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## Basic Optimum Design Diagrams of Highway Plate Girders

Diagrammes fondamentaux pour le calcul optimum de ponts-poutres à âme pleine

Grunddiagramme für den optimalen Entwurf von Vollwand-Brückenträgern

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### 1. INTRODUCTION

In the structural system optimization process multistage systematic comparisons and decisions in such as behavior analysis, fabrication, erection, appearance etc. have to be made with regard to a number of optimized solutions of a wide variety of different structural schemes. For the progress of such a design process, the developments of the optimum design programs for specific structures are the essential efforts. The design approaches centered on this problem have been developed principally based upon the use of a combination of mathematical programming or rigorous numerical search techniques and structural analysis method. However the man-machine communication using these sophisticated optimization programs causes numerical complexity and large computer costs considerably in the large scale structural systems, accordingly, to avoid the difficulties and to make a realistic large scale system optimization process reliable, more practical and efficient design approaches have been expected [7,8].

On this problem a simplified optimum design process for minimum cost design of constant-depth highway plate girders is presented in this paper on the basis of the data manipulation of system optimum design diagrams developed by the graphical optimum design method [1,2] and the fundamental characteristics of the diagrams are discussed.

### 2. DESIGN CONDITIONS

The bridge types to be objective in this paper are 1st class simple span, 2 and 3 span continuous highway welded plate girders with constant web height throughout the bridge length. The range of bridge length, width, span ratio and the number of segments are indicated in Table 1. The arrangements of the main girders and thicknesses of reinforced concrete slabs are assumed as shown in Fig. 1 and Table 2 with due regard to the load distributions and appearances of the bridge superstructures.

The design variables to be determined are the steel type,  $M$ , all sectional dimensions,  $X$ , and the length,  $l$ , to be used for each girder segment.

Design criteria imposed on the steel girder section are constraints on allowable stresses, plate thicknesses for stabilities of the girder and minimum rigidities of vertical and horizontal stiffeners, and which, as well as the displacement constraints of the girder, are taken from the "Specifications for Steel Highway Bridges" [5]. Discrete constraints on commercial availability of plate thicknesses are also considered. The steel types available for the design are assumed as SS41, SM50 and SM58 steels which physical and economical characteristics are described in the Specifications and the "Table of Prime Costs for Steel Highway Bridges" [6] respectively.

The total cost of the girder,  $TCOST$ , is assumed to consist of material cost,  $CM$ , fabrication cost,  $CFF$ , and welding cost,  $CWF$ , which are evaluated with reference to Ref. [6].

$$TCOST = \sum_{i=1}^{NS} COST_i \times l_i$$

$$= \sum_{i=1}^{NS} [CM_i(X, BP, SP, EX) + CFF_i(X, KS, CP) + CWF_i(X, WL, CP)] \times l_i \quad (1)$$

in which NS = number of segments, COST = minimum cost per unit length of girder segment, BP = base price of steel plate per unit weight, SP = extra charge for steel type, EX = extra charge for plate thickness and width, KS = number of workmen to fabricate unit weight of steel plate except welding, CP = daily wage of a workman, WL = welding length in specific size. SP and the charge for pre-heating in welding vary with steel type used.

### 3. EFFECTS OF PRICE RATIOS

Since the optimum solution and the total cost of a girder with specific bridge length and width are complex functions of the design factors such as X, BP, EX, CP, KS, etc. as described in the previous section, it seems to be considerably difficult to presume the effects to the optimum solutions due to the changes of the design factors. However the ratio of CP to BP may be supposed to be the most effective factors. Consequently at first stage of this study the effects due to the changes of unit price ratios CP/BP to the optimum solutions are investigated.

Table 3 shows the examples of optimum solutions for the girders at various price ratios. As seen clearly in this table the optimum steel type, M, moment of inertia, I, and length, L, to be used for each girder segment are extremely insensitive to a wide variety of the unit price ratios except the case CP/BP=0.0 which is an unconsiderable case in the practical design problems. These phenomena can be seen at every span lengths and bridge types in the minimum cost design of highway plate girders and it has been cleared that the optimum design diagrams presented in the following sections may be applicable for a wide range of the ratios of CP to BP with sufficient accuracy.

