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**Design of reinforced and prestressed concrete Structures for Fire Resistance**

Calcul et conception des structures en béton armé ou précontraint en vue de leur résistance à l'incendie

Bemessung von Stahlbeton- und Spannbetonbauwerken gegen Brandeinwirkungen

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*How to design prestressed Concrete for a specific Fire Endurance***INTRODUCTION**

Building elements such as beams, slabs, columns, and walls, react to fire in accordance with certain laws of nature. By the use of accepted engineering principles, it is possible to predict through calculations the behavior of prestressed concrete elements which are exposed to fire. The calculation procedures involve the use of physical properties of concrete and steel at high temperatures together with the temperature history and temperature distribution within the element.

**STRUCTURAL BEHAVIOR**

Simple Support: To design a simply supported slab of prestressed concrete for a specific fire endurance, one must know the span length, the loading, the types of steel and concrete, the slab thickness, and the position of the prestressing steel.<sup>(1)\*\*</sup> Because fire endurance of a slab is determined experimentally by exposing the underside of the slab to a standard fire,<sup>(2)</sup> it should be assumed for design purposes that such conditions exist. Under such conditions, the temperature distribution within the slab can be calculated or otherwise determined (e.g., from published fire test data<sup>(3)</sup>) for the fire endurance time. For a given location of the reinforcement, the steel temperature is thus known. The strength of the steel can be determined for that temperature from experimental data relating temperature and strength.<sup>(4,5)</sup> Similarly, the strength of the concrete in the compressive zone can be determined.<sup>(6)</sup> By using the strengths of the steel and concrete thus determined, calculations for ultimate moment capacity can then be made utilizing formulas similar to those for normal temperature conditions.<sup>(1)</sup> By equating the applied moment to the ultimate moment capacity, the required amount of prestressing steel can be calculated. If the amount of steel results in an over-reinforced section, the design must be modified, e.g., by making the slab thicker.

Continuity: For continuous slabs or beams subjected to fire, thermal deformations occur that cause a redistribution of applied bending moments, i.e., the moments over the supports increase until the top reinforcing steel yields and the moments near midspan decrease.<sup>(7,8)</sup> The top reinforcement must be sufficiently ductile and long enough to permit moment redistribution without causing a shear failure.

Restraint to Thermal Expansion: When a fire occurs beneath the floor slab of an interior bay of a multi-bay building, the heated portion tends to expand and exert forces on the surrounding slab. The surrounding slab, in turn, pushes against the heated portion and in effect, externally pre-stresses the slab. When the fire starts, the line of action of the force (called restraint force or thermal thrust) acts near the bottom of the slab.<sup>(9)</sup> The effect of this force on the capacity of the slab is similar to that of "fictitious reinforcement" located at the line of thrust.<sup>(10)</sup> In general, such restraint to thermal expansion greatly increases the slab's fire endurance.<sup>(11)</sup>

Fig. 1 shows diagrammatically the effect of thermal restraint on the behavior of a uniformly loaded member. The thrust,  $T$ , is treated as fictitious reinforcement.

#### CALCULATION PROCEDURES

Procedures for calculating fire endurance of simply supported and continuous beams and slabs follow accepted engineering principles and have been explained elsewhere.<sup>(1,12)</sup> A search of the literature shows a lack of examples of calculations dealing with restraint of thermal expansion of beams and slabs exposed to fire. Such an example is included in this paper, following a discussion of some basic concepts.

Selvaggio and Carlson<sup>(13)</sup> showed that the magnitude of the thermal thrust force,  $T$ , for a given expansion,  $\Delta\ell$ , varies with the "heated perimeter,"  $s$ , and the modulus of elasticity of the concrete:

$$\frac{T_1}{T_0} = \frac{s_1 E_1}{s_0 E_0} \quad \dots(1)$$

where  $T$  = thrust in kips

$s$  = heated perimeter, i.e., the portion of the perimeter of the cross section, normal to the direction of thrust, that is exposed to fire, in inches

