Examples of Computer-aided optimal Design of Structures

Exemples de calculs d'optimisation à l'aide de l'ordinateur

Beispiele des Computer-Einsatzes bei der Optimierung

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Applied Structural Optimization

I. INTRODUCTION

The total process for the design of a sophisticated structure is a multistage procedure which ranges from consideration of overall system requirements down to the detailed design of individual components. While all levels of the design process have some greater or lesser degree of interaction with each other, the past state-of-the-art in design has demanded the assumption of a relatively loose coupling between the stages. Initial work in structural optimization has tended to maintain this stratification of design philosophy, although this state of affairs has occurred, possibly, more as a consequence of the methodology used for optimization than from any desire to perpetuate the delineations between design stages.

In recognition of this stratification, a possible hierarchy of design variable classes has been postulated.[1] The partitioning implied in this manner is not rigid but is representative of possible or probable design capabilities compared to total design requirements.

The hierarchy is

    a) Member Sizes
    b) Configuration
    c) Material Properties
    d) Construction or Topology

In the first class, all geometric details of the structure are fully defined and only member sizes are to be chosen by a design process. Although apparently a very restricted class of problems, this actually represents (i) the limit of most of the optimization capability available to date and (ii) an extremely wide class of structural problems. It is a fact that in many structures the location and configuration of a great deal of the primary structure is mandated by nonstructural considerations. Likewise materials and construction will frequently be dictated by environment, design codes, cost, etc. There are many other structures for which the above does not apply.

By configurational variables, location but not number of principal components is implied. Hence, the first two members of the hierarchy may be regarded as continuous variables*, whereas the latter two categories clearly involve discrete noncontinuous variation of parameters. Because of the difficulties encountered in dealing with noncontinuous variables within a mathematical framework, principal attention has been generally confined to the first two classes of variables, with maximum attention on the member sizes.

The principal approaches to the optimization of structural systems for minimum weight in the past have been based upon the use of a combination of mathematical programming or other rigorous numerical search techniques and an equally rigorous structural analysis method. There have been many variations on this theme, but the essential combination of methods has remained the same. For analysis, finite element methods have been the most frequent choice, while the numerical search techniques have run the gamut from linear programming to Monte Carlo.[2,3,4] While this type of combination of methods is valid and appropriate for certain classes of problems, within the individual strata of the overall design process, it has led to certain intractable situations.

The rigor and sophistication of both the analysis and search procedures inevitably mandate numerical complexity and large computer costs for the optimization of anything approaching a representative large scale system. This, in turn, has cast considerable doubt upon the economic value of some optimization concepts. While many difficulties have been encountered using traditional methods of mathematical programming, there have been significant developments in new approaches to structural optimization which have overcome some of these difficulties for selected classes of problems.[5,6,7,8]

While mathematical programming methods are fairly rigorous and extremely general in their range of applicability, computer programs developed along these lines tend to be effective for the optimization of small scale systems only. When expanded for the optimization of realistic large scale structures, such approaches tend to become excessively costly and also of doubtful reliability and accuracy. The major problems seem to arise from a large increase in the number of analysis iterations with increase in the number of design variables. In addition, the explicit or implicit need to calculate numerical approximations to derivatives of constraints with respect to all variables means that each iterative step itself becomes lengthy.

Some new developments in mathematical programming have tended to overcome some of the difficulties but others remain. In addition, the possibility of further new developments in both analysis and numerical search techniques cannot be overlooked.

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*It is recognized that in many branches of structural engineering, principal members may only be selected from standard sizes and are not strictly continuous variables. This problem is usually treated by considering section properties as continuous variables and then selecting the nearest standard sizes for the final designs.

One approach to the problem which apparently avoids many of the pitfalls of mathematical programming is through the use of optimality criteria formulations.[5,6]

The basic concept behind otimality criteria is the rejection of the generality of mathematical programming and the utilization of the physical characteristics of the structural optimization problem to generate an approach of somewhat limited applicability but of the greatest computational efficiency.

In the optimality criteria approach, preconditions regarding the optimum structural system are generated based upon a physical, mathematical or even intuitive understanding of the problem. A simple search procedure is then developed to find the design satisfying these specified criteria.

