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Minimum Weight Plastic Design of Regular Rectangular Plane Frames

Calcul plastique pour un poids minimum de cadres plans rectangulaires

Plastische Bemessung auf Minimalgewicht für rechteckige, ebene Rahmen

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1. INTRODUCTION

Structural engineers have been concerned more with practical computational techniques for optimum structural designs than with theoretical results. On the other hand, scientific investigations on optimality criteria and optimal structures have been carried out mostly by researchers in the field of structural or applied mechanics. These two approaches are mutually compensating in order to develop more rational methods of structural designs.

The introductory report by A.B.Templeman has been primarily concerned with the hierarchy of optimum structural design problems and the corresponding computational techniques. Reference is made in his report to the linear theory of minimum weight plastic design and to the advantage of linear programming. It should also be recognized that the theoretical results on optimality criteria and optimal structures not only have the scientific significance but also lay the foundations and stimulate new ideas for developing practical computational techniques.

The purpose of this discussion is to call attention to the recent results [2-10] by the author and his colleagues on some general solutions derived analytically in closed forms to the problems of minimum weight plastic design of regular rectangular plane frames of practical interest and then to point out the theoretical and practical significances of those solutions.

2. FRAME MOMENT FOR REGULAR RECTANGULAR FRAME

Fig.1 shows a regular rectangular plane frame and one set of vertical and lateral design loads. The geometrical regularity in such a frame not only is reflected in design loads but also characterizes its structural behaviors and optimal plastic designs. In many practical design problems, the story shear resultant increases rapidly from the top floor toward lower stories as compared with the variation of vertical gravity loads. In those countries where frames must withstand against strong-motion earthquakes and strong gusts, *fairly large* lateral design loads are assigned. Under these circumstances, the first step of analytical treatment of a Foulkes-type problem is to assume an extremely deteriorated collapse mecha-

