Ultimate Strength Design Formulae for Simple Tubular Joints

Formules du calcul à la résistance limite pour les noeuds simples
de profilés circulaires

Formeln für die Ermittlung der Traglast von einfachen Knoten-
punkten in Rohrprofilen

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1. INTRODUCTION

One of the Committees in the Architectural Institute of Japan is carrying out revision of the "Specification for Design of Tubular Structures in Steel" that was first published in 1962. Although the Specification is applicable to the building structure, it may provide a good deal of information for the design of offshore structures of tubular members.

This report discusses about the experimental grounds of those provisions for the design of the tubular X, T, Y and K-joints for static loadings which are presently under deliberation in the Committee. The provisions are based on the ultimate strength formulae that were selected for such use from the results of a series of regression analyses of the test data obtained in Japan and the U.S.A. since 1963.* The derivation of the formulae is described in detail in Reference [1].

All the existing ultimate strength formulae for these joints have been sub-jected to two questions as follows:
1. Most of the existing ultimate strength formulae tend to overemphasize the strength of the T, Y and K-joints when the diameter to thickness ratio of the chord (D/T) becomes greater than about 50.
2. The strength of the K-joint increases as the two braces intersect and then overlap with each other. This behavior is not adequately taken into account in the existing formulae.

To overcome these difficulties the reanalyses of the test data were carried out as a continuation of the past studies by the authors and their colleagues[2],[3].

The strength of the tubular joints under static loads is an influencing factor in determining the design of any tubular structure and yet it still covers some areas that are not fully understood at the present stage, which may be clear from the later discussions in this report. In this regards the authors wish to welcome any comment on the proposals presented in this report.

2. DEFINITION OF ULTIMATE STRENGTH

The ultimate strength referred herein is the maximum axial compressive force applied at the brace ends when a joint fails as a result of excessive local bend-ing deflections of the chord walls. The strength of a joint that fails owing to

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* Additional test data are now being gathered from Europe through the activity of the Subcommission XV-E of the IIW.

failures in a member, such as fracture of the tension brace or local instability of the compression brace, is outside the scope of the present definition of the ultimate strength of the joint.

The local failure of the chord walls occurs also at the points where tension braces are attached. The final rupture of these joints is controlled by cracking of the chord and/or brace walls at the toes of the brace to chord welds. The joints that fail in this manner always attain a far greater strength than that in the former case where the braces are under compression. The ultimate strength data for the joints under tension should therefore be treated separately and are excluded from the regression analyses in Referenc [1].

Table 1 Proposed Ultimate Strength Formulae

Type of Joints	Predicted Ultimate Strength, P_u
X-Joint	$P_u = \dfrac{6.57}{1 - 0.810\, d/D}\, \sigma_y T^2$ (a)
T-Joint	$P_u = 6.43\, [1 + 4.60\,(\tfrac{d}{D})^2]\, \sigma_y T^2$ (b)
T, Y and K-Joints $\bar{N} = N/\sigma_y A$ $A = \pi (D-T)T$	$P_u = 2.11(1 + 12.1\tfrac{d}{D})\, f_g\, f_\theta\, f_{\bar{n}}\, \sigma_y T^2$ (c) $f_g = 1 + 3.88(1 - 20.9\tfrac{T}{D})(1 - 0.530\tfrac{d}{D}) \cdot$ $[1 + \tfrac{2}{\pi} \tan^{-1}(0.237 - 0.183\,\tfrac{g}{T})]$, but not less than 1.0 $f_\theta = (1 - 0.167\cos\theta + 0.049\cos^2\theta)/\sin\theta$ $f_{\bar{n}} = 1 + 0.262\,\bar{N} - 0.391\,\bar{N}^2$ ($\bar{N}$: positive for tension)