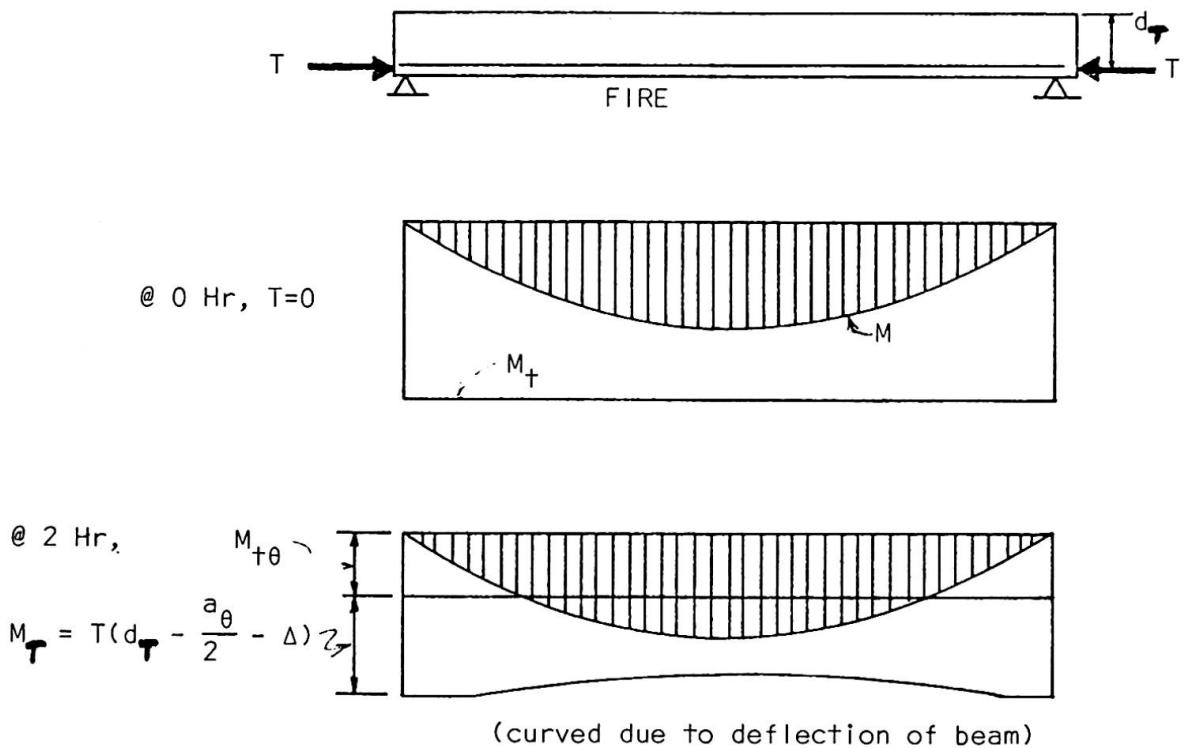


Fig. 1 -- Moment diagrams for axially restrained beam during fire exposure. Note that at 2 hr  $M_{T\theta}$  is less than  $M$  and effects axial restraint permit beam to continue to support load.

$E$  = concrete modulus of elasticity in ksi and subscripts 0 and 1 refer to the reference specimens, i.e., those described in references 9 and 13, and the member in question, respectively.

Issen et al.<sup>(9)</sup> refined Eq. 1 by introducing a term  $z = A/s$ , so that

$$\frac{T_1}{A E_1} = \frac{T_0}{A_0 E_0} \left( \frac{z_0}{z_1} \right) \quad \dots (2)$$

To facilitate calculation of  $T_1$ , Issen et al.<sup>(9)</sup> introduced two dimensionless terms,  $T/AE$ , the thrust parameter, and  $\Delta\ell/\ell$ , the strain parameter, in which  $\ell$  is the heated length in inches. Reference 9 also gives nomograms, shown in Fig. 2, relating the thrust parameter, the strain parameter and  $z$ . Thus for a given member it is possible to estimate  $T_1$  for any particular value of  $\Delta\ell$ , without use of the specific values of  $T_0$ ,  $E_0$ , or  $A_0$ . Note that the nomograms can be used for reinforced as well as pre-stressed concrete of either normal weight or lightweight concrete.