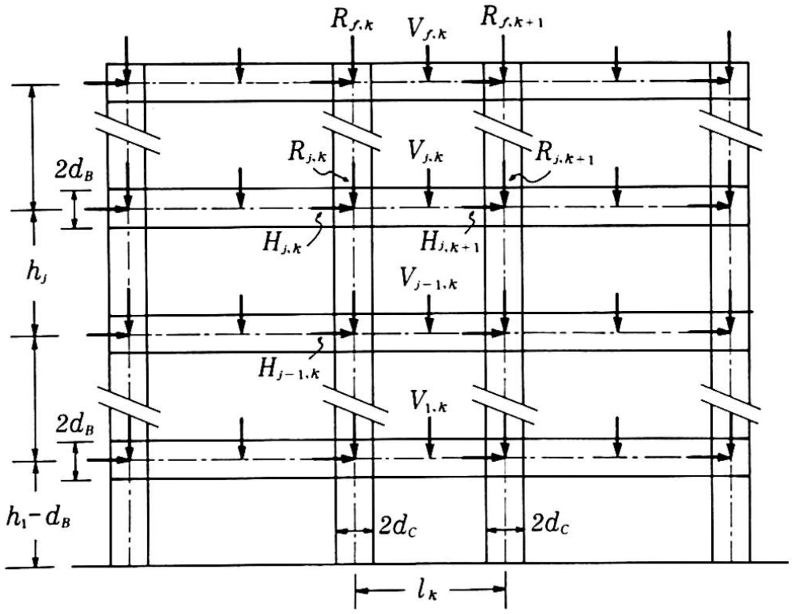


Fig.1
Regular
rectangular
frame

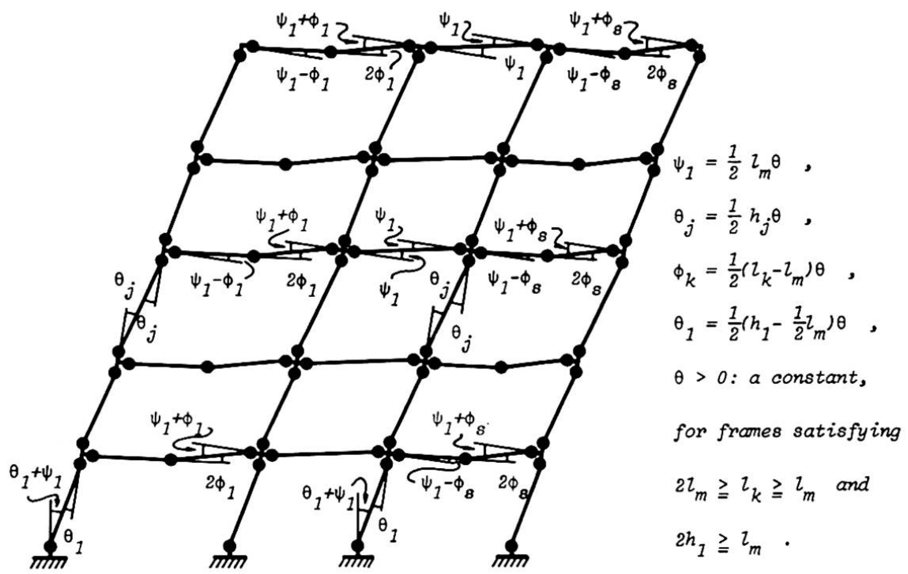


Fig.2
A Foulkes
mechanism
for
regular
rectangular
frame

nism shown in Fig.2, in which *simple* plastic hinges have formed at almost all the potentially critical sections except at the midspan sections of some particular bay(s) to be found as a part of the solution. Fig.3 shows that the corresponding bending moment diagram at plastic collapse may be conceived as the result of two-fold superpositions of decomposed diagrams. Each decomposed diagram is such that the moment equilibrium is maintained at the four corners with the same absolute value in the manner shown in Fig.3. This equal corner moment associated with this elementary moment diagram is called a "frame moment". A restricted minimization may then be carried out analytically in terms of the frame moments, and some statical conditions are derived under which the assumed bending moment diagram corresponds indeed to a general solution. It is then shown that the Foulkes mechanism condition can also be satisfied for a class of frames satisfying two simple geometrical conditions as shown in Fig.2.