According to the past tests, most tubular joints reached the maximum load after full plastic deflections of the chord walls were produced at the local portions where the braces were attached, and then unloading took place. A typical load-deflection curve of such joints is shown by the curve 1 in Fig. 1. In some joints, however, the overall stiffness increased again after they sustained full plastic deformations of the chord walls and eventually carried a greater load than the first maximum load. The load-deflection curves of the latter type are shown by the curves 2 and 3 in Fig. 1. The ultimate strengths used for the analyses were the first maximum loads that were attained by the joints after sufficient areas of the chord walls yielded.
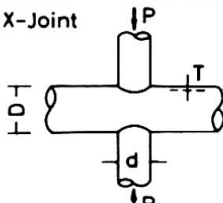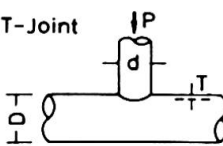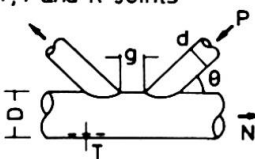
K-joints often fail in a combination of various failure modes depending upon the dimensions of the joints. Even though the final failure of a K-joint was governed by one of the other modes than the excessive local deflections of the chord walls, the ultimate strength of the joint was included in the data so far as the joint sustained full plastic deflections of the chord walls under the compression brace at the maximum load.

3. PROCEDURE OF REGRESSION ANALYSIS
    The multiple regression analyses were carried out to develop the best-fit equations of the ultimate strengths of the joints. In the process of building a mathematical model for a prediction equation, it was first assumed that the joint was able to be replaced by a simple and fictious structure of which ultimate strength would represent the ultimate strength of the actual joint.

    Such a simplified model structure is a ring with an effective width Be. The ring has the same diameter D, thickness T and yield stress σ_y as those of the actual joint and is subjected to concentrated forces acting at d distant points, where d is the outer diameter of the brace. These concentrated forces represent the axial compressive force P in the brace.

According to the simple plastic theory, the collapse load of the ring Pu is given by the equation,

$$Pu = \frac{Be}{a} f_0\left(\frac{d}{D}\right) \sigma_y T^2 \qquad (1)$$

where a is the mean radius of the chord. Be and $f_0(d/D)$ are functions of geometrical parameters of the joint and vary with the type of the joint.

Therefore, the model may be written in the form,

$$Pu = f_0\left(\frac{d}{D}\right) f_1\left(\frac{d}{D}, \frac{T}{D}, \frac{g}{T}, \theta, \overline{N}\right) \sigma_y T^2 \varepsilon$$

$$(2)$$

Fig. 1   Examples of Load-
          Deflection Curves

in which Pu is the ultimate strength of the joint, g is the clear space (gap) between the two braces, θ is the angle of intersection between the compression brace and the chord and $\overline{N}$ is the dimensionless axial stress in the chord (See Table 1). ε is the error term, which was assumed to be multiplicative rather than additive because the model (2) consists of multiplicative terms of the influencing factors each of which has a certain physical meaning [1].   It is assumed here that errors $\ln(\varepsilon)$ are independent random variables with mean zero.

Since the postulated model was nonlinear in the parameters, the linearization and iterative techniques were exercised to fit the model by the method of least squares [4].   A series of such analyses were performed with several alternative models for the functions f_0 and f_1 of Eq. 2.   The resultant regression equations were compared on the basis of the "multiple correlation coefficient R^2".   The final selection of an equation was made such that the selected equation would explain the variation of the ultimate strength data better (attain a larger R^2) with less predicting variables.

In Reference [1] are shown all the data used for the analyses and also the reference sources of them.   An effort was made to utilize as far as possible measured values rather than nominal values for the independent variables.   Although the yield stress in the circumferential direction may be more meaningful in this model, such yield stress is not usually measured in most experimental works. σ_y,s adopted herein are the longitudinal yield stresses measured on tensile coupons cut from the as-rolled chord materials.

4. RESULTS OF ANALYSES AND DISCUSSIONS

The three equations shown in Table 1 were selected as the ultimate strength prediction equations for the tubular X, T, Y and K-joints.   Eq. c is applicable to any of the T, Y and K-joints, where g is infinitely large in the T and Y-joints and becomes negative when the braces overlap in the K-joint. R^2 was of 92%, 95% and 91% in Eqs. a, b and c, respectively.