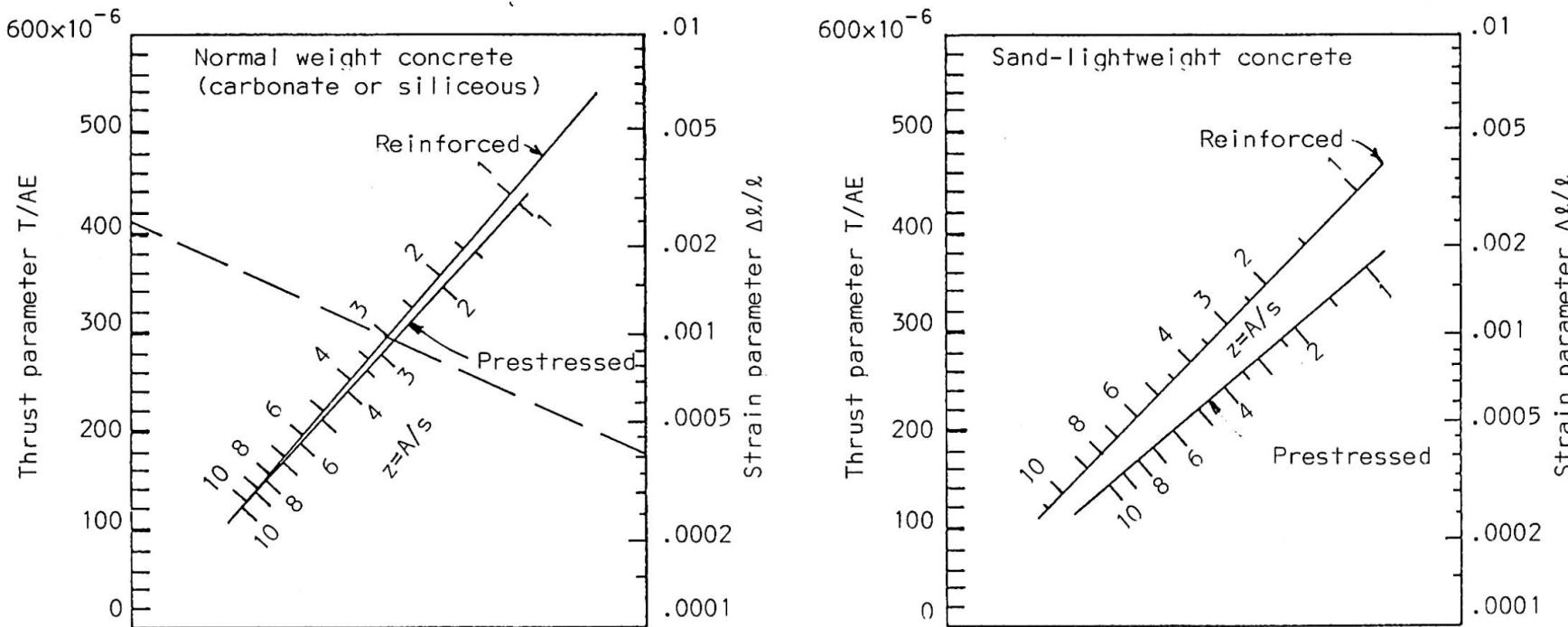


Fig. 2 Nomograms relating thrust parameter, strain parameter, and ratio of cross-sectional area to heated perimeter.<sup>(9)</sup>

In design problems involving restraint forces, it is necessary to estimate the deflection of the fire-exposed member. This can be done by making use of data developed during fire tests of the reference specimens. Fig. 3 closely represents the deflection of the reference specimens in which minimal restraint occurred.

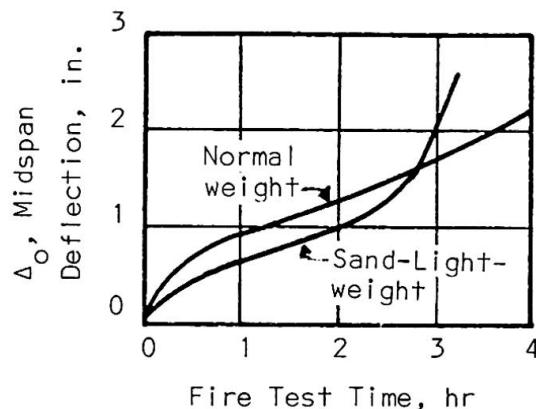


Fig. 3 -- Midspan deflection,  $\Delta_o$ , of reference specimens

The deflection of members can be estimated:

$$\Delta_1 = \frac{\ell_1^2 \Delta_o}{3500 y_{b1}} \quad \dots (3)$$

in which  $\Delta_o$  is obtained from Fig. 3 in in.

$\ell$  = heated length of member in in.

$y_b$  = distance between the centroidal axis of the member and the extreme bottom fiber in in.

A procedure for estimating the thrust requirements for a given fire endurance for simply supported slabs or beams follows:

1. Determine the retained moment capacity,  $M_{t\theta}$ , for the required fire endurance. The units for  $M_{t\theta}$  are in.-kips.
2. If the applied moment,  $M$ , is greater than  $M_{t\theta}$ , estimate the deflection,  $\Delta_1$ , assuming that minimal restraint occurs. Use Fig. 3 and Eq. 3. (If  $M_{t\theta} > M$ , no thrust is needed.)
3. Estimate the location of the thrust line at the supports. For minimal restraint, the thrust line is near the bottom of the

member at the support and can be assumed to be  $0.1 h$  above the support where  $h$  is the overall depth of member.

4. Calculate the magnitude of the required thrust  $T_1$  using the formula:

$$T_1 = (M - M_{+0}) / (d_T - \frac{a_\theta}{2} - \Delta_1) \quad \dots (4)$$

where  $d_T$  is the distance between the thrust line and the top of the member at the supports.

5. Calculate the thrust parameter,  $T_1/A_1 E_1$  and  $z_1 = A_1/s_1$ .
6. Enter Fig. 2 with  $T_1/A_1 E_1$  and  $z_1$  and determine the strain parameter,  $\Delta\ell/\ell$ .
7. Calculate  $\Delta\ell$  by multiplying the strain parameter by the heated length  $\ell$ .
8. Determine if the surrounding structure can withstand the thrust,  $T_1$ , with a displacement no greater than  $\Delta\ell$ . If the structure cannot withstand  $T_1$  with a displacement no greater than  $\Delta\ell$ , either  $M_{+0}$  must be increased or the surrounding structure must be made stiffer.