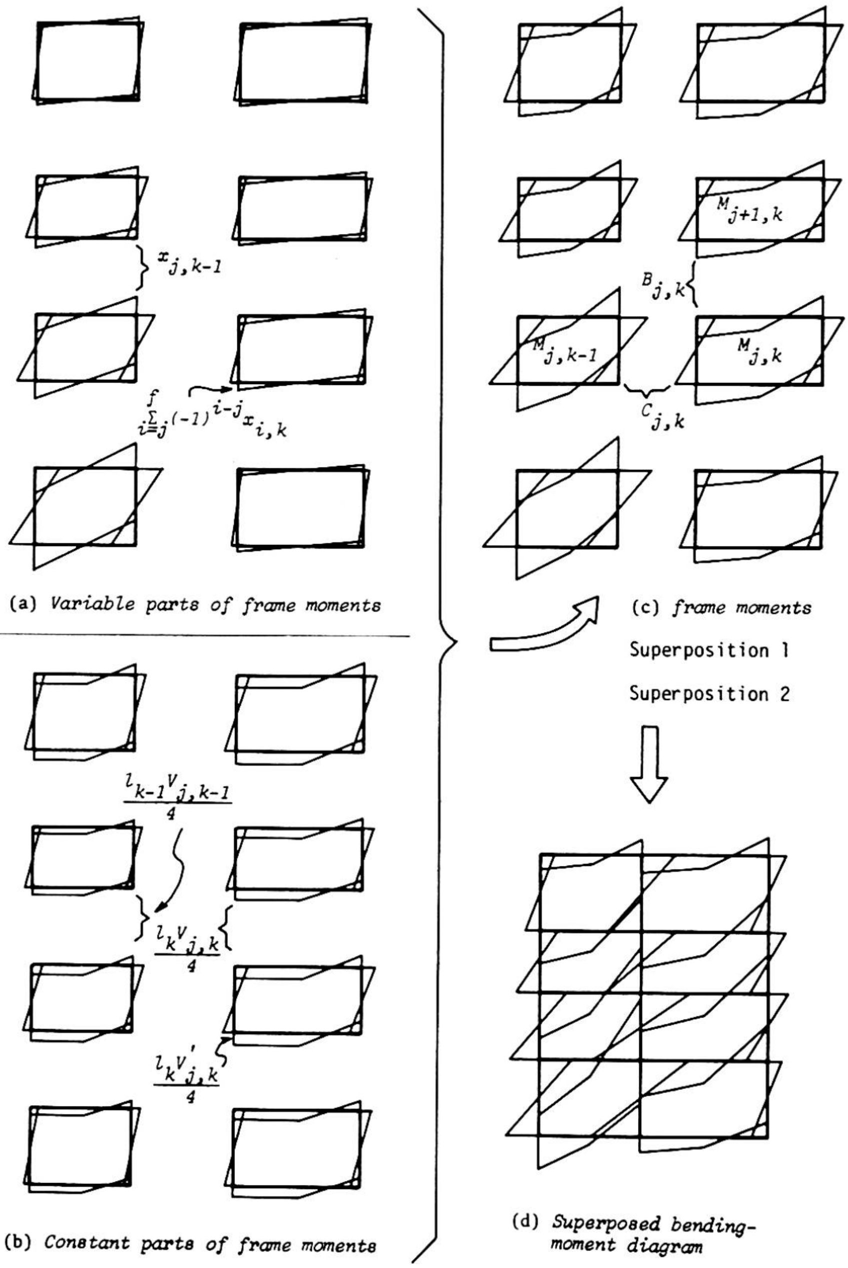


Fig.3 Frame moments and two-fold superposition procedure

3. APPLICATION OF FRAME MOMENT DECOMPOSITION TECHNIQUE

The two general solutions mutually exclusive and compensating on a design chart [2] not only clarify the general features of the classes of the minimum weight designs, but also provide a basis on which some modified general solutions can be derived to problems formulated more realistically by incorporating the axial force-bending moment interaction yield conditions for idealized beams and columns [3] shown in Fig.4. Fig.5 shows a part of modified Foulkes mechanism in a theory [4] in which only the idealized columns are required to satisfy the interaction yield condition shown in Fig.4. The regularity in the frame geometry enables one again to derive the general solutions and the statical and geometrical conditions analytically in closed forms [3, 4].

For the problem where reaction constraints have been incorporated within the framework of Foulkes' theory, a *bay shear distribution law* has been derived in [5] also on the basis of the concept of the frame moment and of the afore-mentioned two-fold superposition procedure.

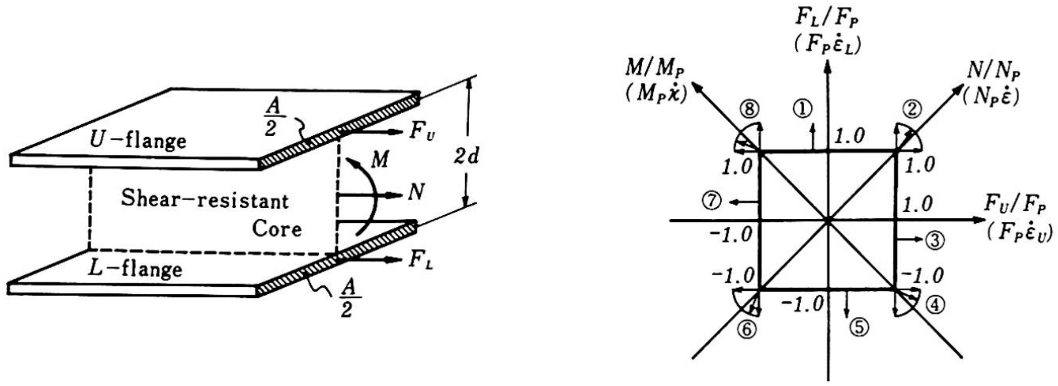


Fig.4 Axial Force-Bending Moment Interaction Yield Condition for an Idealized Sandwich Member