The residuals provided by Eq. c are plotted overall in a form of a frequency histogram (Fig. 2).   It appears that the residuals follow a normal distribution.   According to a chi-square goodness of fit test this assumption of normality was found to be acceptable at the 0.05 level of significance.   From this it may not be unreasonable to assume that, if the model is correct, errors $\ln(\varepsilon)$ are normally distributed.

Since the models are nonlinear, statistical tests that are true for the linear case do not apply.   However, since the number of observations n is large, the 95% confidence limits for an
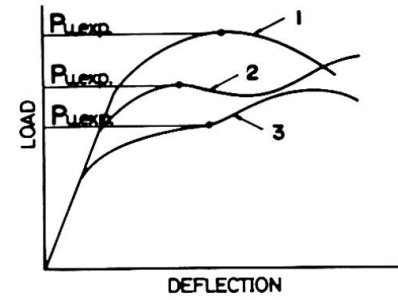

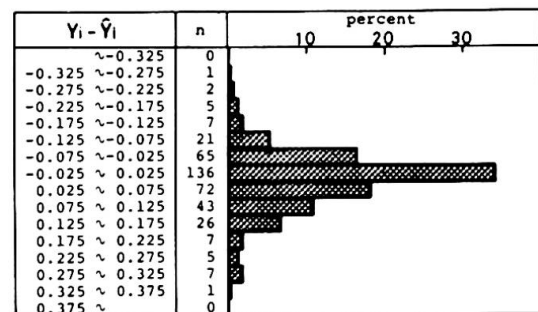| $Y_i - \hat{Y}_i$	n	percent
～-0.325	0	
-0.325 ～-0.275	1	
-0.275 ～-0.225	2	
-0.225 ～-0.175	5	
-0.175 ～-0.125	7	
-0.125 ～-0.075	21	
-0.075 ～-0.025	65	
-0.025 ～ 0.025	136	
0.025 ～ 0.075	72	
0.075 ～ 0.125	43	
0.125 ～ 0.175	26	
0.175 ～ 0.225	7	
0.225 ～ 0.275	5	
0.275 ～ 0.325	7	
0.325 ～ 0.375	1	
0.375 ～	0	

Fig. 2   Histogram of Residuals

individual predicted value are approximately given by

$$Pu \cdot e^{\pm 2s} \tag{3}$$

where s is an estimate of standard deviation and is approximately obtained by the equation,

$$s = \sqrt{\frac{(\text{residual sum of squares})}{n - p}}$$

in which p is the number of parameters in the regression equation. The above statement is valid only when the errors $\ln(\varepsilon)$ are normally distributed.

The predicted mean value Pu and the approximate 95% confidence limits according to (3) are compared with the test results "Pu,exp" in Figs. 3 through 6. These plots indicate no strong abnormality in the residuals and the present regression analyses would not appear to be invalidated.

It is important to note that the formulae in Table 1 are applicable only within the ranges of variation of the predicting variables. Figs. 8, 9 and 10 illustrate how the predicting variables varied in the test data.

The Japanese Specification referred to earlier tentatively assumes a factor of safety of 2 on the predicted ultimate strengths of the X, T, Y and K-joints. The allowable force may be increased 50% above Pu/2 when the joint is under combined permanent and temporary loadings. This safety factor appeared to be conservative from Figs. 3 through 6. In order to calculate a probability of failure for a joint, however, it is necessary to know probability distributions of loads and the yield stress of the materials.