Example Problem: A parking structure consists of multi-story reinforced concrete columns, L-shaped spandrel beams, and 8-ft wide, 57-ft span double tees with a cast-in-place topping. In much of the structure continuity can be achieved with reinforcement in the topping, but ramp areas consist of a single span. Determine if adequate restraint can be achieved in ramp areas to achieve a 2-hr fire endurance.

For double-tee floor (4-ft wide section):

$$A_{ps} = 7(0.153) = 1.071 \text{ in.}^2$$

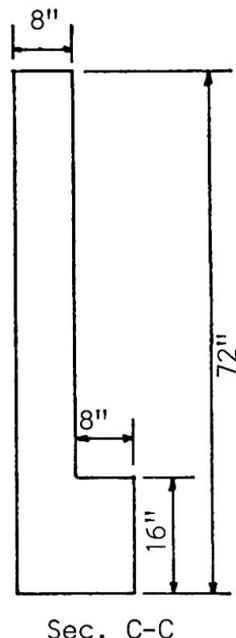
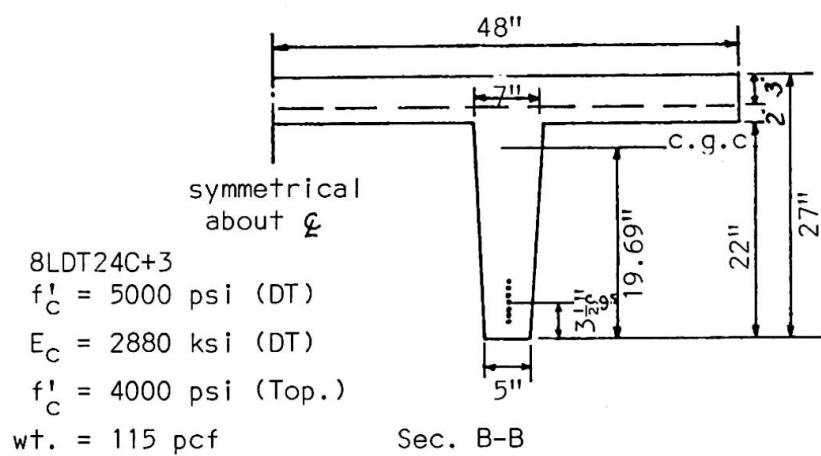
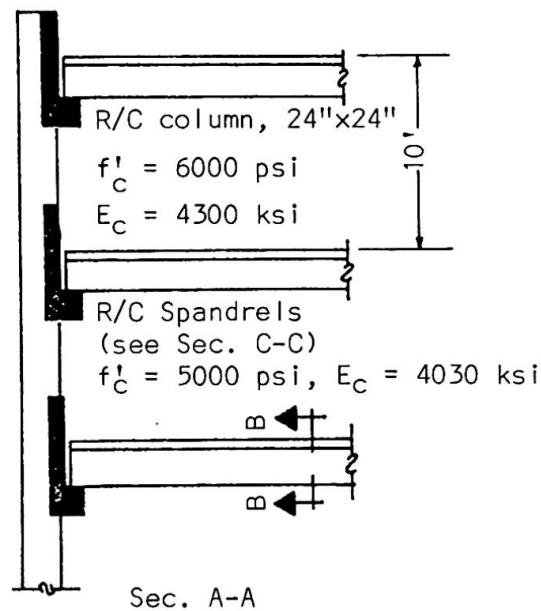
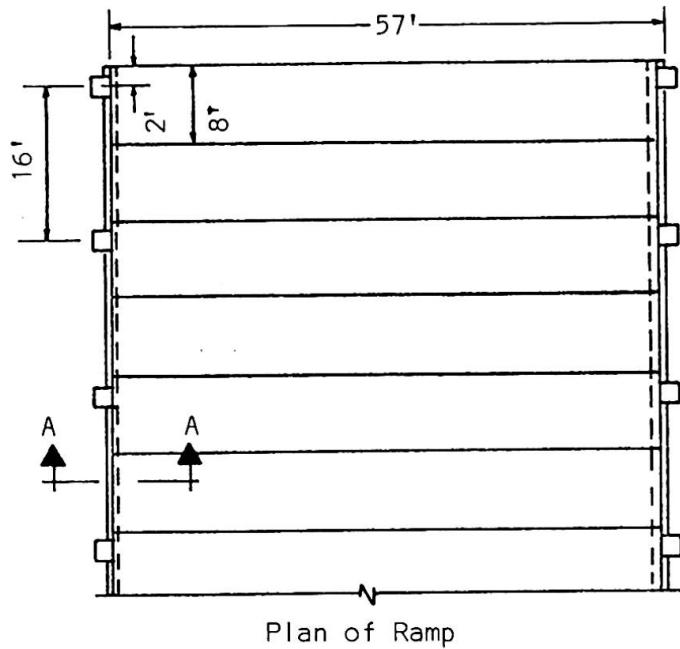
$$A_c = 5(48) + 22(7 + 5)/2 = 372 \text{ in.}^2$$

$$w_d = 372(115)/144 = 297 \text{ lb/ft}$$

$$w_\ell = 75 \text{ psf} = 300 \text{ lb/ft}$$

$$M = (0.297 + 0.300)(57)^2/8 = 242.4 \text{ ft-k} = 2909 \text{ in.-k}$$

$$d = 27 - 3.5 = 23.5 \text{ in.}$$



Solution:

(1) Estimate  $M_{t\theta}$  for double tee at 2 hr

(1.a) Determine temperature of prestressing steel,  $\theta_s$ , from Fig. 4.