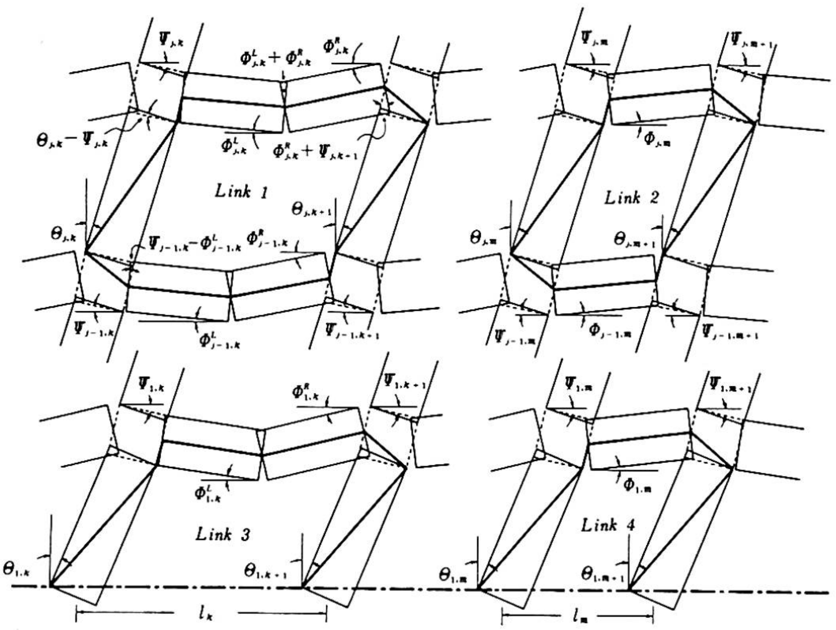


Fig.5
A modified
Foulkes
mechanism

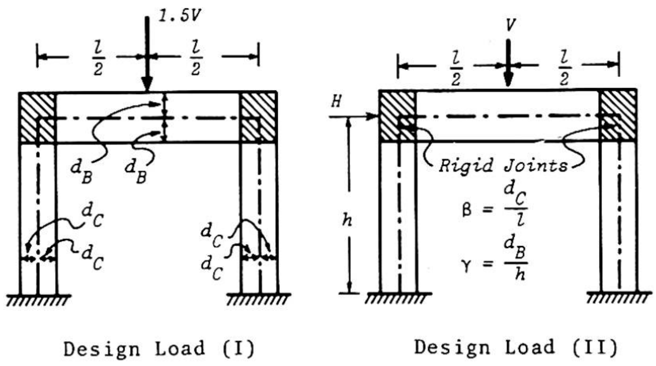
For the problem of multi-story multi-span frames to be designed for five sets of design loads, a kinematical restricted maximization procedure has been developed in [6] by combining the primal-dual method of LP with a semi-inverse approach similar to [2]. Some general solutions have thereby been derived analytically in closed forms. Fig.6 shows a portal frame obeying an idealized interaction yield condition and subjected to two sets of design loads. Fig.7 shows a fundamental design chart for this frame. This chart together with the theory in [7] constitutes the foundation for a possible analytical attempt of incorporating the result of [3] and [4] in [6].

4. SIGNIFICANCE OF THE CLOSED FORM GENERAL SOLUTIONS

The theoretical significance of these general solutions are now obvious. Each general solution provides a basis for developing practically useful general solutions to problems of more realistic formulations, though some modifications may become necessary for the topmost few stories. The afore-mentioned results may be said to provide ample grounds for the fruitfulness of this successive refining process.

For practical application, these solutions must first be modified for the effect of inelastic stability and member design requirements. The author and his colleagues have already clarified to a certain extent through numerical large-deflection analyses that minimum weight frames can indeed withstand against static al-

