,$$

wobei

β_i – unbekannte Faktoren,

x_i – zuberücksichtigende Parameter,

n – Anzahl von Parameter,

Werden wir die einfachsten Beispiele für Konstruktion der Verhältnisse wie (1) betrachten.

1. Der frei gelagerte Überbau ohne Querscheiben von Autobahnbrücken aus Stahlbeton mit gleichen Trägern.

Beim angegebenen Durchfahrtsprofil kann das rechnerische Biegemoment von der Verkehrslast in Hauptträgern des Überbaues mit der Formel

$$M_B = A_1 + \frac{l}{K}\left(A_2 + A_3 l + A_4 t\right) \qquad (2)$$

ermittelt,

wobei

M_B – rechnerischer Extrembiegemoment von der Verkehrslast (es werden Lasten HK-80, H-30 und Träger für Fussgängerstege unter Berücksichtigung des Überlastungsfaktors und des dynamischen Faktors betrachtet),

l – Spannweite,

K – Anzahl von Hauptträgern,

t – Länge der Fussgängerauskragungen,

A_i – unbekannte Koeffizienten.

2. Der freigelagerte Plattenüberbau.

Beim angegebenen Durchfahrtsprofil können Biegemomente mit den Formeln:

$$M_x = (B_1 + B_2 \ell)\ell + B_3 q_c \ell^2 + (B_4 q_n + B_5 P_t)\ell^2,$$
$$M_y = (C_1 + C_2 \ell)\ell + C_3 q_c \ell^2 + (C_4 q_n + C_5 P_t)\ell \qquad (3)$$

ermittelt,

wobei

M_x, M_y – rechnerische extremale Quer – und Längsbiegemomente;

q_c – Eigengewicht,

q_n – Belastung aus Fahrbahndecke,

P_t – Gewicht der Fussgängerstege,

B_i, C_i – unbekannte Koeffizienten,

A_i, B_i, C_i – Koeffizienten sind mittels mathematischen Bearbeitung der gewonnenen Ergebnisse der räumlichen Berechnung für verschiedene Durchfahrtsprofile gewonnen. Analogisch sind auch Abhängigkeiten zur Ermittlung von anderen Arten der Spannungen und Verschiebungen erhalten.

Die Genauigkeit der mittels Regressionsanalyse gewonnenen Formeln hängt wesentlich vom Umfang der gespeicherten Information ab. Daraufhin, sind Resultate aller nach Programm SPIKA durchgeführten räumlichen Berechnungen im langzeitlichen Speicher von EDV für nachfolgende mathematische Verarbeitung gesammelt.

Die Formeln wie (2,3) finden ihren Einsatz in der Anfangsstufe des Projektierens, wenn alle Varianten untersucht werden, alle Kombinationen und Ausmasse von Konstruktionsbauteilen vorgesehen werden und mehrmalige wiederholte Berechnungen nötig sind.

Die Anwendung solcher Formeln beim optimalen Projektieren von Brückenüberbauten lässt stark die Wirksamkeit des Suchens nach optimalen Lösungen steigern.

1. Ulizkij B.J., Potapkin A.A., Rudenko W.I., Ssacharowa I.D., Jegoruschkin J.M. "Räumliche Brückenberechnungen (unter Anwendung von EDV)." M. Verkehrsverlag, 1967.

ZUSAMMENFASSUNG

Es werden einige Optimierungsprobleme beim Projektieren von Stahlbetonbrücken mitgeteilt, die auf einer neuen Auffassung der Ermittlung des Spannungs- und Formänderungszustandes der Brückenkonstruktion basieren. Dabei erzielt man reduzierten Berechnungsaufwand und erhöhte Wirksamkeit beim Suchen nach optimalen Lösungen.

SUMMARY

Some optimization problems in the design of concrete bridges are solved with a new approach for predicting stress-strain state of bridges. This method reduces to a considerable degree time consuming calculations and increases the efficiency of search of optimal solution.

RESUME

Quelques problèmes d'optimisation sont résolus grâce à une nouvelle conception de l'état contraintes-déformations des ponts. Cette méthode permet une diminution importante du temps de calcul et une augmentation d'efficacité de l'optimisation.

II c

Exemples de calculs d'optimisation à l'aide de l'ordinateur

Beispiele des Computer-Einsatzes bei der Optimierung

Examples of Computer-aided optimal Design of Structures

Structural Optimization through Sensitivity Coefficients

Optimisation des structures au moyen des coefficients de sensitivité

Optimierung der Tragwerke mittels Sensitivitätskoeffizienten

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1. INTRODUCTION

There are basically two approaches to the solution of a structural optimization problem. In one approach which is followed by Schmit [1] and by Schmit and Fox [2], both kinds of variables namely the design variables and the behaviour variables are treated as unknowns in the programming problem. In the other approach which is followed by Romstadt and Wang [3] and by Vanderplaats and Moses [4] the solution procedure consists of a series of analysis-programming cycles. In each programming stage the most recent set of behaviour variables is treated as known and the design variables are treated as unknowns. The advantage of the first approach is that the programming problem is to be solved only once. The size of the programming problem depends on the number of nodes, the number of members and the number of load conditions. Generally for any practical case the size of the problem becomes almost unmanageable. In the second approach though the programming problem has to be solved many times the size of the problem is smaller. Hence the second approach is preferred to the first.

In the present paper the second approach is followed but the solution procedure consists mostly of solving a series of programming problems. It is observed that in the conventional approach (e.g. that followed by Romstadt and Wang [3]) the time required for the solution of a problem increases rapidly with the increase in the statical indeterminancy of the structure. In such cases the proposed approach is more economical.

2. THEORETICAL ANALYSIS

2.1 Formulation and solution

Formulation of the structural optimization problem as a programming problem has been very well brought out by several authors such as Vanderplaats and Moses [4] or by Brown and Ang [5]. It is therefore assumed here that a structural optimization problem can be formulated as the following non-linear programming problem.

$$\left.\begin{array}{l}\text{Minimize } F(X)\\ \text{Subject to } G_j(X,Y) \leq 0 \quad j=1,\ldots,m\end{array}\right\} \quad (1)$$

where X is a vector of design variables and Y is a vector of behaviour variables. The objective function can be any function which can be expressed as a function of the design variables. Usually weight of the structure is treated as the objective function. The constraints can be any inequalities which have to be satisfied by the structure such as stress limitations, size limitations or the deflection limitations. In this formulation no distinction is made between the constraints and the restraints.

The optimization process (see fig. 1) is started with an initial set of design variables, X_o. The analysis of the structure (by stiffness matrix method) is carried out to give the associated set of behaviour variables Y_o. After this analysis is over it is found out what is the change in each behaviour variable due to a 100 per cent change in each of the design variables. This information is stored in a matrix called sensitivity matrix which is denoted by CH. A general element CH_{ij} of this matrix stands for the change in i^{th} behaviour variable due to a unit change in j^{th} design variable. The structural optimization problem is then formulated in terms of X_o and Y_o to yield new set of design variables $\bar{X}_o$ (see block A). The corresponding set of behaviour variables is now found from the matrix equation

$$Y_1 = Y_o + CHX(\bar{X}_o - X_o) \qquad \text{(see block B)}$$

The next programming cycle then makes use of this vector Y_1 to yield new solution of the design variables $\bar{X}_1$. The process thus continues till the difference between the design vectors obtained from two successive programming cycles is found to be smaller than a predetermined vector ϵ. It is clear that the sensitivity coefficients calculated for the initial design will not be useful if the structure is statically indeterminate to a high degree and the original design has undergone a lot of change. In that case the sensitivity coefficients are recalculated (see block C).

The optimization of the structure is thus consisting of mostly the solution of a series of programming problems.

2.2 Calculation of the Sensitivity Coefficients

With the initial set of design variables the stiffness matrix of the structure is assembled in half-band form. If one design parameter is changed by 100 per cent the new stiffness matrix is obtained by recomputing the element stiffness matrix only for one member and then making the appropriate changes in the overall stiffness matrix. Knowing the original set of displacements, the external forces and the new stiffness matrix; new displacements are computed using Jacobi iteration [6]. With new nodal displacements known the new set of behaviour variables and hence the change in each of them due to 100 per cent change in one design variable is computed. These changes when divided by the original value of the design variables give one column of the sensitivity matrix. Before computing the next column the stiffness matrix is reduced to the original matrix.