### 4. SL(BL)-OPTIMUM WH RELATIONSHIPS

The minimum total cost of the plate girder with specific SL, BW and NS is changed concavely with web height on the whole, but locally several minimum solutions exist as noted in Ref. [1,2]. The optimum web height, WH, therefore should be decided by comparing the results in each discrete web

Table 1. Bridge Types, Lengths, Widths, Span Ratios and Segment Numbers

BRIDGE TYPE	BRIDGE LENGTH	BRIDGE WIDTH	SPAN RATIO	SEGMENT NUMBER
SIMPLE SPAN G.	20-40 <sup>m</sup>	6,7,8,9 <sup>m</sup>	—	5,7,9
2 SPAN CONT. G.	40-80	6,7,8,9	1.00:1.00	8,10,12,14
3 SPAN CONT. G.	60-120	6,7,8,9	1.00:1.00:1.00 1.00:1.30:1.00	13,15,23

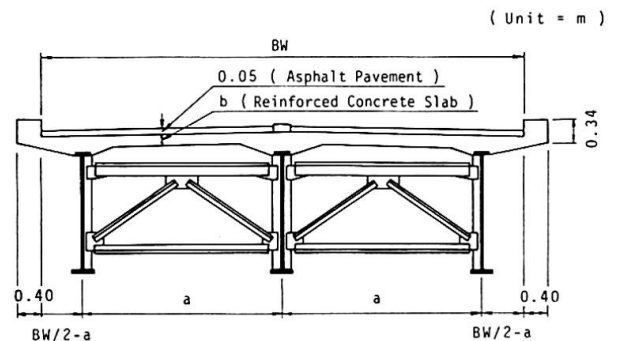
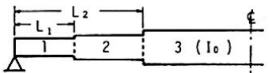
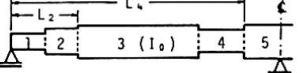


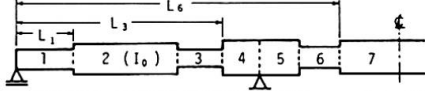
Fig. 1. Cross Section of Girder Bridge

Table 2. Girder Spaces (a) and Thicknesses of Reinforced Concrete Slabs (b)

Bridge Width = BW (m)	6.00	7.00	8.00	9.00
a (m)	2.4	2.8	3.3	3.8
b (cm)	19.0	20.0	22.0	23.0

Table 3. Examples of Optimum Solutions for the Girders at various Price Ratios (BW = 8.00m, WH = 170cm)

1 SPAN (SL=30m)							2 SPAN (SL=30m)						
													
CP/BP	Seg. 1			Seg. 2			Seg. 2			Seg. 4			
	M <sub>1</sub>	I <sub>1</sub> /I <sub>0</sub>	L <sub>1</sub> /BL	M <sub>2</sub>	I <sub>2</sub> /I <sub>0</sub>	L <sub>2</sub> /BL	M <sub>2</sub>	I <sub>2</sub> /I <sub>0</sub>	L <sub>2</sub> /BL	M <sub>4</sub>	I <sub>4</sub> /I <sub>0</sub>	L <sub>4</sub> /BL	
0.0	SS41	0.656	0.098	SM50	0.963	0.228	SS41	0.984	0.097	SS41	0.882	0.447	
0.023	SM50	0.501	0.103	50	0.966	0.230	SM50	0.749	0.102	SM50	0.678	0.447	
0.045	50	0.504	0.104	50	0.969	0.231	50	0.746	0.101	50	0.708	0.444	
0.068	50	0.531	0.110	50	0.964	0.229	50	0.748	0.101	50	0.709	0.444	
0.082	50	0.532	0.111	50	0.963	0.229	50	0.752	0.103	50	0.664	0.451	
0.098	50	0.533	0.111	50	0.967	0.230	50	0.757	0.104	50	0.665	0.450	
0.136	50	0.502	0.103	50	0.946	0.223	50	0.757	0.104	50	0.665	0.451	