To enter Fig. 4, the beam width at the steel centroid,  $b_o$ , and the distance between the steel centroid and the bottom of the beam,  $u$ , must be determined. [Note that Fig. 4 applies only to the temperatures along the vertical centerline of beams, thus if the steel is located away from the centerline, the beam isotherms would have to be plotted to determine the steel temperature more accurately.]

$$u = 3.5 \text{ in.}$$

$$b_o = 5.0 + \frac{3.5}{22} (7 - 5) = 5.3 \text{ in.}$$

From Fig. 4,  $\theta_s = 895^{\circ}\text{F}$

(1.b) The strength of prestressing steel at  $895^{\circ}\text{F}$  is 35.5% [From Fig. 5] of its strength at normal temperatures. Thus

$$f_{pu\theta} = 0.355 f_{pu} = 0.355(270) = 95.8 \text{ ksi}$$

(1.c) The stress in the steel at ultimate is

$$f_{ps\theta} = f_{pu\theta} \left(1 - \frac{0.5A_{ps} f_{pu\theta}}{bd f'_c}\right) = 94.7 \text{ ksi}$$

$$\text{and } a_{\theta} = \frac{A_{ps} f_{ps\theta}}{0.85 f'_c b} = 0.62 \text{ in.}$$

$$(1.d) \quad M_{t\theta} = A_{ps} f_{ps\theta} \left(d - \frac{a_{\theta}}{2}\right) = 2352 \text{ in.-k}$$

$M_{t\theta} < M$ , so added capacity is needed.

(2) Estimate midspan deflection of double tee assuming minimal restraint at 2 hr for sand-lightweight concrete.