2.3 Solution of the Programming Problem

The programming problem stated in (1) is solved by using

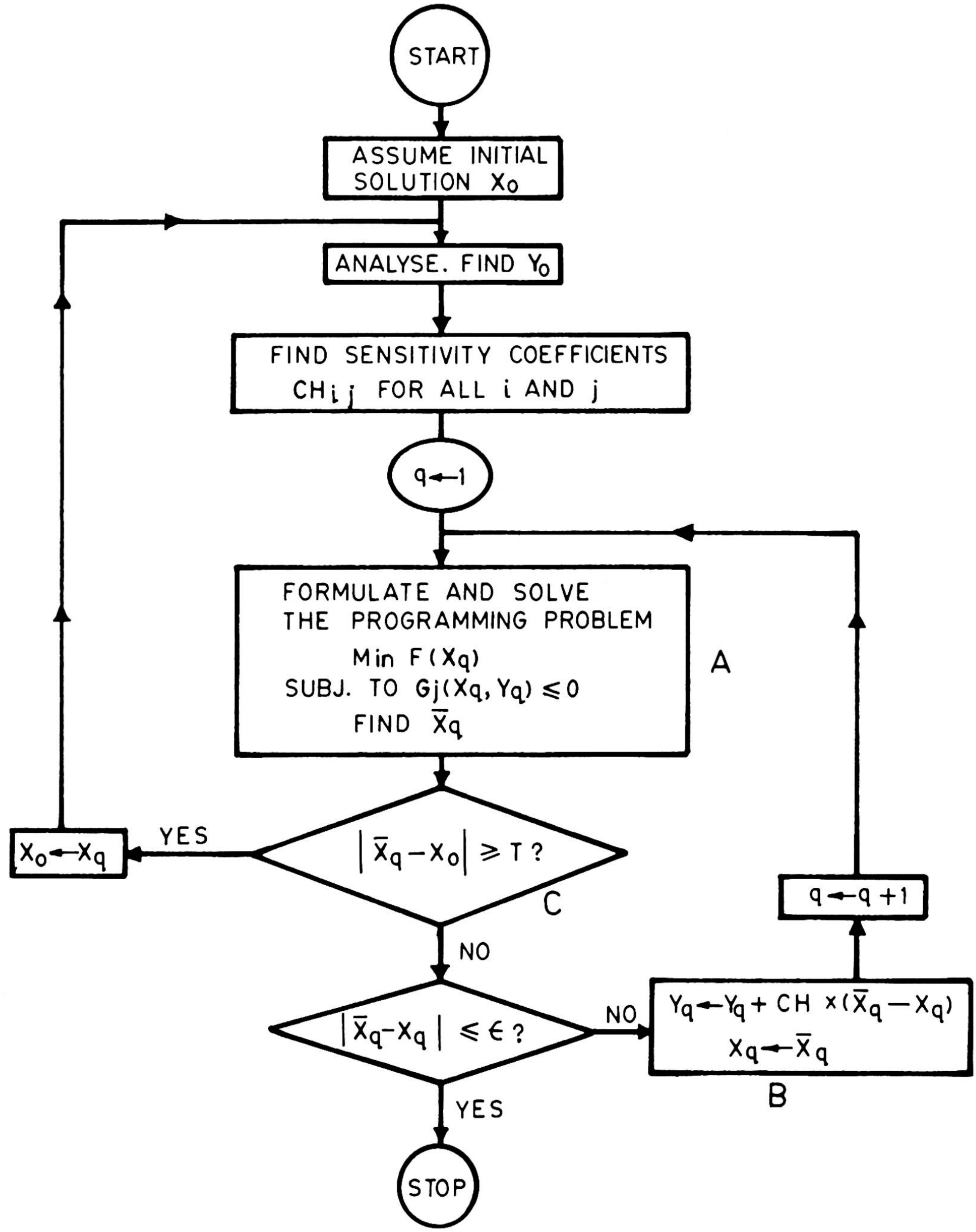

FIG.1. FLOW CHART FOR THE OPTIMIZATION PROCESS

exterior penalty function [6] method which consists of solving a sequence of unconstrained minimization problem:

$$\text{Min } \Phi(X,Y,r) = F(X) + r \langle G_j \rangle^2$$

for increasing values of r which is called penalty. The unconstrained minimization method which is found to be most efficient is the Davidson-Fletcher-Powell [6] method of variable metric and the unidirectional approach that is used is the direct root method. If the number of components in X is large the programming problem given in (1) may not lead to convergence. Hence the vector X is split into subvectors X_1, X_2, ... etc. where each of the subvectors is of a much smaller dimensions than the original vector X [7]. The solution of the programming problem (1) then consists of solving a series of smaller programming problems where only one of the subvectors such as X_1, X_2 are treated as unknowns. The use of exterior penalty function is found to be better when solving such partitioned problems.

3. COMPUTER PROGRAMMING AND NUMERICAL WORK

A general computer program based on the proposed method is written separately for trusses and for frames. The program is written in FØRTRAN IV language and is compatible with IBM 360, CDC 3600, DEC 10 and EC 1030 (called Ryad in some countries) computer systems. Several truss and frame problems for minimum weight design have been solved.

4. CONCLUSIONS

A general optimization algorithm for any structural problem has been suggested.

Acknowledgement

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SUMMARY

Structural optimization problem is generally solved as a sequence of analysis-programming cycles. In this paper it is shown how this problem can be treated as a series of programming problems. The relation between the changes in the behaviour variables due to a specified change in each of the design variables is found and stored in the form of "Sensitivity Matrix". This matrix directly gives the solution corresponding to a given set of design variables. The availability of this matrix dispenses with the frequent analysis.

RESUME

Le problème d'optimisation des structures est résolu généralement par une série de cycles dans un programme de calcul. On montre comment ce problème peut être traité par une série de problèmes de programmation. La relation entre les changements des variables de comportement et ceux de l'une quelconque des variables du projet est determinée et compilée sous forme de "matrice de sensitivité". Cette matrice donne directement la solution correspondant à un ensemble donné de variables du projet. On peut donc éliminer avec cette matrice de nombreux calculs.

ZUSAMMENFASSUNG

Das Problem der Optimierung von Strukturen wird allgemein als eine Reihe von analytischen Programmierungszyklen gelöst. Im vorgelegten Beitrag wird gezeigt, wie die allgemeine Methode der Optimierung als eine Reihe von Programmierungsproblemen behandelt werden kann. Die Beziehungen zwischen den Verhaltensvariablen und den Entwurfsvariablen werden hergestellt und als "Sensitivitätsmatrix" gespeichert. Aus dieser Matrix ergibt sich direkt die der gegebenen Gruppe von Entwurfsvariablen entsprechende Lösung. Die Benützung dieser Matrix vermeidet eine Wiederholung der Berechnung.

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Earthquake-Resistant Design of the Tower and Pier System of Suspension Bridges

Dimensionnement contre les tremblements de terre du système de pylône et pile des ponts suspendus

Die Erdbebenbemessung des Systems von Pylon mit Sockel bei Hängebrücken

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1. INTRODUCTION

Economical applications of mathematical programming methods in structural optimization are limited to specific structures as mentioned in Introductory Report.[1] In the case of structures with relatively simple and bulky dimensions, the mathematical programming method could be applied efficiently even if the structures are designed under relatively complicated design conditions. Dynamic loading problems are not treated in Introductory Report, and loading conditions appearing in optimal design have been mostly limited to the static ones.

Studies on aseismic design of long-span suspension bridges were carried out for many years in Japan, and the results of investigations were published as the official or individual reports. According to the studies on aseismic design of the suspension bridges, design of the tower and the pier is very important.[2] These parts of the bridge must be investigated as a system because of the interaction of these parts during the earthquake. The tower and pier system of suspension bridges involves rigid, massive, and large pier and relatively flexible and slender tower, so that the system has very complicated interaction.[2] The combination of the methods of mathematical programming and dynamic structural analysis is in fact well suited to the aseismic design of the tower and pier system of suspension bridges.

To formulate earthquake action for aseismic design, the method of response spectrum is employed in the design codes of the long-span suspension bridges in Japan. In this paper, the response spectrum method is mainly applied in the dynamic analysis and design of the system. Another approach based on more probabilistic concepts using power spectrum density of earthquake action and random vibration theory is possible using design constraints for reliability. Some approximation concepts[3] are used to save the computing time and to decrease the design variables in this paper.

2. THE STRUCTURAL SYSTEM

The system to be designed is the tower and pier system of the suspension bridges as shown in Fig.1, height h_T of the tower and h_P of the pier are determined from the environmental attribute of the bridge, and width b_1 of the pier is determined from geometrical relation with the bridge width. The design variables in global sense are, therefore, the longitudinal width b_2 of the pier and the stiffness of the tower. The combination of these two variables induces very complicated dynamic properties of the system.[2)]

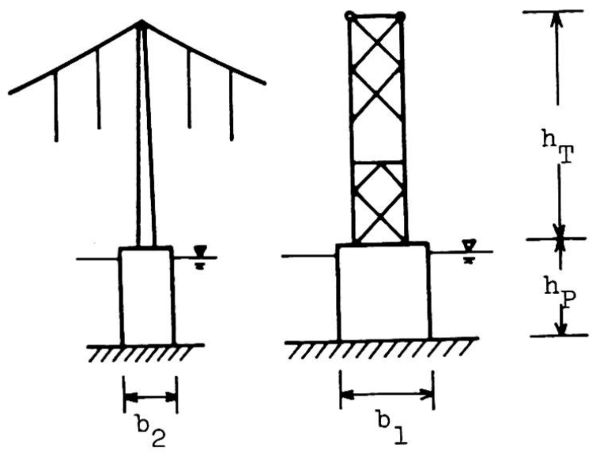

Fig.1 Structural System

3. DYNAMIC ANALYSIS OF THE SYSTEM

Analytical Model The analytical model of the tower and pier system of the suspension bridge treated in this paper is shown in Fig.2. The tower is assumed to be the lumped mass system, and the following assumptions are made:

(1) The foundation has elastic property.
(2) The reaction of the cable at the top of the tower is taken into account by applying the equivalent axial thrust and using an equivalent spring for the cable.[4)]
(3) The pier is assumed to be perfectly rigid and to be a single-degree-of-freedom capable of rocking motion.

Model of Earthquake Excitation Earthquake excitation is represented by response acceleration spectrum. In this study, the standard spectrum as shown in Fig.3 is used which is authorized by Honshu-Shikoku Connection Bridge Authority of Japan. In this figure, the longitudinal axis refers to be response magnification factor β, and standard acceleration in this design is 180 gal.

Dynamic Response Analysis The equation of motion for this multi-degrees-of-freedom-system can be written as:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-[\bar{M}]\{\ddot{z}\}$$

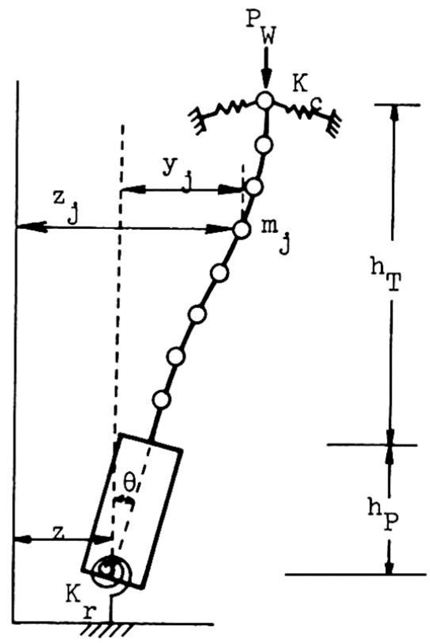

Fig.2 Analytical Model

where [M] is a mass matrix, [C] is a damping matrix, [K] is a stiffness matrix, {y} is a displacement vector, $\{\ddot{z}\}$ is an earthquake acceleration vector. With the aid of modal matrix [Φ] and the generalized displacement vector {q}, where {y}=[Φ]{q}, then the equation of motion rewritten in the following form assuming proportional damping.

$$[I]\{\ddot{q}\}+[\,^{\frown}(2h_i\omega_i)_{\smile}]\{\dot{q}\}+[\,^{\frown}(\omega_i)^2{}_{\smile}]\{q\}=-\{P\}$$

where ω_i refers to natural frequency, and h_i is damping constant of i-th mode.