5. COMPARISON OF PROPOSED FORMULAE WITH EXISTING FORMULAE

The proposed formulae and the test results are compared with the formulae recommended in the AWS and DNV-Codes [5],[6] in Figs. 3 through 6 where factors of safety are not taken into account. The DNV-formula agrees well with Eq. a as well as with the test results for X-joints. However, both the AWS and DNV-formulae are not necessarily consistent with the formulae b and c nor with the test results for T, Y and
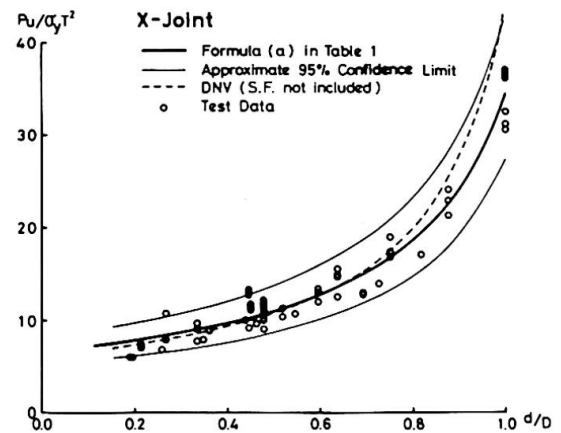

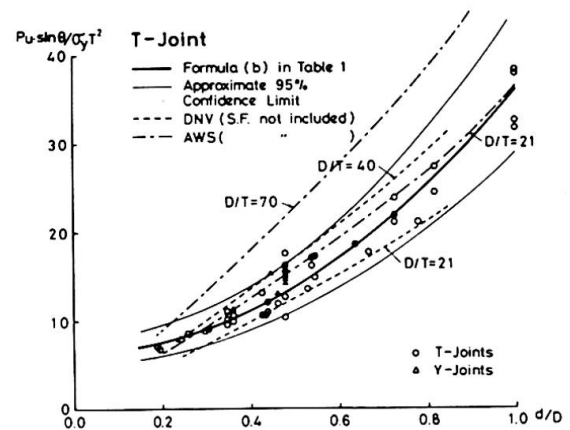Fig. 3 Predicted Ultimate Strengths and Test Results

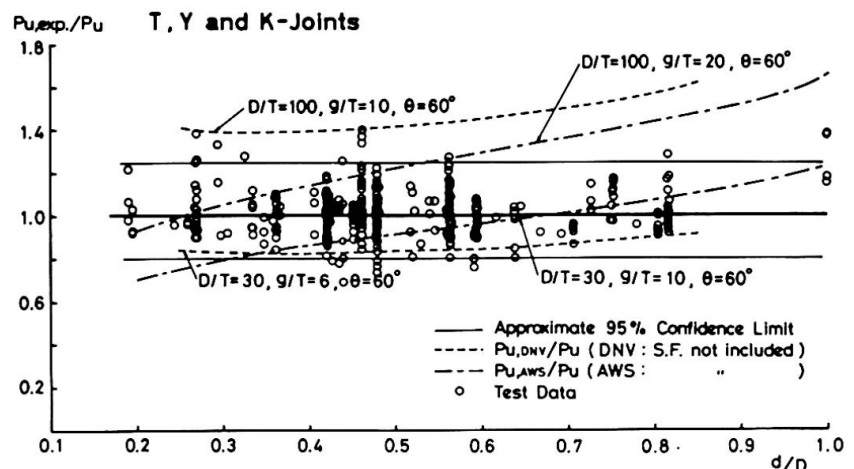Fig. 4 Predicted Ultimate Strengths and Test results for T and Y-Joints

Fig. 5 Ultimate Strengths of T, Y and K-Joints: Comparisons between Formula c, AWS and DNV Formulae and Test Data

k-joints.   In most cases, the AWS and DNV-formulae are risky when D/T=100 and
are too conservative when g/T is less than about 8.   Response of the AWS formu-
la to a variable d/D looks to be different from what is observed in the test re-
sults for K-joints.   Both the formulae are generally applicable to the T and Y-
joints and the K-joints with extended braces, when D/T is less than 40.