3 SPAN (BL=80m, η=1.25)								
								
Seg. 1			Seg. 3			Seg. 7		
M <sub>1</sub>	I <sub>1</sub> /I <sub>0</sub>	L <sub>1</sub> /BL	M <sub>3</sub>	I <sub>3</sub> /I <sub>0</sub>	L <sub>3</sub> /BL	M <sub>7</sub>	I <sub>7</sub> /I <sub>0</sub>	L <sub>7</sub> /BL
SS41	0.564	0.043	SS41	0.725	0.273	SS41	0.969	0.424
SM50	0.566	0.043	SM50	0.755	0.270	SM50	0.976	0.429
50	0.566	0.043	50	0.755	0.270	50	0.976	0.429
50	0.566	0.043	50	0.756	0.270	50	0.977	0.429
50	0.560	0.042	50	0.753	0.269	50	0.974	0.431
50	0.566	0.043	50	0.756	0.270	50	0.977	0.429
50	0.566	0.043	50	0.755	0.270	50	0.978	0.429

BP = Base Price of Steel plate (yen/ton)

CP = Daily Wage of a Workman (yen/(person-day))

M = Opt. Steel Type

I = Opt. Moment of Inertia (cm<sup>4</sup>)

L = Opt. Distance of variation of Sectional Dimensions from Left End Support

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height. Fig. 2 shows  $WH_{opt}$  for every bridge types and bridge widths. They are changed stepwise and keep constant values for fairly wide ranges of span lengths. Discrete web plate thicknesses play an important role in the decision of optimum web heights and the ratios of web heights to thicknesses fairly coincide with the upper limits prescribed by the stability constraints of the girders. It is to be notable furthermore that only 170, 200 and 230cm are selected almost as optimum web heights for the plate girders with nonuniform cross sections with a few exceptions of 190cm at the small ranges of 2, 3 span continuous girders.

### 5. OPTIMUM M, I, L, SDIM

Optimum M, I and L to be used for the segments of the girder with the optimum web height decided in the previous section may be determined efficiently by using graphical optimization algorithms [1,2]. In the algorithms  $M_{opt}$  and  $I_{opt}$  for each segment are determined by comparing the minimum costs of unit length of girder segment for every steel type namely SS41, SM50 and SM58 steels which are evaluated straightly with use of the maximum working bending moment, BM, and the relationships between moment of inertia and maximum resisting bending moments, RBM, and minimum costs, COST, of the girder sections which are provided from the suboptimization of girder elements. According as bending moment increases, higher strength

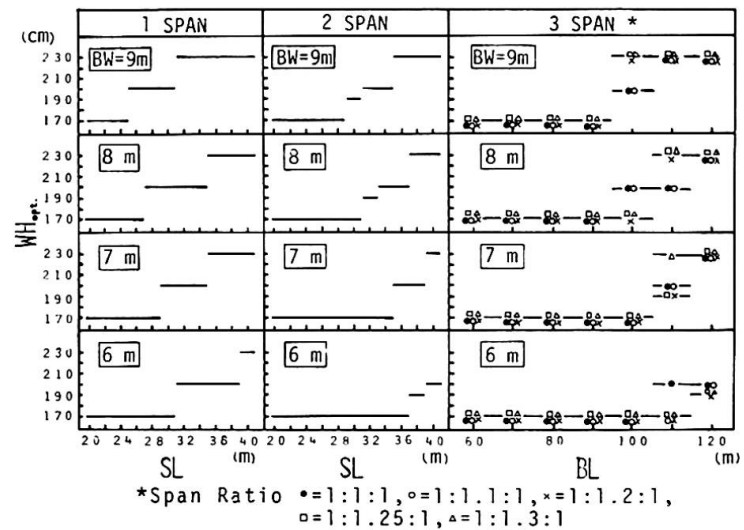


Fig. 2. SL(BL)- $WH_{opt}$  Relationships for 1,2,3 Span Girders

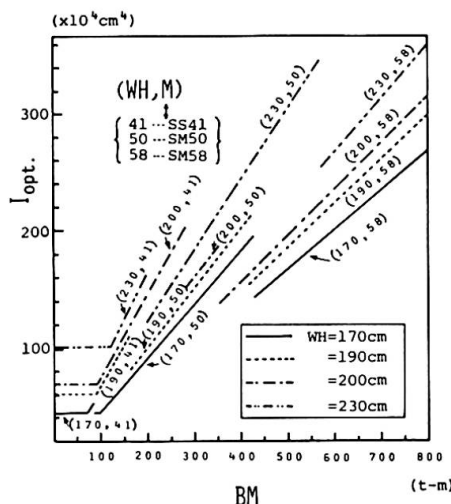


Fig. 3. BM-Opt. M and I Relationships (WH=170,190,200,230cm)