From Fig. 3,  $\Delta_o = 1.0 \text{ in.}$

$$\text{From Eq. 3, } \Delta_1 = \frac{\ell^2 \Delta_o}{3500 y_{b1}}$$

$$\Delta_1 = \frac{(57 \times 12)^2 (1.0)}{3500 (19.69)} = 6.79 \text{ in.}$$

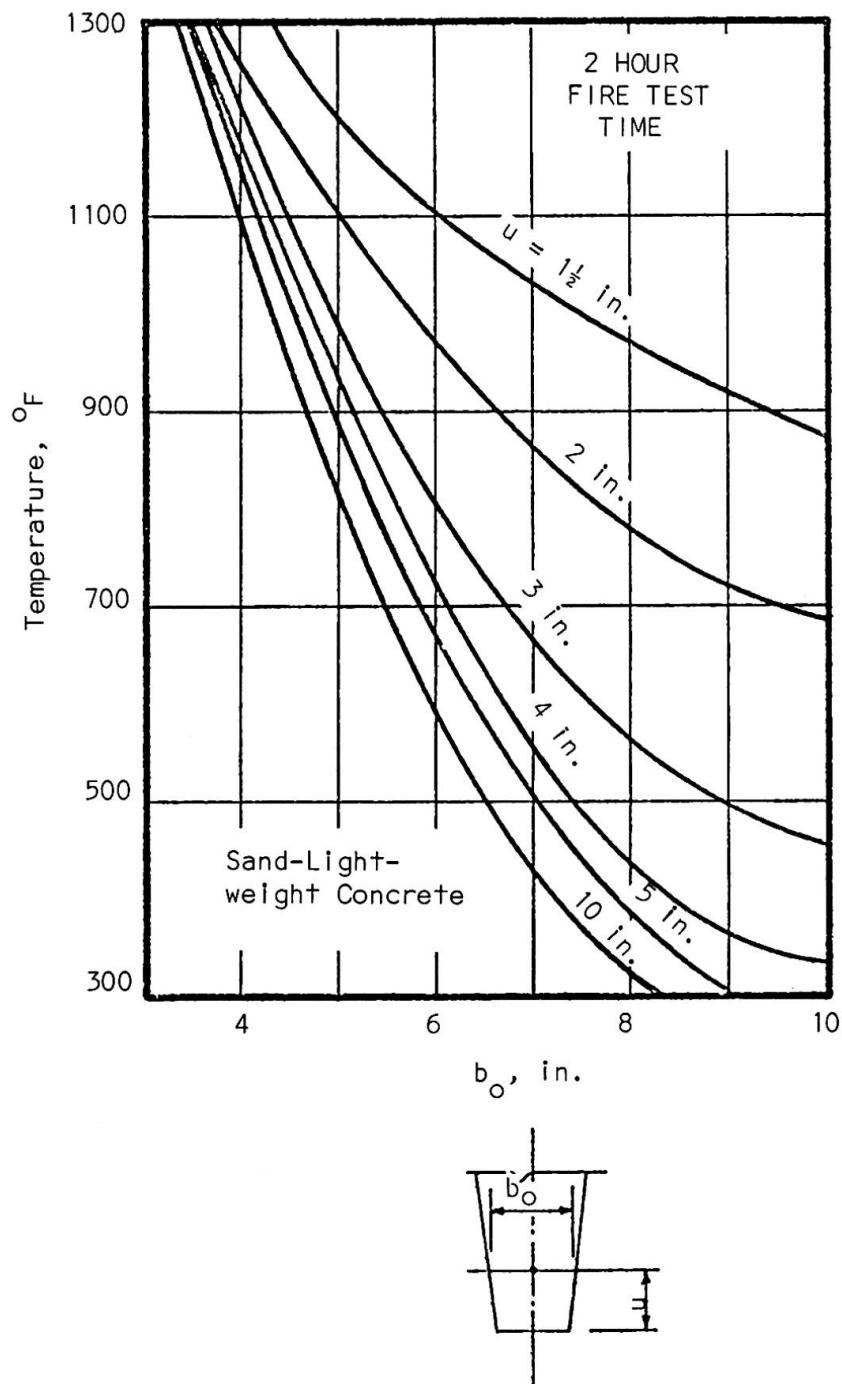


Fig. 4 -- Temperatures at 2 hours along the vertical centerline of beams made of sand-lightweight concrete.

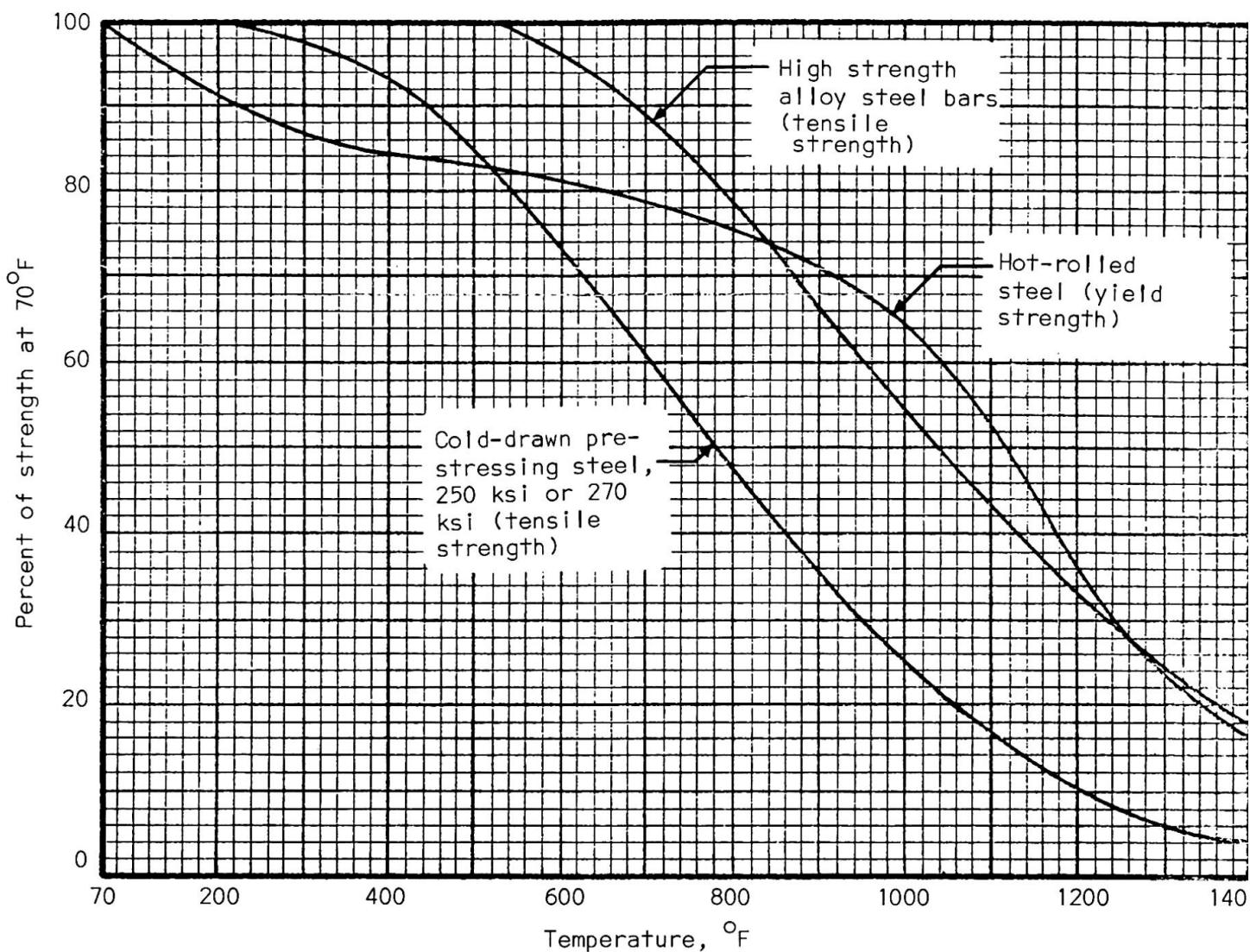


Fig. 5 -- Temperature-strength relationships for hot-rolled and cold-drawn steels.