Another comparison is made between Eq. c and the formula by Okumura et al
[7] in Fig. 7.   The ultimate strengths of T, Y and K-joints predicted by the
Okumura's formula are scattered between the two dashed lines in the figure (when
θ=60°).   This formula does not overestimate the strength of these joints with
large D/T ratios, but it again is too conservative for a majority of K-joints
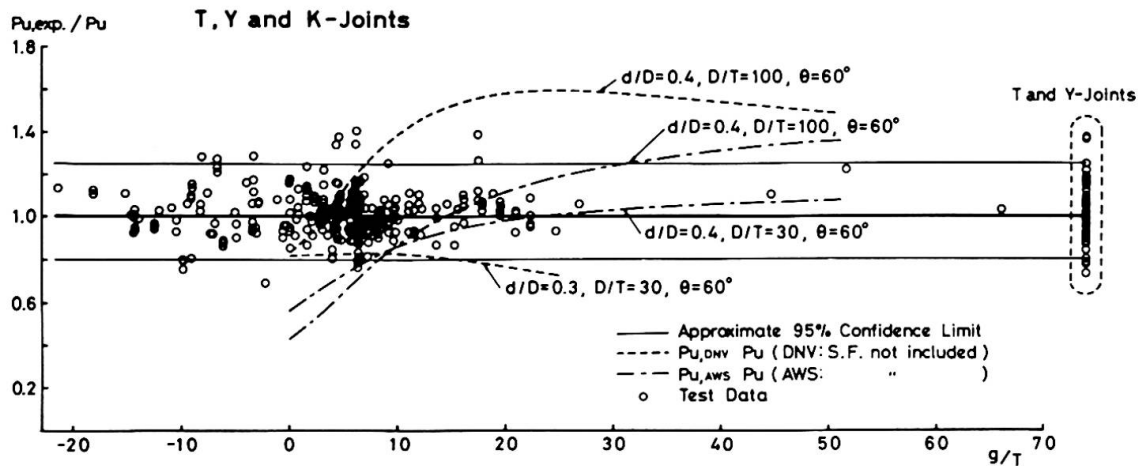with intersecting braces.

Fig. 6   Comparisons between Formula c, AWS and DNV Formulae and Test Data
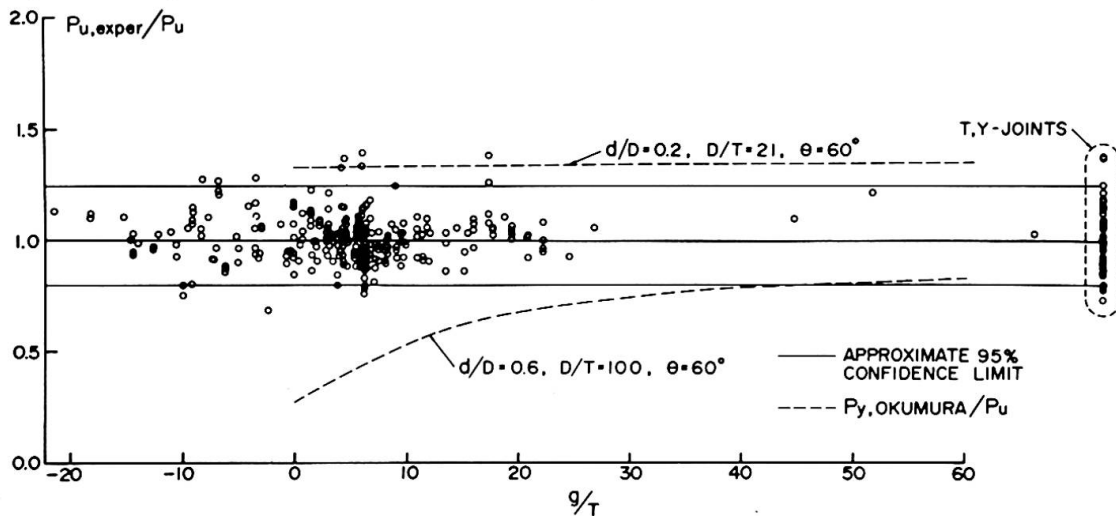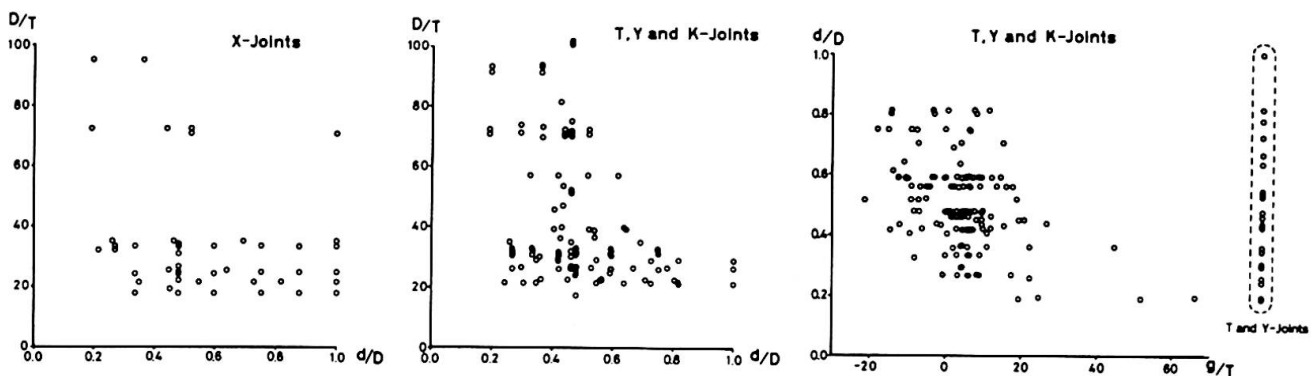