A full discussion of this approach to one facet of structural optimization is given in Section II along with examples of the applicability.

Even with the development of optimality criteria programs and other similar approaches, these methods still suffer in many cases from severe limitations with regard to class and ranges of design parameters which can be treated as variables in a search for an optimum system.

It is this latter fact which has tended to maintain the stratification of the design process. It has been simply not possible or practical to mix variables of the different hierarchy classes in any rigorous search procedure. The major handicap has been the lack of continuity of variation of some parameters. While the concept of fixing configuration, mode of construction and materials at the outset of design may be acceptable for some structures, it will certainly fall far short of a goal of overall system optimization. Attempts have been made, with varying degrees of success, to incorporate configurational variables.[2,3] Generally, the stumbling block to the use of configurational and other variables (apart from computational costs) has been the requirement for continuity of variation in the parameters, due to the need for derivatives to provide search directions in a continuum space. With configurational variables this may be marginally possible provided the topology is undisturbed but to effect continuous variation in such concepts as material properties, construction mode and topology is beyond the capabilities of the vast majority of mathematical programming techniques.

For the optimization of large scale systems where many or all of the above parameters are initially undefined, more flexible and more general approaches have been sought. An additional consideration has been to develop an approach which would avoid the high computational costs of the more rigorous formulations, providing thereby an economic tool for ready use in design trade-off studies.

One new approach to the determination of the minimum weight of complex structural systems involving material, constructional and configurational variables in addition to the more conventional design variables has been developed and is labelled the "sieve-search" technique.[9] In this new procedure, which sacrifices

some degree of rigor for economy and generality of application, an attempt has been made to consider the effects of detailed design on the overall configuration of the total system and thus tie together hitherto uncoupled design stages.

In performance of optimization studies using the sieve-search technique, the guiding philosophy is the generation of an optimal arrangement of pre-optimized components. In this approach, the detail components of a structure are optimized first using local loading conditions and then the major configurational parameters are varied in order to find the optimal arrangement of the locally optimized components. The optimal design is obtained by a sequential comparison of the individual designs based on discrete values of configurational and constructional design variables. The above procedure is labeled a sieve-search since all nonoptimum designs are eliminated by the sequential comparisons leaving only the least weight design. The process can be labeled "discrete" in contrast to the more classic approaches wherein continuous variables are treated.

The sieve-search method was developed initially for an applied to the design of an extensive class of surface effect ships. Section III discusses the basic philosophy behind this approach to structural optimization using the surface effect ship as a prime example. The extension of the procedure to other classes of structural design problems is both possible and economically attractive. Its potential use for bridge design is also discussed in Section III.

II. OPTIMIZATION USING OPTIMALITY-CRITERIA

As discussed previously, there are a number of basically different approaches to the problem of overall structural optimization. While some of the variations in the approaches stem from differences in the classes and types of systems which are being optimized, there are also problems for which two or more methods of solution are available.

A classic problem, of great practical interest, is the optimization of a structural system whose overall geometry is fully defined and fixed by a set of external conditions but whose member sizes are to be selected optimally. The structure will usually be subjected to a multiplicity of loading conditions (no one of which is uniquely critical) and in addition to known limitations on the strengths of individual components, stiffness of the system may be of critical importance. Also fabricational constraints or other codes may mandate minimum sizes for constituent members.* For this type of problem which is encountered frequently in engineering design, the primary approaches to optimization developed during the 1960's were based upon the use of mathematical programming

*In discussing a structure, the concept of an assemblage of individual elements is used. This is generally consistent with the idea of a finite element model which is usually used for the actual structural analysis. If a continuun is considered, it, too, would be represented as an assemblage of discrete elements,which may be viewed as separate variables in an optimization process.

formulations.[4]   Because of the computational difficulties en-
countered with mathematical programming, this approach has been
largely abandoned for large scale structures and newer methods
based on the concepts of optimality criteria have been developed.