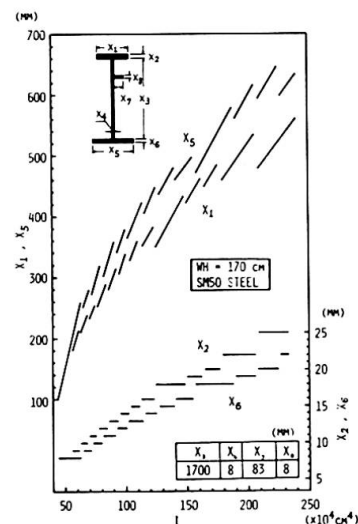


Fig. 4. I-SDIM Relationships for WH=170cm, SM50 Steel

Table 4. Opt.  $L_i/SL(BL)$  for the Bridge Types and Number of Segments

Type of Girder	Number of Seg.	$\xi (=L_i/BL)$ (Symmetric)							
SIMPLE SPAN G.	5	0.104	0.238	0.500					
	7	0.061	0.134	0.234	0.500				
	9	0.053	0.118	0.194	0.298	0.500			
2 SPAN CONT. G.	8	0.070	0.323	0.447	0.500				
	10	0.069	0.330	0.434	0.468	0.500			
	12	0.046	0.101	0.302	0.378	0.453	0.500		
	14	0.046	0.101	0.333	0.431	0.457	0.478	0.500	
3 SPAN CONT. G.	1:1.00:1	13	0.049	0.237	0.309	0.334	0.376	0.455	0.500
		15	0.031	0.067	0.233	0.305	0.334	0.384	0.463
	1:1.10:1	13	0.047	0.220	0.295	0.323	0.363	0.439	0.500
		15	0.030	0.066	0.216	0.291	0.323	0.367	0.466
	1:1.20:1	13	0.046	0.203	0.280	0.313	0.351	0.429	0.500
		15	0.029	0.064	0.198	0.279	0.313	0.349	0.434
	1:1.25:1	13	0.046	0.194	0.273	0.308	0.345	0.424	0.500
		15	0.028	0.062	0.192	0.272	0.308	0.344	0.428
	1:1.30:1	13	0.029	0.063	0.180	0.209	0.262	0.287	0.308
		23	0.354	0.410	0.441	0.500			
	1:1.30:1	13	0.042	0.188	0.263	0.303	0.340	0.420	0.500
		15	0.027	0.060	0.183	0.264	0.303	0.340	0.424

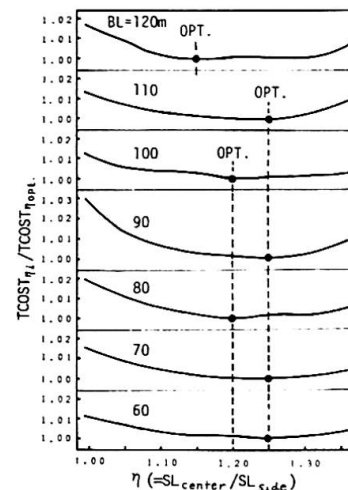


Fig. 5. Span Ratio-TCOST Ratio Relationships for 3 Span Continuous Girders

steels namely from SS41 to SM50 and SM58 are selected as optimum steel types and the corresponding optimum I are therefore varied discretely. The relationships between BM and optimum M, I for  $WH_{opt} = 170, 190, 200, 230\text{cm}$  are shown in Fig. 3. Horizontal lines of the relationships are caused by the constraints on minimum widths and thicknesses for flange plates. The arrangement of optimum steel types and moment of inertias of the girders therefore differ with values of the maximum bending moment diagrams. On the other hand, the ratios of optimum segment lengths do not change so much with span lengths and web heights, as tabulated in Table 4, they may be represented by a particular set of the ratios for a specific segment number and girder type with quite few exceptions. It has been cleared that the increments of total costs of the girders designed with these segment length ratios are less than 0.3% at most compared with the correct solutions. Optimum sectional dimensions, SDIM, of the girder segments can be decided directly from the moment of inertia and optimum sectional dimension diagrams for the web height and steel type as shown in Fig. 4 for  $WH=170\text{cm}$  and  $M=SM50$  steel. Flange widths  $X_2$  and  $X_6$  are increased like as sawteeth with discrete changes of flange plate thicknesses as seen in the figure.