The maximum displacement of point j, $y^{(j)}_{max}$, can be evaluated by root mean square method:

$$y_{max}^{(j)}=\sqrt{\Sigma(F_i\phi_i^{(j)}q_{i\ max})^2}$$

where F_i refers to the participation factor of i-th mode, $\phi_i^{(j)}$ refers to the relative displacement of point j in i-th mode. $q_{i\ max}$ is obtained from response spectrum given in Fig.3:

$$q_{i\ max}=\beta_i\ddot{z}_{max}/\omega_i^2$$

where $\ddot{z}_{max}$ is the maximum earthquake acceleration.

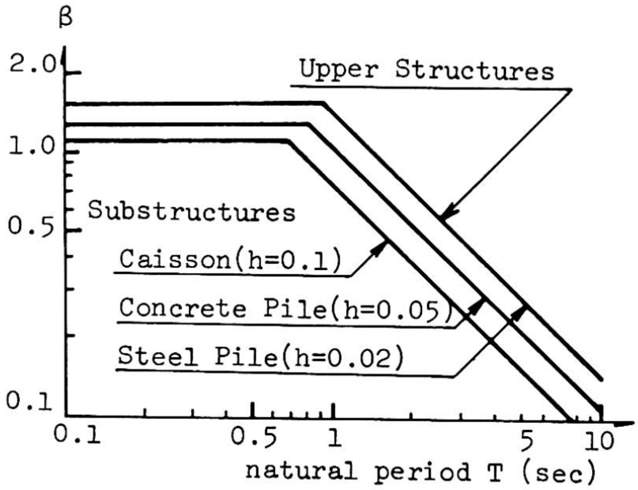

Fig.3 Earthquake Excitation Model

4. DESIGN MODEL

Approximation Concepts of the Tower To save culculation time and to improve reliability of solution, two design variables of the system are selected: One is the moment of inertia of the tower, the other is the longitudinal width of the pier. Other variables of the system are defined by approximation concepts.[3]

Let I, A and Z refer to the moment of inertia, the cross sectional area and the section modulus respectively, the empirical relation such as following may hold:

$$A=1.21*I^{0.33}$$

$$Z=0.55*I^{0.75}$$

The moment of inertia of the tower can be varied along the height in two ways: One is linearly varied; the other is stepwise varied into two portions. These design models are shown in Fig.4.

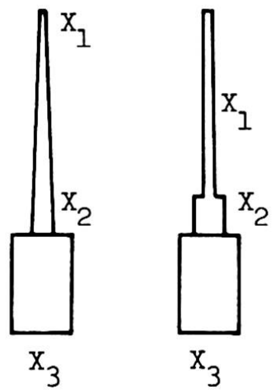

Fig.4 Design Model

Foundation Model The modulus of elasticity of the foundation is denoted by E. In the result of the past studies,[2] complicated dynamic phenomena due to the foundation condition, width of the pier, and the rigidity of the tower were observed. In the cases where two of the natural frequencies are very close, the coupled vibration of the tower and pier occurs, and the structural systems of such cases should be avoided. In this study, the modulus of elasticity ranges from $10*10^4$ ton/m^2 to $150*10^4$ ton/m^2 taking into account wide variety of foundation conditions.

Damping Constant The damping constant is assumed to be 0.1 for the mode where the vibration of the pier is predominant, and to be 0.02 for the vibration of the tower. For the coupling modes 0.05 for both modes is assumed.

5. OPTIMIZATION

Objective Function The generalized cost, W, is selected to be the objective function:

$$W=W_T+k*W_P$$

where W_T represents the weight of the tower and W_P, of the pier, and k refers to the ratio of unit cost of the pier to that of the tower.

Constraints The following constraints are considered:

(1) Stress of the tower shaft does not exceed the given allowable stress defined against earthquakes.
(2) Displacement of the pier top does not exceed the given allowable value.
(3) Tower shaft is safe against buckling.
(4) Pier is safe against overturning.
(5) Other geometric conditions.

Optimization Technique Objective function and constraints obtained in this way become non-linear and undifferential type, so SUMT by Powell's direct search method without differential is employed as optimization technique.

6. NUMERICAL EXAMPLE AND INVESTIGATIONS

As a numerical example, the tower and pier system shown in Fig.5 is considered, and the results of the computation are shown in Table 1,2. These computation were performed using the design model with stepwise varied cross section. In making Table 1, the following data was used:

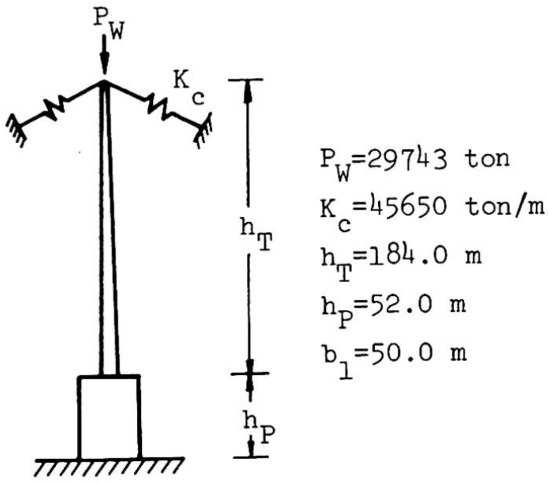

Fig.5 Tower and Pier System

cost ratio 0.2
maximum acceleration .... 180 gal
allowable value of pier top displacement .... 0.05 m
allowable stress of steel .... 37700 ton/m^2

From Table 1, the following investigations may be made:

(1) When elastic modulus of foundation, E, is small, the design of the system is determined only by the displacement constraint at the pier top. When the value of E is large, it tend to be determined by overturning of the pier and buckling of the tower, and the pier width tends to decrease. It shows that the pier width is closely related with E.
(2) The generalized cost is greatly affected by the modulus of elasticity of the foundation. Thus, the investigation of the foundation is very important.
(3) When E is large, the effect of earthquake response tend to decrease, and stiffness of the tower becomes uniform along the height of the tower. From this, when E is large enough, it is not necessary to increase the cross section of the lower part of the tower.

The design is controlled severely by the constraint of the displacement of the pier top in the range of small E. When this constraint is relieved to 0.065 m, the results are shown in Table 2. From these Tables, the following remarks may be made:

(1) In the range of small E, when the constraint of the pier top displacement is relieved slightly, the generalized cost decrease considerably. This result shows that the allowable value of the displacement of the pier top has a significant effect.
(2) In the range of large E, the result is not so affected by the constraint on displacement.

7. CONCLUSION

Table 1

E (10^4 ton/m^2)	I (m^4)		b_2(m)	W	Constraints					
					Pier		Tower			
	Upper	Lower			(1)	(2)	Top	(3)	Base	(4)
10	11.22	62.26	38.97	52146	X					
20	16.52	65.97	24.45	36077	X					
30	5.81	24.90	19.91	27865	X					
50	4.78	21.80	14.37	20896	X	X				X
70	4.75	4.77	14.35	20509		X				X
150	4.75	4.77	14.35	20509		X				X

(1): Displacement of the pier top (2): Overturning
(3): Stress of the tower shaft (4): Buckling

Table 2

E (10^4 ton/m^2)	I (m^4)		b_2(m)	W	Constraints					
					Pier		Tower			
	Upper	Lower			(1)	(2)	Top	(3)	Base	(4)
10	8.15	32.71	28.76	39081	X					
20	13.65	24.43	18.56	27053	X					
30	4.75	8.41	14.54	20842	X					X
50	4.77	4.78	14.35	20511		X				X
70	4.77	4.78	14.35	20511		X				X
150	4.75	4.77	14.35	20508		X				X

(1): Displacement of the pier top (2): Overturning
(3): Stress of the tower shaft (4): Buckling

The optimal design of the tower and pier system on the elastic foundation subjected to earthquake excitation is studied by using response spectrum and modal analysis. Investigation in this study shows that necessity or importance of displacement condition of the pier top must be discussed more precisely from the point of safety of the structure in the range of small E, and that necessity of earthquake-resistant design must be discussed more precisely from the dynamic response of the structure in the range of large E.

PROBABILISTIC APPROACH

Probabilistic approach using power spectrum density for earthquake and based on random vibration theory can be formulated as follows.

Earthquake load is represented by power spectrum density function shown in Fig.6.[5] As earthquake is assumed to have zero mean and to be stationary probabilistic process, variances of the displacement and of the velocity can be evaluated based on the random vibration theory.

Failure probability can be com-

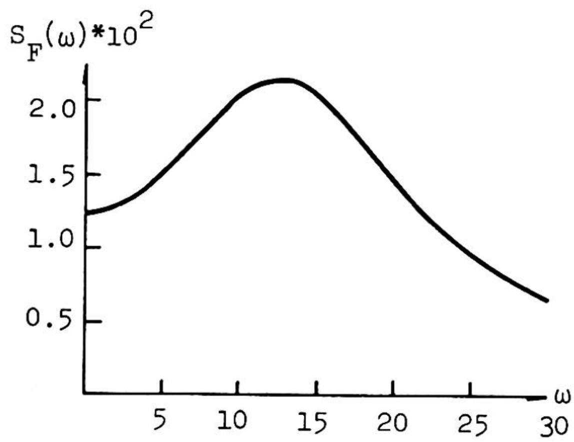

Fig.6 Power Spectrum Density

puted through dynamic reliability theory using displacement and velocity variances. Thus, it is possible to formulate optimization by probabilistic approach using failure probability as constraints.

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SUMMARY

Effective application of the structural optimization method is limited to some specific types of structures in civil engineering structures. In the case of structures with relatively simple and bulky dimensions, the mathematical programming method could be applied efficiently. In this paper, the authors carried out the optimal design of the tower and pier system of suspension bridges on the elastic foundation subjected to earthquake ground motion using response spectrum and dynamic analysis.