The underlying concepts behind optimality criteria methods
can best be illustrated by considering the contrasts between
optimality criteria and mathematical programming. In mathematical
programming approaches, sets of rules are established for numerical
search procedures which will determine an optimal solution in a
strictly empirical manner. The set of search rules will guarantee
a continuous and monotonic decrease in a prescribed merit function,
essentially without regard to the physical (or sometimes even
mathematical) nature of that merit function. The search will
regard constraints, if such exist, and will continue searching
until no further improvement in the merit function is possible. No
preconditions concerning the nature of the optimum are specified
beyond the criterion that it is impossible or uneconomic to deter-
mine a further design which will be an improvement on the present
design. This approach may be labelled post hoc since the optimum
is identified essentially only on an after-the-fact basis. Both
the strength and weakness of mathematical programming reside in
this concept. The strength is the generality which this independ-
ence of mathematical formulation imparts, with the resulting wide
range of applicability; the weakness is that no use is made of any
of the physical characteristics of the problem and hence frequently
an unnecessarily long and costly solution process results. The
antithesis of this situation arises in an approach which recognizes
the physical nature of the structural optimization problem per se
and sets out to take fullest advantage of the restricted class of
problem. In this approach, some conditions are established ini-
tially concerning the nature of the design which will be regarded
as optimal. These conditions, which are defined before initiation
of the redesign process, may be rigorously exact, approximate or
even intuitively assumed. The essential requirement is that their
application will lead to a relatively simple (usually iterative)
algorithm for a redesign process converging on the design which
satisfies the initially prescribed criteria. This approach is
then labelled a priori, since the characteristics of the optimal
system are specified initially.

The classic and most obvious example of an optimality criteria
approach is the time-honored fully-stressed-design. Every practi-
cal engineer is fully aware of and would probably support the basic
idea that the most efficient (optimal) design is one in which every
member is used to its fullest extent under at least one loading
condition. Prior to the advent of computers and the development
of advanced methods of structural analysis using finite elements,
generations of structural engineers have traditionally attempted
to inject some degree of optimality into designs by analyzing a
trial structure, using some appropriate quasi-classical procedure,
and adjusting member sizes to eliminate over- or under-stressing.
The more ambitious engineers might even have re-analyzed and re-
sized the structure one or more times. Probably very few practic-
ing engineers ever wondered whether anything is invalid with this
rather natural 'calibrated-eyeball' approach. Accurate analysis
of indeterminate structures presented a difficult problem prior to

the introduction of computers, discouraging the repeated use of more elaborate schemes, while approximate analyses were somewhat insensitive to the crucial effects of rerouting internal force distributions resulting from resizing iterations.

With the appearance of computers in the fifties, the first attempt at automated optimum sizing was the computerized version of the above procedure, initially still relying on time-honored approximate analysis methods. The ensuing development of the finite element methods by the early sixties made rather accurate analyses possible for indeterminate structures of virtually any form or shape. Instead of just two or three resizing cycles now a much larger number of cycles became feasible, at least for numerical experimentation by researchers, even if not in practice.

This simple and intuitive concept was eventually formalized as the fully-stressed-design (f.s.d.). To achieve f.s.d., the most commonly used algorithm, although not the only one available[10], is the simple stress-ratio. In the stress-ratio algorithm, it is assumed that the gross forces in any member of the structure will not vary with member size and hence the member properties may be adjusted directly in the ratio of the actual to the allowable stress. In indeterminate structures, changing member properties generally effects some redistribution of internal forces, so that an iterative process is required to achieve a f.s.d. The most important feature of the stress-ratio, and other similar algorithms is, that, in marked contrast with direct numerical search procedures, the number of re-analyses needed to reach an apparently converged design is usually small and independent of the size of the problem. This intuitive approach fulfilled a need for automated sizing for strength requirements and the strength optimization problem seemed to be under control.[11,12] No such simple and efficient method existed at that time for stiffness related problems.