Fig. 7   Formula by Okumura et al Compared with Test Results (T, Y and K-Joints)

(from left) Figs. 8, 9, 10   Independent Variables in Test Data

6. CONCLUSIONS

The ultimate strength formulae summarized in Table 1 is applicable to the tubular X, T, Y and K-joints with a wide range of variation of each of the geometrical parameters.   The formula c in the table can also applies to the K-joints with overlapping braces, which may be the first of such examples.

The test data, however, are still unavailable for some important ranges of variation of predicting variables.   Examples of the areas that require further studies are:

1. The joints with a very heavy chord (D/T<20) and with a very light chord (D/T>100).   The joints in these two extremities are often used in Jack-up and semi-submersible type offshore rigs, respectively.
2. The K-joint with large braces (d/D≃1).
3. Resistance of the joints under bending at the brace ends.   It is to be noted that in the regression analyses an effect of secondary bending moments on the strength of the K-joint was treated merely as a factor that induces random errors.
4. The joints in high strength steels.   The effects of material properties and heat treatments are still unaccountable factors that require additional work.   Most of the test results were obtained through the joints made of cold formed tubes in mild steels or in low alloy medium strength steels.

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4. N. R. Draper et al, "Applied Regression Analysis," John Wiley & Sons (1966)
5. AWS-Structural Welding Code, Sect. 10, AWS D1.1-72 (1972)
6. DNV, "Rules for the Design, Construction and Inspection of Fixed Offshore Structures," Det Norske Veritas (1974)
7. Okumura et al, "Estimation of Strength of Tubular Joints." Preliminary Report, 10th Congress of IABSE, Tokyo (Sept. 1976)

SUMMARY

This report presents the design formulae for the tubular X, T, Y and K-joints under static loads. It also discusses about the experimental grounds on which the formulae are based. The proposed formulae are compared with various existing formulae.

RESUME

Ce rapport présente les formules du calcul pour les noeuds de profilés circulaires, en forme de X, T, Y et K sous l'influence des charges statiques. Il présente aussi les bases expérimentales qui ont permis l'établissement de ces formules. Les formules proposées sont comparées avec d'autre formules existantes.

ZUSAMMENFASSUNG

Dieser Bericht enthält Bemessungsformeln für X, T, Y und K-Knoten von Hohlprofilen, unter statischen Belastungen. Die den Formeln zugrundeliegenden experimentellen Daten werden angegeben. Die Formeln werden schliesslich mit verschiedenen bereits bekannten Formeln verglichen.