RESUME

Une application pratique de la méthode d'optimisation structurale est limitée à certaines structures du génie civil. Dans le cas de structures relativement simples et de grandes dimensions, la méthode de programmation mathématique peut être appliquée efficacement. Dans cet article les auteurs ont fait le calcul d'optimisation du système de pylône et pile des ponts suspendus sur fondation élastique subissant le tremblement de terre, à l'aide d'une analyse dynamique.

ZUSAMMENFASSUNG

Die Anwendung der Tragwerks-Optimierung ist auf einige spezielle Strukturarten im Bauingenieurwesen begrenzt. Bei Strukturen mit einfachen und massigen Abmessungen lässt sich das mathematische Programmierungsverfahren erfolgreich verwenden. In diesem Aufsatz wird eine Optimierung des Pylonsystems auf elastischem Untergrund unter Erdbebenlast entwickelt. Hierbei werden Verhaltensspektren und dynamische Analysen angewendet.

Total Cost Optimum of I-Section Girders

Dimensionnement de poutres à section en I en vue d'un coût total optimal

Optimierung von I-Stahlträgern bezüglich der Totalkosten

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1. INTRODUCTION

The optimum design for I-section girders has been investigated considerably, and most of the investigations aim at a minimum-weight-design or a design including only the cost of shop welding in the fabrication cost. Considering all of the fabrication costs, however, the optimum values of design variables will vary remarkably. Therefore, a total cost optimum design, in which an objective function considers material cost as well as fabrication cost including costs of full-scale-drawing, machining, shop welding, shop assembly and shop painting, has to be done. Since such variables as plate thickness, surface area and weight of members, material grade, etc., are included in fabrication costs, the optimum value of the objective function may not be exactly computed, if some of variables are omitted. Therefore, the design variables to be used in this investigation include almost all dimensions of a cross section.

At the present study, a computer-aided, optimum design for single simply supported girders is carried out by SLP method (Sequence of Linear Programming method).[1),2)] If their upper and lower lateral bracings and sway bracing are designed and their dimensions are determined, it is posible to do an automated design by the use of an automated drawing machine.

2. OPTIMUM DESIGN FOR I-SECTION GIRDERS

Material S, cover plate thickness T_c, cover plate width B_c, flange plate thickness T_f, flange plate width B_f, web plate thickness T_w, web plate height B_w, and segment length of a girder section C_ℓ are selected as design variables. Concerning S, steel of 41kg/mm^2 in tensile strength is expressed with 4, 50kg/mm^2 is expressed with 5 and 58kg/mm^2 with 6, and an intermediate value is set on a continuous function.

The constraints contain limit of stress, limit of deflection, limit of plate width to thickness, as specified at the Specifications, limit of flange width to web height, namely $B_f/B_w=1/3\sim1/6$, and upper and lower limits of the values of design variables, which are also used as move limits.

When an allowable tensile stress and an allowable compressive stress of a material are given by σ_{at} and σ_{ac}, respectively, and a ratio of height to thickness of web plate is given by γ, σ_{at}, σ_{ac} and γ are expressed as a function of S as follows:

$$\sigma_{at}=\sigma_{at}(S), \qquad \sigma_{ac}=\sigma_{ac}(S) \qquad \cdots\cdots\cdots\cdots (1)$$

$$\gamma=\gamma(S) \qquad \cdots\cdots\cdots\cdots (2)$$

Then, an objective function Z is expressed with

$$Z=\sum_j \rho\cdot V_j\cdot C\cdot CM+\sum_j\sum_i H_{ij}\cdot SMH+\sum_k\sum_\ell \tilde{H}_{k\ell}\cdot SMH, \qquad \cdots\cdots\cdots\cdots (3)$$

where V_j: volume of the j-th element, ρ: unit weight of steel material, C: coefficient for unit cost of steel material, CM: unit cost of steel material, SMH: unit cost for one man hour work, H_{ij}:work man hour of the i-th manufacturing operation of the j-th element as a function of design variables, $\tilde{H}_{kl}$: work man hour of the k-th manufacturing operation of the l-th element as a fixed value. When, C is considered as a function of T, S and B, and C_1, C_2 and C_3 indicate the case of the function of T, C, the case of the function of S, C and the case of the function of B, C, respectively, the following expressions are obtained:

$$\left.\begin{aligned} &C_1(T)=0.0348T^2-0.0845T+1.2091 \\ &C_2(S)=0.27S-0.08 \\ &C_3(B)=1.0 && \text{for } B<200\text{(cm)} \\ &C_3(B)=1.0+(B-200)/(0.3\ CM)\times 0.01 && \text{for } B\geqq 200\text{(cm)} \end{aligned}\right\} \cdots\cdots\cdots\cdots (4)$$

where T: plate thickness, B: plate width. H_{ij} can be considered as a function of S, T, W and A_r, where W: weight, A_r: surface area. HA is the coefficient of work man hours depending on S, and the following equation may be obtained:

$$H_{ij}=H_{ij}\times HA(S), \qquad \cdots\cdots\cdots\cdots (5)$$

where

$$HA(S)=0.04S^2-0.29S+1.52. \qquad \cdots\cdots\cdots\cdots (6)$$

The coefficients of Eqs. (4) and (6) are obtained on the basis of actual examples at a bridge fabricating shop in Japan. In the case of welded joints, the work man hours of butt welded joint, H_{ij}, and fillet welded joint, H'_{ij}, become a function of total welded length, but their calculation is to be made with a ratio, η, of equivalent welded length to 6^{mm} fillet. Assumed as a function of T, H_{ij} and H'_{ij} are calculated by the following equations, that is, in the case of butt welds,

$$H_{ij}=H_{ij}(L_1),\ L_1=L_1\times\eta_1(T),\ \eta_1(T)=1.2T^2+3.8T+1.3 \qquad \cdots\cdots\cdots\cdots (7)a$$

and in fillet welds,

$$H'_{ij}=H'_{ij}(L_2),\ L_2=L_2\times\eta_2(T),\ \eta_2(T)=0.0476T^2+0.1952T+0.7572 \qquad \cdots\cdots\cdots\cdots (7)b$$

where L_1, η_1: in the case of butt welds, total equivalent welded length and ratio of equivalent welded length, respectively; L_2, η_2: in the case of fillet welds, the same as L_1, η_1. H_{ij} for marking and painting may be considered a function of A_r.

The procedure of this optimum design is shown by a block flow chart as in Fig.1, in which X shows the variables to be computed by the simplex method and X^0 shows their initial values.

At this study, single main girders without lateral bracings and sway bracing are treated, because omitting of the bracings does not affect the optimum value of total cost.

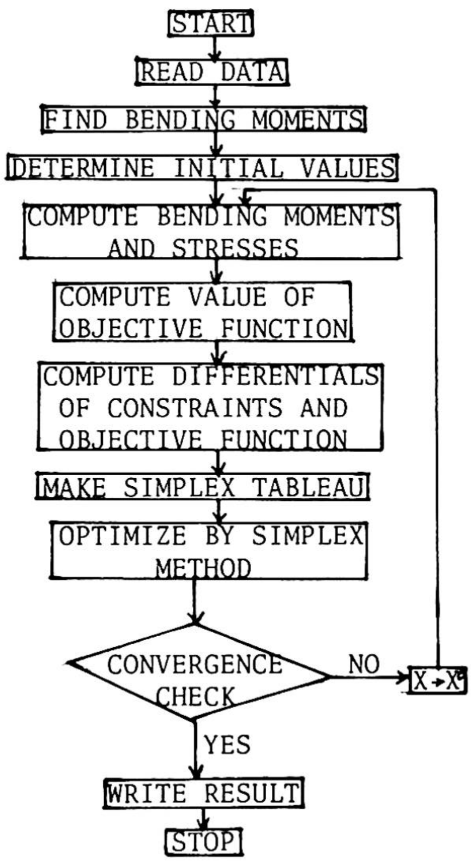

Fig.1. Flow chart of optimum design of girders

3. EXAMPLE OF OPTIMUM DESIGN

3.1 The conditions of design are given as follows:
1) type: I-shaped and deck-type welded railway plate girder, 2) live load: KS18 specified at the Railway Bridge Specifications in Japan, 3) span length: 5 kinds of span length, 16^m, 19^m, 22.3^m, 25.5^m and 30^m, 4) specifications: the Japanese Specifications for Design of Steel Railway Bridges.[3)]

It is assumed that a girder can be provided with three kinds of variation of sections as seen in Fig.2 with NA=2, 3 and 4, in which NA means the number of different girder sections. The upper cover and flange plates are symmetrical with the lower plates.

σ_{at}, σ_{ac} and γ are respectively given at the Specification as:

$$\left.\begin{aligned} &\sigma_{at}=(0.125S^2-0.792S+2.168)\times 1300 \\ &\sigma_{ac}=50S^2+50.5S+199-(0.2S^2-1.3S+2.5)\times(\ell/B_f)^2 \end{aligned}\right\} \cdots\cdots\cdots\cdots (1)a$$

Table 1. Comparison of optimum values for span length of 2230cm

SMH (1000 yen)	NA	S_1	S_2	S_3	T_c (cm)	T_{f1} (cm)	T_{f2} (cm)	T_{f3} (cm)	T_w (cm)	B_c (cm)	B_{f1} (cm)	B_{f2} (cm)	B_{f3} (cm)	B_w (cm)	$SLEG_1$ (cm)	$SLEG_2$ (cm)	$SLEG_3$ (cm)	$SLEG_4$ (cm)
0.0	2	4.0			2.00	1.86			1.19	48.1	44.7			183.8	740.2	1115.0		
1.6	2	4.0			1.83	2.09			1.15	43.9	50.2			178.5	672.5	1115.0		
3.2	2	4.0			1.62	2.33			1.11	38.9	56.0			172.4	589.2	1115.0		
0.0	3	4.0	4.0		1.95	2.00	1.40		1.15	46.8	48.0	29.6		177.8	712.6	1027.2	87.8	
1.6	3	4.0	4.0		1.80	2.29	1.40		1.07	43.2	54.9	27.8		166.5	644.1	1027.2	87.8	
3.2	3	4.0	4.0		1.65	2.35	1.40		1.09	39.7	56.4	28.2		169.3	597.6	1027.2	87.8	
0.0	4	4.0	4.0	4.0	1.58	2.29	1.86	1.40	1.14	38.0	54.9	44.7	29.5	177.2	583.0	766.3	150.0	199.1
1.6	4	4.0	4.0	4.0	1.45	2.41	1.83	1.40	1.12	34.9	57.9	44.0	29.0	173.7	533.3	788.9	137.8	188.3
3.2	4	4.0	4.0	4.0	1.30	2.68	1.82	1.40	1.11	31.2	57.3	43.6	28.6	171.8	472.7	800.4	132.3	182.3