(3) Determine the position of the thrust line at the supports.

Assume that the thrust line is 0.1 h above support

$$d_T = 27 - 0.1(27) = 24.3 \text{ in.}$$

(4) Calculate required thrust  $T_1$

$$T_1 = \frac{(M - M_{+θ})}{(d_T - 0.5a_{θ} - Δ_1)}$$

$$a_{θ} = \frac{2909}{2352}(0.62) = 0.77 \text{ in.}$$

$$T_1 = \frac{(2909 - 2352)}{(24.3 - 0.39 - 6.79)} = 32.5 \text{ k/beam}$$

(5) Calculate the thrust parameter  $T_1/A_1 E_1$  and  $z_1 = A_1/s_1$   
 $T_1/A_1 E_1 = 32.5/372(2880) = 30 \times 10^{-6}$   
 $z_1 = 372/[48 + 2(22)] = 4.0 \text{ in.}$

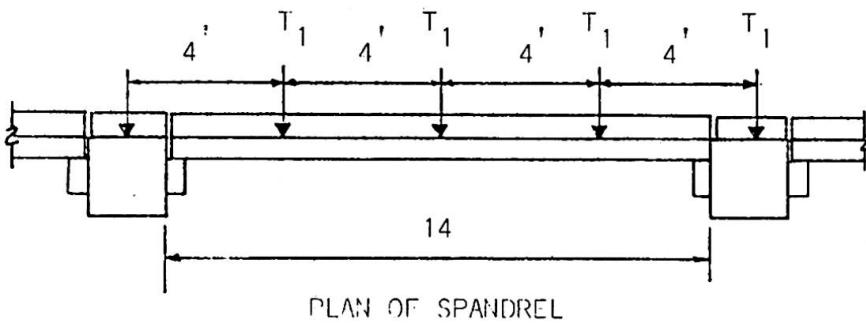
(6) Determine the strain parameter,  $\Delta\ell/\ell$ , from Fig. 2:

$$\Delta\ell/\ell = 0.0026$$

(7) Determine  $\Delta\ell$ :

$$\Delta\ell = 0.0026(57 \times 12) = 1.78 \text{ in.}$$

(8) Determine if spandrel can withstand thrust from the double tees:



(8.a)  $M$  at spandrel midspan: (assume spandrel span = 13 ft 8 in. = 164 in. center to center of supports)

$$M = 1.5T_1(82) - T_1(48) = 75T_1 = 75(32.5) = 2438 \text{ in.-k}$$

Spandrel must withstand  $M = 2438 \text{ in.-k}$  with a load factor of 1.4 because the thrust occurs early and remains essentially constant. Determine the amount of reinforcement needed. Assume  $d = 14 \text{ in.}$ ,  $a = 4 \text{ in.}$ , and  $f_y = 60 \text{ ksi}$ :

$$A_s = \frac{1.4M}{f_y(d - a/2)} = \frac{1.4(2438)}{60(14 - 2)} = 4.74 \text{ in.}^2$$

Check reinforcement index,  $\omega$ :

$$\omega = \frac{A_s f_y}{bd f'_c} = \frac{4.74(60)}{16(14)(5)} = 0.25 < 0.30 \text{ O.K.}$$

Check  $a$ :

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.74(60)}{0.85(5)(16)} = 4.17 \text{ in.} \approx 4 \text{ in. O.K.}$$

(8.b) Estimate lateral deflection of spandrel due to thrust. Assume cracked section,  $I_{cr} = 3300 \text{ in.}^4$ ,  $E_c = 4030 \text{ ksi}$ .

$$\Delta = \frac{T_1}{48EI} [(164)^3 + 6(34)(164)^2 - 4(34)^3] = 0.50 \text{ in.}$$

(8.c) Estimate column deflection assuming fire beneath top level.

For column,  $I = 27,600 \text{ in.}^4$ ,  $E_c = 4300 \text{ ksi}$ .