Fig.6
Portal frame
and
the two design
loads



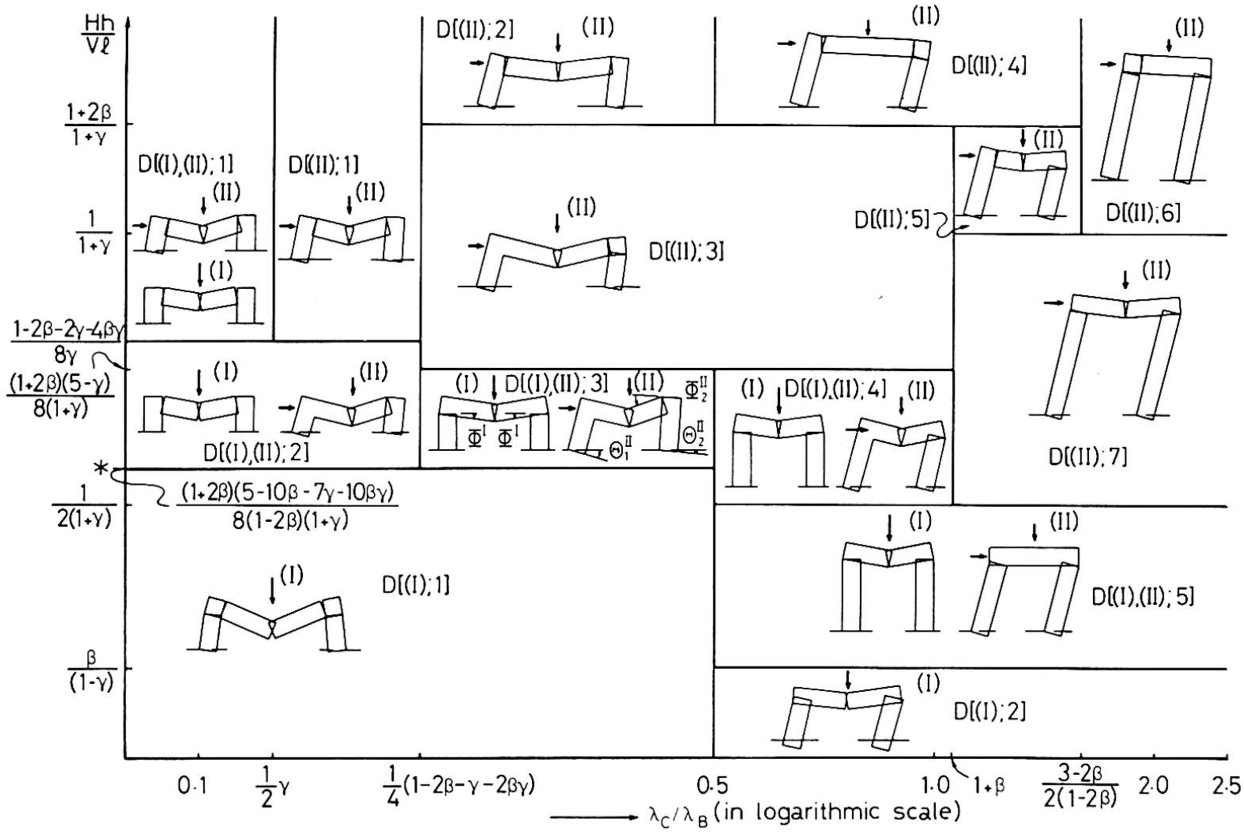


Fig.7 Design chart for a portal frame subjected to two sets of design loads

ternating lateral loads [8] and strong-motion earthquake disturbances [9] and are not particularly imperfection sensitive [10]. It should also be noted that the afore-mentioned solutions are the necessary consequences of the one-sided optimization using an approximate "failure" design criterion aside from the "serviceability" design criterion to be satisfied in practice.

Yet it can be said that the afore-mentioned solutions have the following significances: (i) they clarify the intrinsic features of the minimum weight plastic designs of regular rectangular frames at various levels considerably well; (ii) they will provide good initial solutions, if properly incorporated in a program, to start a numerical search for an optimal solution under additional constraints and may also be utilized as some standards for program verification. (iii) It may be well expected that the closed form solutions will be useful for seeking for optimum span length combinations analytically.

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SUMMARY

The frame moment decomposition technique due to the author and its applications to several more realistically formulated problems have been briefly described. The theoretical and practical significances of the analytical approach to the minimum weight plastic design problems have been explained in reference to the papers by the author.

RESUME

On décrit la méthode de décomposition des moments du cadre, proposée par l'auteur, et ses applications pratiques. La valeur théorique et pratique de cette méthode de calcul plastique, pour un poids minimum, est discutée.

ZUSAMMENFASSUNG

Die vom Autor entwickelte Methode der Momentenzerlegung sowie deren praktische Anwendungsmöglichkeiten werden beschrieben. Der theoretische und praktische Wert dieser Methode der plastischen Bemessung auf Minimalgewicht wird untersucht.