SMH (1000 yen)	NA	δ_1 (cm)	δ_2 (cm)	Z (1000 yen)	Weight (ton)	β	σ_{a1} (Kg/cm²)	σ_1 (Kg/cm²)	σ_{a2} (Kg/cm²)	σ_2 (Kg/cm²)	σ_{a3} (Kg/cm²)	σ_3 (Kg/cm²)	σ_{a4} (Kg/cm²)	σ_4 (Kg/cm²)	α	$\frac{SLEG_1}{0.5L}$	$\frac{SLEG_2}{0.5L}$	$\frac{SLEG_3}{0.5L}$
0.0	2	2.21	1.85	288.3	4.92	1.01	1155	1155	1166	1166					12.1	0.664		
1.6	2	2.30	1.93	555.6	4.91	0.97	1170	1170	1174	1174					12.5	0.603		
3.2	2	2.40	2.01	804.9	4.95	0.93	1184	1184	1179	1179					12.9	0.528		
0.0	3	2.30	1.92	284.1	4.83	0.96	1165	1165	1171	1171	1122	517			12.5	0.639	0.921	
1.6	3	2.48	2.08	571.5	4.83	0.88	1182	1182	1178	1178	1111	607			13.4	0.578	0.921	
3.2	3	2.45	2.05	841.0	4.83	0.90	1185	1185	1179	1179	1114	583			13.2	0.536	0.921	
0.0	4	2.34	1.96	270.1	4.58	0.96	1182	1182	1178	1178	1166	1166	1122	1122	12.6	0.523	0.687	0.134
1.6	4	2.40	2.01	578.2	4.57	0.94	1188	1188	1180	1180	1165	1165	1119	1119	12.8	0.478	0.708	0.124
3.2	4	2.42	2.03	873.7	4.58	0.92	1186	1186	1180	1180	1165	1165	1117	1117	13.0	0.424	0.718	0.119

$\gamma=2.5S^2-47.5S+305$ ··(2)a
where ℓ represents a distance between fixed points of a flange plate. If α is assumed to be a value of a span length divided by a web height, the initial values of B_W, B_C, T_C, B_f and T_f are calculated under the assumption of α, but the initial values of S and locations of joint are given as constant values independently of α. Now, the calculation of the initial values by a computer, makes it possible to do an automated design.

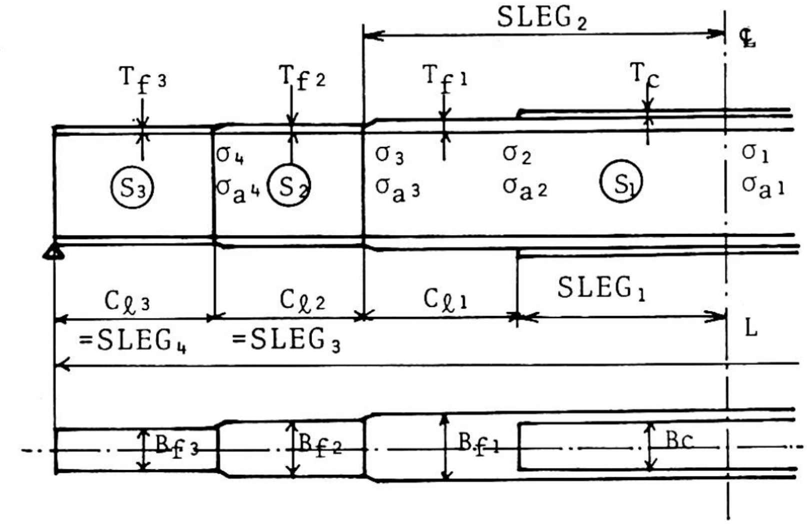

Fig.2, Notations for deck plate girder

3.2 Results of Calculation

As an example of the results of calculation, the case of span length of 22.3^m are summarized in Table 1, in which SLEG: values shown in Fig.1, δ_1:deflections due to live load and dead load at the span center, δ_2: deflections due to live load at the span center, Z: values of objective function, β: coefficients to be given later.

3.3 Discussion

As the result, the followings are discussed:

(1) The materials were considered as the design variables too, but the calculation shows that the case of S=4, namely SS41 steel will give optimum values.

(2) In the case of material cost only, the value of an objective function becomes the cheaper, with an increase of the number of different sections. On the other hand, in the case of material cost and fabrication cost, the value of the objective function becomes the higher and the girder weight becomes the lighter, with an increase of the number of different sections.

(3) Conventionally a web height B_W used to be expressed in terms of the following relation:

$$B_W=\beta\sqrt{\frac{M}{\sigma\cdot T_W}} \qquad \cdots\cdots\cdots\cdots(8)$$

where β: coefficient, M: bending moment. The values of β,calculated by the optimum values, are shown in Table 2. They do not change greatly as to span lengths, but generally become the larger , the longer the span length is.

Table 2. Values of coefficient β

SMH	NA \ L	1600^{cm}	1900^{cm}	2230^{cm}	2550^{cm}	3000^{cm}
	2	0.96	0.97	1.01	1.02	1.00
0.0	3	0.93	0.94	0.96	0.99	1.00
	4	0.92	0.94	0.96	0.98	1.03
	2	0.92	0.94	0.97	0.97	1.01
1.6	3	0.89	0.88	0.88	0.91	1.01
	4	0.89	0.93	0.94	0.93	1.04
	2	0.92	0.93	0.93	0.95	1.02
3.2	3	0.89	0.90	0.90	0.90	1.03
	4	0.87	0.89	0.92	0.94	1.05

(4) The values of SLEG/0.5L are shown in Table 3, where L: span length. In the table, $SLEG_1$ is the shorter, the higher SMH is, while flange plate lengths do not change greatly.

Table 3. Values of SLEG/0.5L

SMH	NA	SLEG \ L	1600^{cm}	1900^{cm}	2230^{cm}	2550^{cm}	3000^{cm}
0.0	2	$SLEG_1$	0.651	0.661	0.664	0.671	0.681
	3	$SLEG_1$	0.625	0.631	0.639	0.645	0.658
	3	$SLEG_2$	0.921	0.921	0.921	0.921	0.921
	4	$SLEG_1$	0.492	0.505	0.523	0.539	0.558
	4	$SLEG_2$	0.668	0.679	0.687	0.696	0.698
	4	$SLEG_3$	0.140	0.140	0.134	0.131	0.117
1.6	2	$SLEG_1$	0.554	0.582	0.603	0.619	0.641
	3	$SLEG_1$	0.537	0.565	0.578	0.594	0.611
	3	$SLEG_2$	0.921	0.921	0.921	0.921	0.921
	4	$SLEG_1$	0.447	0.455	0.478	0.492	0.489
	4	$SLEG_2$	0.700	0.698	0.708	0.715	0.693
	4	$SLEG_3$	0.115	0.124	0.124	0.128	0.118
3.2	2	$SLEG_1$	0.492	0.489	0.528	0.558	0.588
	3	$SLEG_1$	0.485	0.506	0.534	0.548	0.556
	3	$SLEG_2$	0.921	0.921	0.921	0.921	0.921
	4	$SLEG_1$	0.356	0.373	0.424	0.440	0.396
	4	$SLEG_2$	0.694	0.717	0.717	0.707	0.673
	4	$SLEG_3$	0.148	0.138	0.120	0.131	0.143

(5) Except for the value of σ_3 in NA=3, the other maximum working stresses reach up to the full allowable stresses. σ_3 does not become fully-stressed, because the flange plate at this position is determined by its minimum thickness 1.4^{cm} and by its width calculated at $B_f \geqq B_w/6$.
(6) T_c and B_c are the smaller, the higher the fabrication cost is. On the other hand, T_{fi} and B_{fi} are the larger, the higher the fabrication cost is. There is no remarkable difference due to the difference of fabrication cost at the dimension of flange section at the other positions.
(7) The leg length at fillet welds is the smaller and the fabrication cost is the cheaper, the wider the flange plate is and the thinner the flange plate is.
(8) At the present example of design, the optimum dimensions of section for 5-kinds of the span length are calculated, but they can be calculated for the other span lengths by the following procedure. B_w is calculated from Eq. (8) by assuming σ and T_w, and using M and β. Then, the position of joint is obtained from Table 3, and except T_{f2} and B_{f2} in NA=3, the dimension of girder section at the span center and all of the positions of joint can be found by the fully stressed design. However, in NA=3, $T_{f2}=1.4^{cm}$ and $B_{f2}=B_w/6$ or $B_{f2} \geqq 24^{cm}$ are applied.
(9) As seen in the value of β, for the case of material cost only the optimum girder height varies, but for variable unit fabrication costs it does not greatly vary.

4. CONCLUSION

It is indicated that it is possible to carry out the optimum design considering material cost and total shop fabrication cost by means of a program for computer design which is presented at the present study, and it will be possible to extend this program to the computation for a girder with different upper flange section from lower flange section, a composite girder and a continuous girder; and a part of this program has been completed already.

At the present program, transportation and erection costs depending on site conditions are omitted, but in the future, in the case of a specific or individual bridge, it would be necessary to investigate on an overall cost optimum design containing the transportation and erection costs.

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SUMMARY

A program of optimum design considering material cost and total shop fabrication cost for the most foundamental I-section girders of steel bridges is presented with design examples. The influence of 16 design variables on the total cost is discussed, to improve the computer-aided automated optimum design.

RESUME

Un programme de dimensionnement optimal tenant compte du coût des matières et des coûts de fabrication est appliqué aux plus élémentaires sections en I des poutres de pont métallique. Des exemples sont donnés. L'influence de 16 variables de dimensionnement sur les coûts totaux est étudié afin d'améliorer le dimensionnement optimal à l'ordinateur.