For top level,

$$\Delta = \frac{Ph^3}{3EI} = \frac{4T_1 h^3}{3EI}$$

where  $h$  is the unsupported story height =

$$10 \text{ ft} - (27 \text{ in.}) = 120 - 27 = 93 \text{ in.}$$

$$\Delta = \frac{4(32.5)(93)^3}{3(4300)(27,600)} = 0.29 \text{ in.}$$

(8.d) For top level, displacement at midspan of spandrel =

$$2(0.50 + 0.29) = 1.58 \text{ in.} < 1.78 \text{ in.}$$

Thus adequate restraint will occur to achieve 2-hr fire endurance provided that the space (total) between the ends of the double tees and the vertical face of the spandrel beams is less than  $1.78 - 1.58 = 0.20 \text{ in.}$

#### NOTATION

$a$  = depth of equivalent rectangular stress block at ultimate load, and is equal to  $A_{ps} f_{ps} / 0.85 f'_c b$  or  $A_s f_y / 0.85 f'_c b$  (in.)

$A$ ,  $A_c$  = cross sectional area of a member subjected to thrust (in. $^2$ )

$A_s$  = area of reinforcing steel (in. $^2$ )

$A_{ps}$  = area of prestressing steel (in. $^2$ )

$b$  = width of compression zone (for use in flexural calculations) (in.)

$b_o$  = width of a beam or joist at centroid of reinforcement (for use in estimating temperature during fire exposure) (in.)

$d$  = distance between centroid of reinforcement and extreme compression fiber (in.)

$d_T$  = distance between line of action of thrust at the supports and extreme compression fiber (in.)

$E$ ,  $E_c$  = modulus of elasticity of concrete (ksi)

$f'_c$  = compressive strength of concrete (ksi)

$f_{ps}$  = stress in prestressing steel in flexural member at ultimate load (ksi)

$f_{pu}$  = ultimate strength of prestressing steel (ksi)

$f_y$  = yield strength of hot-rolled steel (ksi)

$h$  = overall depth of flexural member (in.)

$h$  = unbraced height of column (in.)  
 $I$  = moment of inertia of cross section (in.<sup>4</sup>)  
 $I_{cr}$  = moment of inertia of cracked cross section of flexural member (in.<sup>4</sup>)  
 $l$  = heated length a flexural member (in.)  
 $\Delta l$  = increase in length due to thermal expansion (in.)  
 $M$  = service load bending moment (in.-k)  
 $M_t$  = theoretical moment strength (in.-k)  
 $M_T$  = ultimate moment due to thrust resulting from restraint of thermal expansion (in.-k)  
 $s$  = heated perimeter of a member, i.e., that portion of the perimeter of a section of a member, normal to the direction of the thermal thrust, which is exposed to fire (in.)  
 $T$  = thermal thrust (k)  
 $u$  = distance from bottom of slab or beam to a point within the member, e.g., the distance from the underside of a slab to the center of a prestressing strand (in.)  
 $w$  = uniformly distributed load on a flexural member, in general  $w = w_d + w_l$  in which the subscripts  $d$  and  $l$  indicate dead and live loads (k/in. or k/ft)  
 $y_b$  = distance between centroidal axis of flexural member to extreme bottom fiber (in.)  
 $z$  =  $A/s$  (in.)  
 $\Delta$  = deflection (in.)  
 $\Delta l$  = increase in length due to thermal expansion (in.)  
 $\theta$  = temperature ( $^{\circ}$ F)  
 $\theta_s$  = temperature of steel ( $^{\circ}$ F)  
 $\omega = A_s f_y / b d f'_c$

#### Subscripts

$c$  = of concrete  
 $p$  = of prestressing steel  
 $s$  = of reinforcing steel  
 $o, 1$  = of reference specimens and member in question  
 $\theta$  = as affected by temperature  
 $b$  = with reference to the bottom fiber

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## SUMMARY

Building elements such as beams, slabs, columns, and walls, react to fire in accordance with certain laws of nature. By the use of accepted engineering principles, it is possible to predict through calculations the behavior of prestressed concrete elements which are exposed to fire. The calculation procedures involve the use of physical properties of concrete and steel at high temperatures together with the temperature history and temperature distribution within the element.

## RESUME

Les éléments de bâtiments tels que poutres, dalles, colonnes et parois, réagissent au feu selon certaines lois naturelles. Par application de principes de calcul reconnus, il est possible de prédire- et de chiffrer- le comportement d'éléments en béton précontraints, exposés au feu. Les procédés de calcul font appel aux propriétés physiques à haute température des aciers et bétons, ainsi qu'à l'évolution de la température dans le temps et dans les éléments.

## ZUSAMMENFASSUNG

Bauelemente, wie Balken, Decken, Stützen und Wände reagieren auf Feuer entsprechend bestimmten Naturgesetzen. Unter Verwendung angenommener Prinzipien ist es möglich mittels Berechnungen das Verhalten vorgespannter, dem Feuer ausgesetzter Stahlbetonelemente vorauszusagen. Die Berechnungsverfahren enthalten die Anwendung physikalischer Eigenschaften von Beton und Stahl bei hohen Temperaturen sowie die Temperaturrentwicklung und -verteilung innerhalb des Elementes.

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