In the late fifties, nonlinear programming methods were introduced as the correct framework for the general structural optimization problem.[13,14] With the development of these more rigorous methods, which were applicable to both strength and stiffness constraints, it was shown that f.s.d. is not necessarily the correct optimal solution for indeterminate structures. On the other hand, it was also shown that f.s.d. may indeed frequently be a correct solution, or more importantly from an engineering viewpoint, may be a close approximation to the correct solution. Thus with f.s.d. a very efficient but invalid method of strength optimization is provided. Fortunately not too many practicing engineers are inclined to question the rigor and validity of f.s.d. and merely welcome its advantages.

The standard f.s.d. stress-ratio redesign algorithm tends to drive a structure towards a design with the stiffest routing of internal force flow, which may or may not coincide with the optimal force flows. This trend may not become apparent if only a few resizing cycles are performed and because they do usually tend to produce a succession of improved designs, they are of great value to the engineer.

The potential sources of problems with f.s.d. are quite easy
to point out, but the extent to which they are present in any given
situation is extremely difficult to assess. The difficulties can
be demonstrated in two small example research problems, where
comparison with correct solutions, obtained by numerical search,
is possible. The two examples may be regarded as somewhat patho-
logical but even for these problems it is not entirely clear what
the true nature of the pathology is. Hence, it is not possible to
state categorically that any real system does not contain the same
disturbing influences. In the stress-ratio algorithm, only the
constraints (stresses) themselves are considered and no reference
is made to the factors of relevance to the merit condition, such
as density. Thus f.s.d. is completely insensitive to favoring
structural elements according to their strength to weight ratios.
Therefore, f.s.d. tends to break down in structures which contain
materials of different densities or markedly different allowable
stresses. The first example (Figure 1) is of two parallel bars
sharing a single load. One bar is of steel, the other is of alumi-
num but both have the same allowable stress. The stress-ratio
algorithm will increase the size of members with higher material
stiffness and/or lower allowable stress. In this example the alu-
minum bar will vanish and the steel bar will be retained. Clearly
this is a f.s.d. but not a minimum weight design. If both bars
are made of the same material, but with different allowables, the
algorithm will eliminate the higher strength bar, again a poor
design. It should be noted that the optimal solution for these
two problems is the other bar fully stressed. The difficulty here
lies with the stress-ratio algorithm, rather than the concept of
f.s.d.

A second more elaborate example is the 10-bar truss shown in
Figure 2.[15]   The truss has a single loading case and initially
the stress limit in all members is ±25000 psi. The f.s.d. obtained
using stress-ratio weighs 1593 lb which is known to be optimal.
Successively raising the allowable stress in bar No. 10 to
±30000 psi, ±50000 psi and ±70000 psi and again using a stress-
ratio,designs of 1545 lb, 1725 lb and 1725 lb, respectively, are
generated. The 1545 lb design is also known to be optimal but
the last two solutions of 1725 lb are clearly unreasonable and
considerably in error. In these two cases stress-ratio has tried
to eliminate the high strength bar, resulting in the poor designs.
Using mathematical programming techniques[8,16], the optimal
design for the two high strength (50000 and 70000 psi) cases is
known to be 1497 lb. Further examination of the problem reveals
the interesting fact that,in both the stress-ratio (1725 lb) and
the optimal (1497 lb) designs, all members are either at their
full allowable strengths, or at their minimum sizes, except for
bar No. 10. In each case the stress in bar No. 10 is 37,5000 psi,
although both designs are radically different. Assigning an allow-
able stress of ±37,500 psi to bar No. 10 and again applying stress-
ratio, results in a third fully-stressed-design, weighing 1568 lb,
which is quite different from the other two. Clearly the whole
field of f.s.d. needs further research. Some studies have been
conducted and variations on the stress-ratio algorithm have been
proposed[17], but with limited success.

On the other side of the coin, large scale programs have been developed, basically using f.s.d. and these programs have been successfully applied to the design of real structures. Whether or not such structures are truly optimal is somewhat academic when it is realized that such designs obtained at moderate computer cost are undoubtedly superior to those generated by hand.