## 6. SPAN RATIO-TCOST RELATIONSHIPS OF 3 SPAN CONTINUOUS GIRDERS

For the 3 span continuous girder design, the variation in the minimum total cost with span ratio may be one of the interesting features and the relations for the girders with nonuniform cross sections are presented in Fig. 5. According to this result the most economical span ratio for plate girder superstructures with nonuniform cross sections is scarcely varied with bridge lengths and it may be decided as almost  $1.20 \sim 1.25$ . It should be emphasized also, however, the fact that the relative differences of total costs in the range of span ratio from 1.10 to 1.35 are considerably small as less than 0.7%. On the contrary, the optimum span ratio for the girder with uniform cross section reduces with bridge lengths from 1.10 to 0.95 for bridge length  $60\text{m} \sim 120\text{m}$  and the relative differences of total costs are changed so much as the order of several percents.

## 7. ABM-OPTIMUM WH, M, I, SW RELATIONSHIPS FOR THE GIRDER WITH UNIFORM CROSS SECTION

Optimum solutions of the girders with uniform cross section may be decided directly such as example A or B in Fig. 6 by using the relationships between absolute maximum bending moment, ABM, and optimum design variables, WH, M, I and SW without reference to the bridge types. Different from the cases of girders with nonuniform cross sections, in which only one optimum solution is obtained for each span length, two optimum solutions, one for SM50 steel another for SM58 steel, which give the same minimum total cost are existed in a wide range of absolute maximum bending moments. Optimum web heights for the girder with uniform cross section differ with optimum steel types and they are decided almost as 170, 240, 270cm for SM50 steel and 200, 230, 260cm for SM58 steel.



## 8. DESIGN PROCEDURE USING OPTIMUM DESIGN DIAGRAMS

By using the design diagrams stated above as data banks, the minimum cost design of highway plate girders may be carried out simply, and the design procedures are shown in Fig. 7. The design process begins by choice of the segment number, NS, of the girder. The optimum web height and segment lengths are decided from SL(BL)-WH<sub>opt</sub> and SL-L/BL relationships respectively. In the next place assumptions are made of moment of inertia and girder weights at each segment, the girder is then analyzed. The optimum steel type, moment of inertia and weight to resist the maximum bending moment at each segment are decided straightly from BM-Opt. M, I, SW relationships for the optimum web height. The girder analysis should be repeated until maximum bending moment, namely optimum M, I, SW, are converged. Optimum sectional dimensions for each segment are decided directly from I-SDIM relationships for the web height and steel type. The total cost of the optimized girder is computed by using I-COST relationships or eq.(1). The entire looping is then carried out again for other NS if necessary. The girders with arbitrary web heights may be designed similarly without the selections of WH<sub>opt</sub>.

## 9. CONCLUSIONS

This paper has dealt with fundamental optimum design diagrams effective to the minimum cost design of constant depth highway plate girders and an efficient systematic optimum design process based on the data manipulation of the design diagrams has been proposed.

The design diagrams described herein in detail are only that for the optimum web heights at each span length, but the diagrams for other considerable web heights have been developed as well. It has been confirmed also that the diagrams are applicable with accuracy to a wide range of unit price ratios of materials to workmen.