ZUSAMMENFASSUNG

Es wird ein Programm für die Optimierung von I-Stahlträgern mit Rücksicht auf Material- und Herstellungskosten präsentiert und dessen Anwendung an Beispielen dargelegt. Der Einfluss von 16 Entwurfsvariablen auf die Totalkosten wird untersucht, um die computerunterstützte Entwurfsoptimierung zu verbessern.

Optimierung von Eisenbahnfachwerkbrücken

Optimization of railway truss girder bridges

Optimisation de ponts ferroviaires en treillis

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1. Grundsätzliches

Bei den praktischen Optimierungsaufgaben kann man grundsätzlich drei verschiedene Verfahren anwenden [1] .

Bei dem ersten Verfahren werden einige zweckmässig ausgewählte Varianten durchgearbeitet und die Ergebnisse miteinander verglichen. Ein erfahrener Entwurfsingenieur gewinnt auf diese Weise mit erträglichem Arbeitsaufwand eine ausreichende Übersicht.

Bei dem zweiten Verfahren versucht man, die Beziehungen zwischen der vorgegebenen Belastung und Geometrie der Konstruktion einerseits und Konstruktionsabmessungen oder Kosten andererseits mathematisch zu erfassen. Eine ausführliche Beschreibung der dazu anwendbaren mathematischen Methoden ist im Einführungsbericht dargelegt [2]. Jedoch weist dieser mathematischer Weg zwei grundsätzliche Nachteile auf. Durch die unumgängliche Vereinfachung und Idealisierung zu komplizierter mathematischer Beziehungen werden die Ergebnisse in meist unübersehbarer Weise unscharf und gelegentlich sogar fehlerhaft. Ferner ist der praktische Entwurf einer Konstruktion durch die vorgegebene Dispositionsforderungen, das Walzprogramm, die Standartsbestimmungen, verschiedene Konstruktionsrichtlinien und übliche Durchführung der Details usw. weitgehend eingeengt. Die Möglichkeit der Anwendung der allgemeinen mathematischen Methoden [2] , die meist nur durch Einsatz moderner Computer denkbar ist, ist bei praktischen Beispielen oft nicht gegeben.

Daher wurde in letzter Zeit ein drittes Optimierungsverfahren entwickelt [1] , dass die Kapazität moderner Computer in anderer Weise ausnützt und die Vorteile der beiden beschriebenen Methoden vereinigt. Man stellt dabei ein Programm auf, das den Entwurfs- und Bemessungsprozess des untersuchten Konstruktionstypes nachbildet. Dabei ist es nicht schwierig, z.B. die richtigen Werte der Knickzahl, die Abstufung des gültigen Walzprogrammes, verschiedene Richtlinien und Normbestimmungen, übliche Konstruktionsdetails usw. zu berücksichtigen. Durch Variieren der Eingangsparameter stellt man ziemlich leicht den Bereich von optimalen Lösungen fest, die dem angestrebten Minimum der Untersuchten Zielfunktion (Materialverbrauch oder Kosten oder Arbeitsaufwand) nahe liegen.

2. Praktische Anwendung des neuen Verfahrens bei Stahlbrücken

Das neue Verfahren wurde zuerst zur Optimierung der Verbundträger angewandt. Hier hängt der Stahlverbrauch praktisch nur von der Höhe des Trägers und von der Schlankheit seines Steges ab. Daher konnte man hier unter Verbrauch von wenigen Minuten der Computerzeit die optimalen Querschnitte von Eisenbahn- oder Strassenbrücken, für Verbundträger oder auch Verbundkastenträger feststellen.

Bei Fachwerkbrücken war die Anwendung des neuen Optimierungsverfahrens durch die grössere Zahl der Eingangsparameter umständlicher. Es wurden die üblichen Trägerform nach Abb.1, drei Fahrbahntypen (offene Fahrbahn, direkt befahrene mit den Hauptträgern mitwirkende Blechfahrbahn und durchgehendes Schotterbett auf einer mitwirkender Blechfahrbahn), mit geschlossenem oder offenem Brückenquerschnitt, Ein- und Zweigleisbrücken und wirtschaftliche Kombination der Stahlsorten St 37 und St 52 bis zur Spannweite von L 100 m untersucht. Es hat sich dabei eindeutig gezeigt, dass die optimale Trägerform mit dem minimalen Stahlverbrauch, evtl. minimalen Baukosten der tragenden Konstruktion vor allem von der Spannweite L, von der Felderzahl N, von der Trägerhöhe H und von der Höhe v der idealisierten Stabquerschnitte (Abb.2) abhängt, während der Einfluss der evtl. beschränkten Konstruktionshöhe des Fahrbahnrostes und der Grösse des Konstruktionbeiwertes vernachlässigbar klein erscheint.

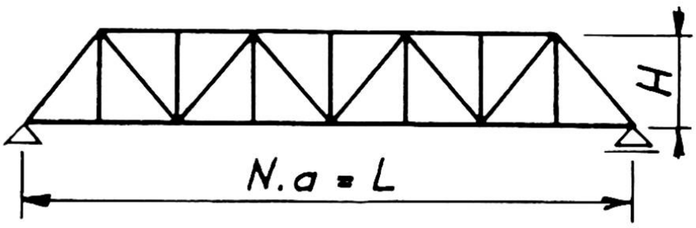

Abb.1

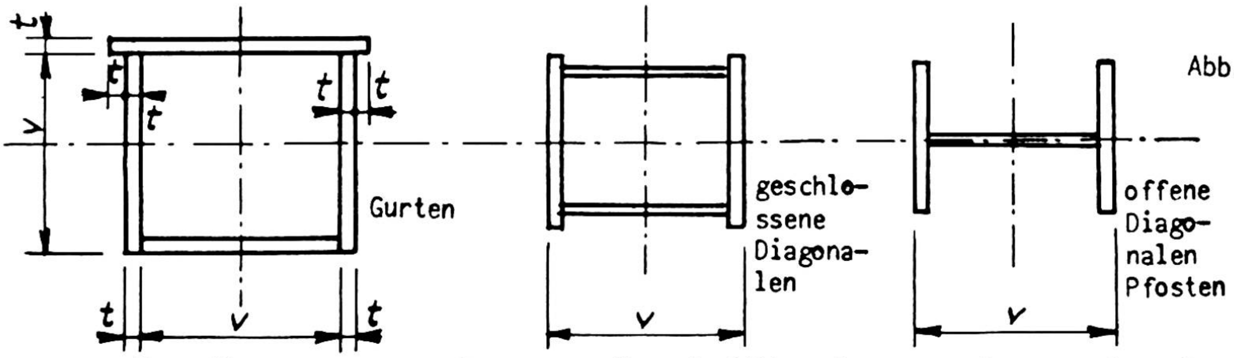

Abb.2

Das Programm wurde so aufgestellt, dass nach der Angabe von L, N und Eigengewicht der Fahrbahn zuerst die geometrische Form für eine ziemlich niedrige Trägerhöhe H berechnet wurde, dann wurden die Längs- und Querträger berechnet und dimensioniert. Nach der Ermittlung von Stabkräften wurden einzelne Stabquerschnitte mit ziemlich kleinem Wert von v dimensioniert. Das resultierende Gesamtgewicht wurde mit dem anfänglichen aus empirischer Formel eingesetzten Wert verglichen; wenn der Unterschied grösser als der vorgegebene Wert war, wurde die ganze Dimensionierung mit korrigierten Werten wiederholt. Zuletzt wurde die Durchbiegung kontrolliert und die Anstrichsfläche festgestellt.

Im weiteren Schritt vergräserte das Computer des Ausgangswert v um Δv, wodurch die Senkung der Zielfunktion Z, d.h.

des Stahlverbrauches oder der Kosten erzielt wurde. Man wiederholte dann die Vergrösserung von v einigemal, bis das Minimum von Z erreicht wurde. Dadurch wurde das Optimum für bestimmte Höhe H festgestellt.

Nachher vergrösserte das Computer den Ausgangswert H um ΔH, wodurch wieder die Senkung des Wertes Z erreicht wurde; diese Iteration wurde so lange wiederholt, bis der Endwert von Z grösser war als sein Wert bei dem ersten Iterationschritt (Abb.3).

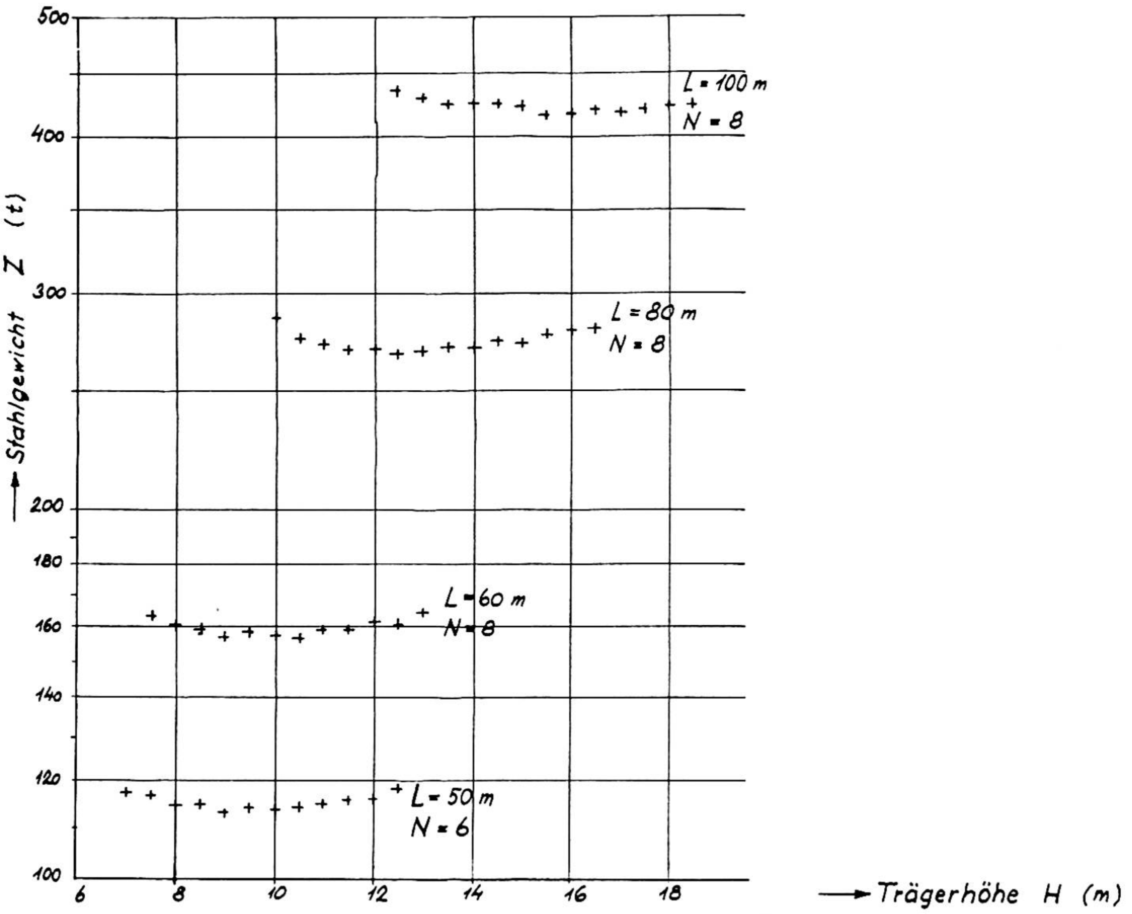

Abb.3

Der Abb. 3 kann man entnehmen, dass die Streung der Werte Z in der Umgebung vom Minimum sehr flach ist und somit die Grösse von Z auf kleine Variationen der Trägerhöhe H nicht empfindlich ist. Deshalb ist es angebracht, nicht von einer optimalen Höhe zu sprechen, sondern von dem Bereich B von optimalen Höhen, dessen Breite durch die Differenz Z festgelegt wird (Abb.4). Zum Beispiel für die Differenz von $\Delta Z = 0{,}02\ Z$ wurden die unteren und oberen Grenzen des Bereiches von optimalen Höhen wie folgt festgestellt :

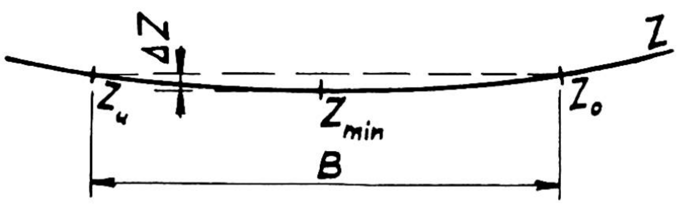

Abb.4

Querschnitt der Brücke	Spannweite L (m)	Feldteilung N (-)	offene Fahrbahn		direkt befahrene Fahrbahn		durchgehendes Schotterbett	
			Grenze des Bereiches der optimalen Höhe H/L					
			untere	obere	untere	obere	untere	obere
offen	50	8	1/8,5	1/6,0	1/9,1	1/6,3	1/9,3	1/5,8
geschlossen	50	8	1/7,1	1/4,9	1/7,4	1/4,9	1/7,5	1/4,8

Was die optimale Kombination des üblichen Stahles St 37 mit Stählen höherer Festigkeit betrifft, ist deren Einsatz nur bei jenen Stäben wirtschaftlich, bei welchen die Stahlverbrauchser - sparnis höher als der zuständige Preisunterschied der fertigen Konstruktion ist. Somit ist es wirtschaftlich, bei Spannweiten von 40 bis 100 m, bei offenen Fahrbahnen beide Gurtungen, bei einer Blechfahrbahn die obere Gurtung und die Längsträger aus St 52 zu entwerfen, sowie auch die "schweren" Diagonalen in der Nähe von Stützen der Brücken mit grösseren Spannweiten.

3. Folgerungen

Es wurde gezeigt und am Beispiel einer Eisenbahnbrücke demonstriert, dass bei den Konstruktionen, deren Kosten nur von wenigen Eingangsparametern abhängen, während viele andere Parameter der Konstruktion mit der Spannweite, mit dem Konstruktionstyp und -zweck zusammenhängen und nicht viel veränderlich erscheinen, vorteilhaft ist, das Berechnungs- und Bemessungsprozess des Entwurfingenieurs in einem Computerprogramm nachzuahmen und den Bereich der optimalen Lösungen durch Variieren der Eingangsparameter festzustellen.

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[1] Schindler, A.: Zur Optimierung von Stahlkonstruktionen. Bauplanung-Bautechnik, 22 (1968), H.8,S. 395-398

[2] Templeman, A., B.: Optimization Concepts and Techniques in Structural Design. Introductory Report of Xth Congress IABSE in Tokyo 1976, Zurich 1975, pp. 41-60

ZUSAMMENFASSUNG

Es werden drei Optimierungsverfahren definiert und die Anwendung des dritten Verfahrens am Beispiel der Eisenbahnfachwerkbrücken erläutert.

SUMMARY

Three ways of optimization are presented. An application is demonstrated on railway truss girder bridges.

RESUME

On définit trois procédés d'optimisation. L'usage du troisième procédé est démontré pour des ponts ferroviaires en treillis.

Total Computer System for Bridges

Système global pour le projet de ponts au moyen de l'ordinateur

Integrales Computersystem für Brückenentwurf

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1. Optimum design and automated design

Automated design has been studied as a part of automation and labor saving problems by those who are engaged mainly in the practical design work, while the optimum design has been researched and developed by researchers who study mainly the mathematical decision method in connection with this design work. However, it is unreasonable to say that design by the automated design system does not have to be optimum design. If optimum design should be used for a practical application, its concept and method should be used for the automated design system and, therefore, we believe that they should be combined.

Under the present conditions, where the method of optimum design is not employed extensively in practical fields, whether the results of study are adopted or not is decided by designers and, in many cases, the designers modify the results before they utilize them. The practical design work is undisciplined and, in most of cases, constraints and objective function are never represented by well arranged formulas and thus contain many factors which depend upon the man's intuition and, therefore, the CAD system is conveniently used for ensuring a smooth execution of design work. At the present time, the practical method by which we can most probably ensure constant and high quality design is the CAD system which is processed in such a way that the method of optimum design is used for deciding algorism of automated design, allowing the system to supply designers with the data necessary for them to make judgements and, according to such data, the system proceeds on the basis of the man-machine relationship.

Whether a designer can accomplish high quality design by using such a system or not depends on (1) whether the ability of the designer who utilizes this system is proper or not or (2) whether the system can conveniently and quickly supply the required data in an easily usable form and if the system can fully carry out "trial and error" in a short time.

Combination of the above methods is indispensable for the improvement of quality of design and the mathematical decision method is also an indispensable factors.

Even if data of the best quality, when viewed from the standpoint of optimum design, is not supplied from the system, it is expected that the designers may be able to accomplish a design of a considerably high quality, if he can use the system conveniently, which means utilizing both mathematical decision method and the judgement of the designers. Under these conditions, the writers of this report have developed the CAD system for bridge design and used it for practical applications. The following describes the design system of a girder bridge.

2. Design system of girder bridge

2.1 Outline

Most ordinary bridges are of the girder bridge type and, therefore, it is necessary to prepare a system which can be used conveniently and withstand the changes, additions and deletions of shape data, designing conditions, manufacturing conditions, etc.

The overall system consists of four subsystems as shown in Fig.1 which are consistently controlled through the data base. Emphasis has been placed on partial optimizing and data that can be used conveniently and utilized easily by designers.

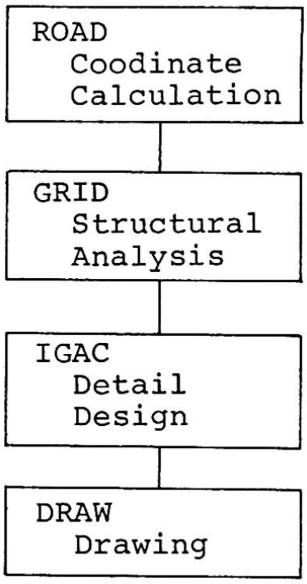

Fig. 1

2.2 ROAD Sub System

This is a universal type system of coordinate calculation. When the form of road, pier layout, main girder and cross beam arrangement are defined, this ROAD Sub System calculates the required values of coordinates. Consequently, the table of values, plan, longitudinal section and cross section are supplied as an output. For the following systems, various figures are filed in phase with each value being taken into consideration.

2.3 GRID Sub System

This system is a structural analysis system which employs a displacement method. When the input of the displacement method is fed independently, the coordinates, stiffness, loads, etc. are mostly fed as input data as far as the GRID is concerned, which is rather complicated for the designers. As for the matters concerning the coodinates, especially, since the results of the above ROAD Sub System are handed from the file, the input load is greatly alleviated.

When girder height is fed into this system as an input, a preliminary analysis is made for a simplified model structure by the stress-method as a preparatory calculation. An assumed stiffness and steel weight are set automatically and, thereafter, the number of input joints is about 200, thus requiring about 20 cards.

2.4 IGAC Sub System

Detailed design is conducted for the main girder section,splices, stiffeners, shear connectors, sway bracings and lateral bracings. As for the coordinates and sectional force, the results of the previous system can be used and, therefore, the designers feed the assembling method of sway bracing and lateral bracing as an input. It is also possible to make various kinds of special designations. Usually, when 10-20 cards are fed as an input, the optimizing process is carried out in the system, one set is decided upon and the design calculation sheet and sectional variation diagrams are produced as an output and filed. However, the designer's personal taste, interchangeability of parts, etc. should also be taken into consideration when the decision is made and, therefore, there arises a demand that some modification should be made after studying the outputs. Meanwhile, questions and modifications can be made by using CRT(IBM 2250).

This system consists of the following three steps;

Step 1; Temporary decision concerning the main girder, cross beam and lateral bracing, preparation of data to be studied(substitute plan included) and filing into Step 2.

Step 2; Question and modification by using CRT device. Filing into Step 3.

Step 3; Preparing a design calculation sheet. Filing into DRAW Sub-System.