By using the optimum design diagrams as data banks, the minimum cost design of highway plate girders may be carried out by the simple

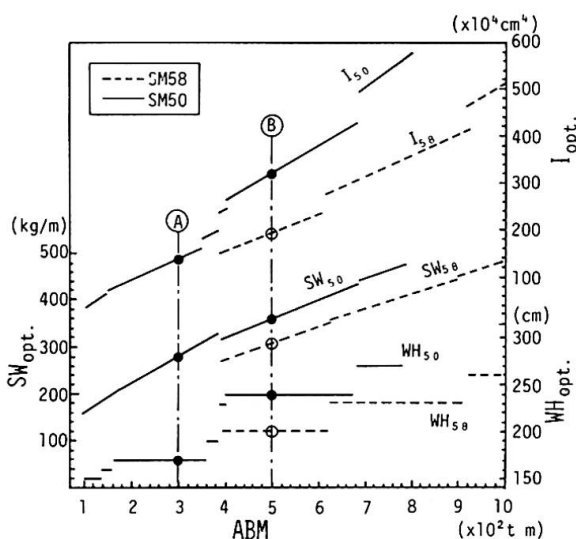


Fig. 6. ABM-Opt. WH, M, I, SW Relationships for the Girder with Uniform Cross Section

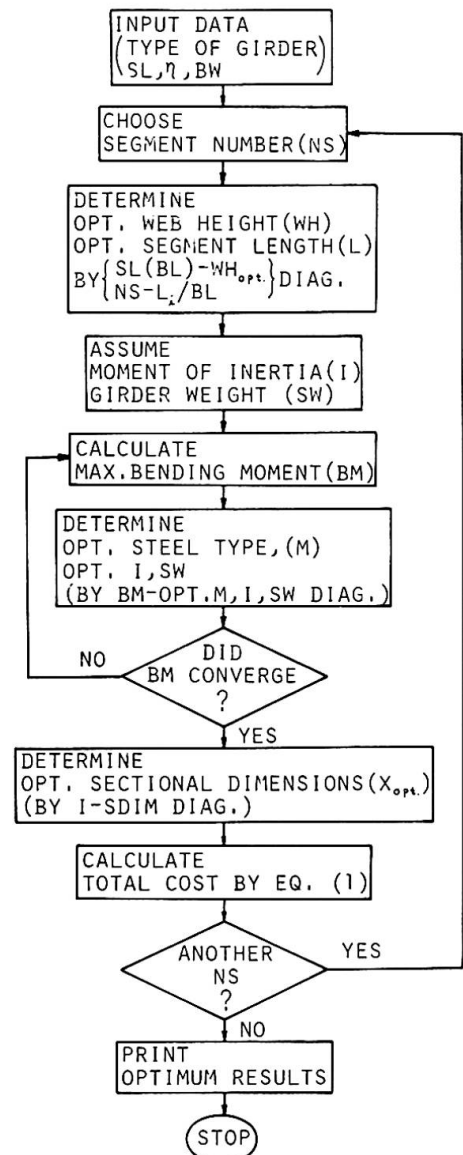


Fig. 7. Flow Chart of the Design Procedures

design procedures. The main computation required for the design is girder analysis only and most part of the design decisions can be made straightly from the diagrams.

The design procedures proposed in this paper may be utilized as one of the element design programs for specific structures in a general purpose system optimization program for highway bridges.

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#### SUMMARY

The basic optimum design diagrams are presented for the purpose of minimum cost design of 1 to 3 span highway plate girders; the fundamental characteristics of the diagrams are clarified. By using the design diagrams as data banks, minimum cost design of highway plate girders may be carried out in detail by a simple design procedure. The design procedure can be utilized as one of the element design programs in a general purpose system optimization program for highway bridges.

#### RESUME

On présente les diagrammes fondamentaux et leurs caractéristiques essentielles, pour un calcul, avec frais minimum, d'un pont-route à poutres à âme pleine de 1 à 3 travées. L'usage de ces diagrammes comme banque de données permet de faire le calcul détaillé selon un procédé simple, qui peut être utilisé comme un des programmes élémentaires d'un système général d'optimisation pour les ponts-routes.

#### ZUSAMMENFASSUNG

Grunddiagramme des optimalen Entwurfes werden mit ihren Charakteristiken dargestellt. Sie erlauben eine Berechnung mit Minimalkosten von 1 bis 3 feldrigen Strassenbrücken aus Stahl-Vollwandträgern. Diese als Datenbank verwendeten Diagramme gestatten ein Minimalkosten-Entwurf bis ins Detail. Das Verfahren lässt sich als eines der elementaren Entwurfsprogramme im allgemeinen System-Optimierungs-Programm für Strassenbrücken benutzen.