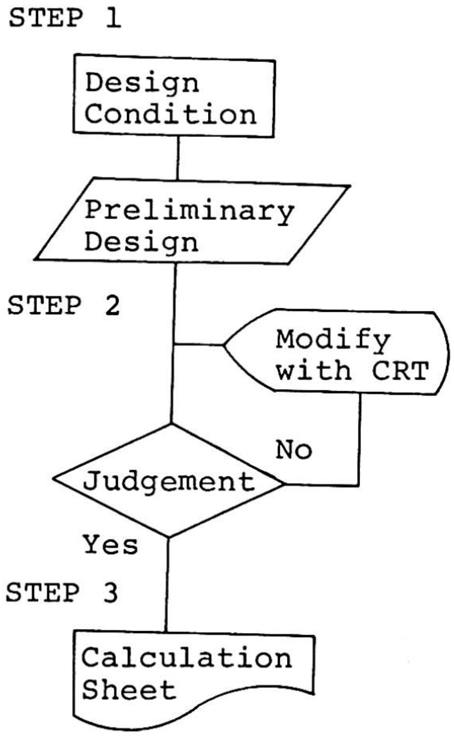

Fig. 2

Step 2 is provided with the CRT pictures of sections, splices, stiffeners, shear connecters, cross beams and lateral bracings. In one particular section, for example;

a. What kind of section can be made if the material at a certain location is changed?
b. What will be the best section if this location is moved 30cm?
c. What will be the thickness of plate when the upper flange width is changed to 50cm?

Various questions such as are listed above are given and if the answers from the system are accepted, the files are renewed accordingly and, thus, the design is modified continuously. Then, the final results are filed for the DRAW Sub System of design drawing.

The final stiffness is filed and the GRID can be reopened by using the file.

Fig.3 An example of the CRT pictures

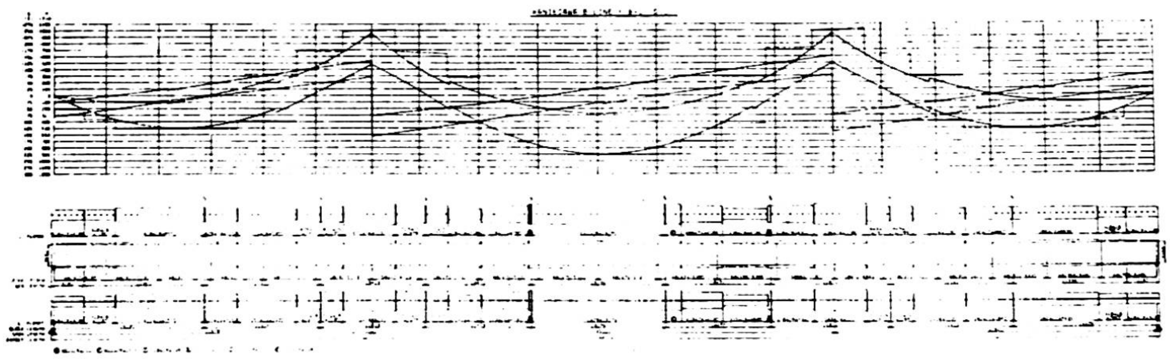

Fig.4 Sectional variation diagram

2.5 DRAW Sub System

This is a system which is used for deciding the details of structures and for making drawings. The results of design calculation are insufficient when they are used as the data for making drawings and, therefore, the mode of structure should be decided in detail. In many cases, however, the details of structure are different depending on each customer. Because of these reasons, the details of structure most often used to meet the standards and design requirements of customers are stored in the system, thus expanding the range of applications. The input designates the items which change the standard of the systm . As for the coordinates, the file is used as a reference and, therefore, the designated item is usually represented by about 10 cards. The outputs are; main girder, cross beam, lateral bracing, detailed design drawing, diagram and the list of steel materials, welding lengths, painting area, etc.

For making the drawings, COM(Computer Output Microfilming) of CALCOMP CO. is used. Unlike the plotter or the drafter in which a pen moves mechanically, this COM is so designed that the locus of an electronic beam is traced on film. One drawing is completed in about five seconds and the operating cost is also very low.

3. Postscript

With this system, the fundamental design(deciding girder arrangement, girder height, etc.) is made after full "traial and error" by means of the ROAD and GRID and, then, the detailed design made by IGAC system is corrected by means of CRT and drawings are made by DRAW. The fundamental design and detailed design are separated, but when the optimum property of design is taken into consideration, we do not believe that there will be much trouble in the actual application if the fundamental values are properly selected.

When this system is used, one designer can complete within one week about 50 drawings, material lists and design calculation sheets for a bridge constructed with five main girders and three span-continuous I girders. Only girder bridges, are described in this report.

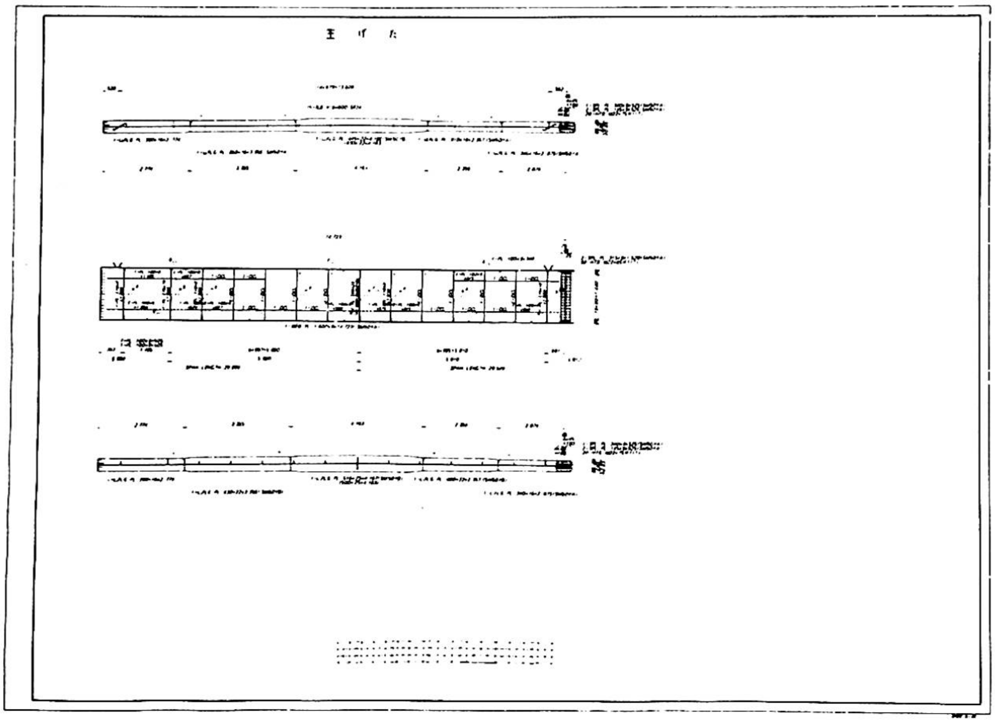

Fig.5 A drawing of main girder by COM

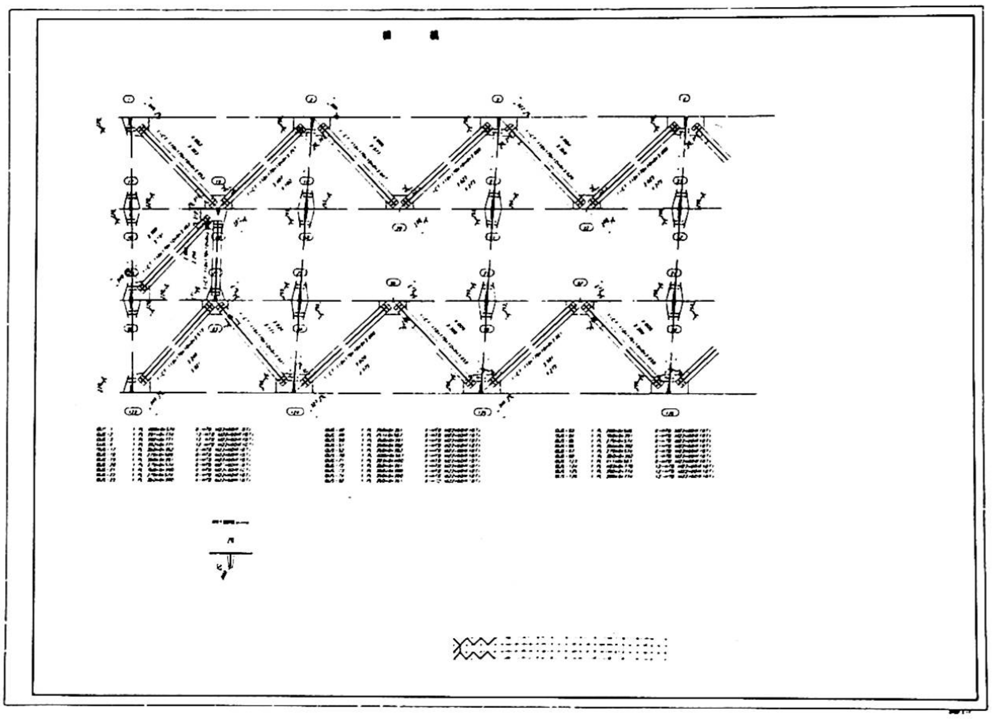

Fig.6 A drawing of lateral bracings by COM

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SUMMARY

A "Total Computer System for Bridges" has been developed, which is aimed at combining the optimum design and the judgement of designers. This system has already been used for actual applications and has produced good results. This report introduces design of a girder bridge in the overall system.

RESUME

Un système global pour le projet de ponts au moyen de l'ordinateur a été développé. Ce système combine le calcul optimal et le jugement de l'ingénieur. Il est déjà utilisé en pratique et donne des résultats excellents. Cet article présente la partie du projet de pont en poutres dans le système global.

ZUSAMMENFASSUNG

Ein totales Computersystem für einen optimierten Brückenentwurf wird entwickelt. Das System verbindet die Absichten des Entwurfes mit einer optimalen Problemlösung; in praktischer Anwendung hat das System bereits gute Resultate geliefert. Am Entwurf von Brückenträgern stellt der vorliegende Beitrag einen Teil des gesamten Computersystems vor.