Figure 3 presents the computer-generated plot of the finite element idealization of a complex wing structure. The total model had 5397 finite elements, 4104 displacement degrees of freedom and 20 separate loading conditions. Redesign studies were performed on the inboard half of this structure starting from various initial designs. The model considered had 3275 finite elements (design variables) and 2520 displacement degrees of freedom. Only two loading conditions were considered critical for sizing. In all cases only three iterations were performed showing acceptable convergence. Six iterations would have been sufficient for accurate production work. The program used for this optimization was ASOP[18] and for the three iterations required 6000 seconds CPU time on a CDC6600 computer.

The preceding discussion has dealt rather extensively with f.s.d. because this is the classic example of an optimality criterion, and it is an approach to optimization which is widely recognized and accepted. It is, nevertheless, very limited in its use. Its role as an optimality criterion, per se, would probably not have been recognized, if there had not been a pressing need for the development of suitable and efficient optimization procedures for stiffness constraints. The driving motivation for the exploration of optimality criteria methods for stiffness constraints was the excessive cost of using direct numerical search methods. What was sought was an approach as simple as stress-ratio but for displacement constraints. Optimality criteria were investigated since such concepts, by definition, contain gradient related information as a result of their derivation. By taking full advantage of the special structural properties of the problem, these criteria should lead rapidly and efficiently to the solution.

The actual development of a practical method for stiffness constraints was a multistage process in which many researchers individually contributed key concepts[19,20,21,5]. It is not of relevance here to discuss all the stages in this development progression; a fuller description may be found in Reference 22.

The essential step in the development of the currently used approach to stiffness constraints was the formulation of a single displacement constraint problem using a Lagrangian multiplier. In a structural system with fixed geometry, A_i, the characteristic sizes of constituent members, are considered to be design variables. If $W(A_i)$ is the merit function for the structure and $F(A_i)$ is a single displacement which is to be constrained to have a magnitude C, then values of A_i which minimize W, while satisfying the equality constraint can be determined by use of a Lagrangian multiplier formulation. The expression

$$W^* = W(A_i) - \lambda \left[F(A_i) - C \right] \qquad (1)$$
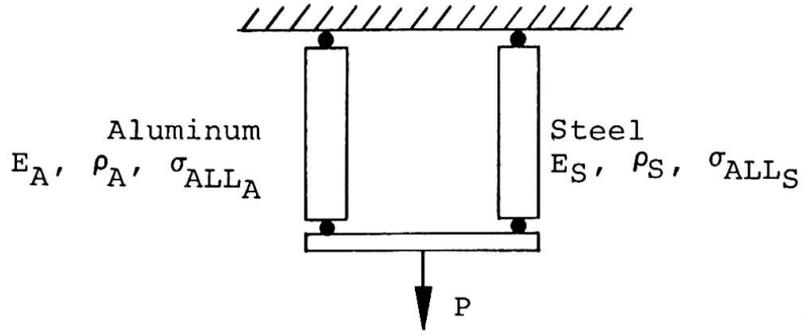
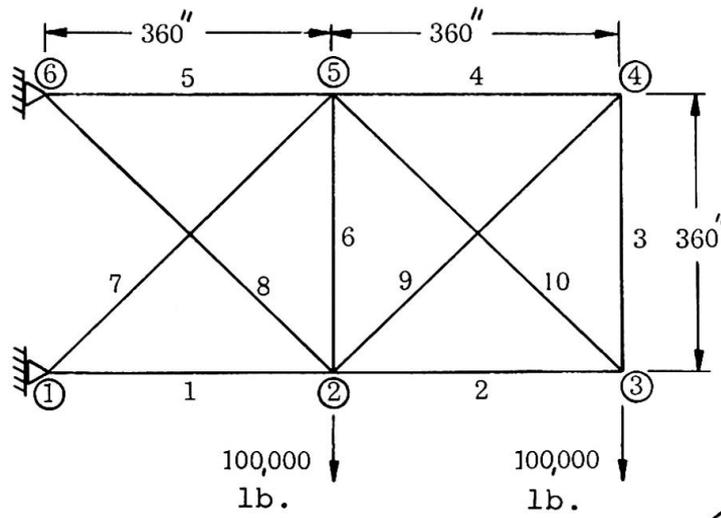Figure 1.   Parallel Bars of Aluminum and Steel
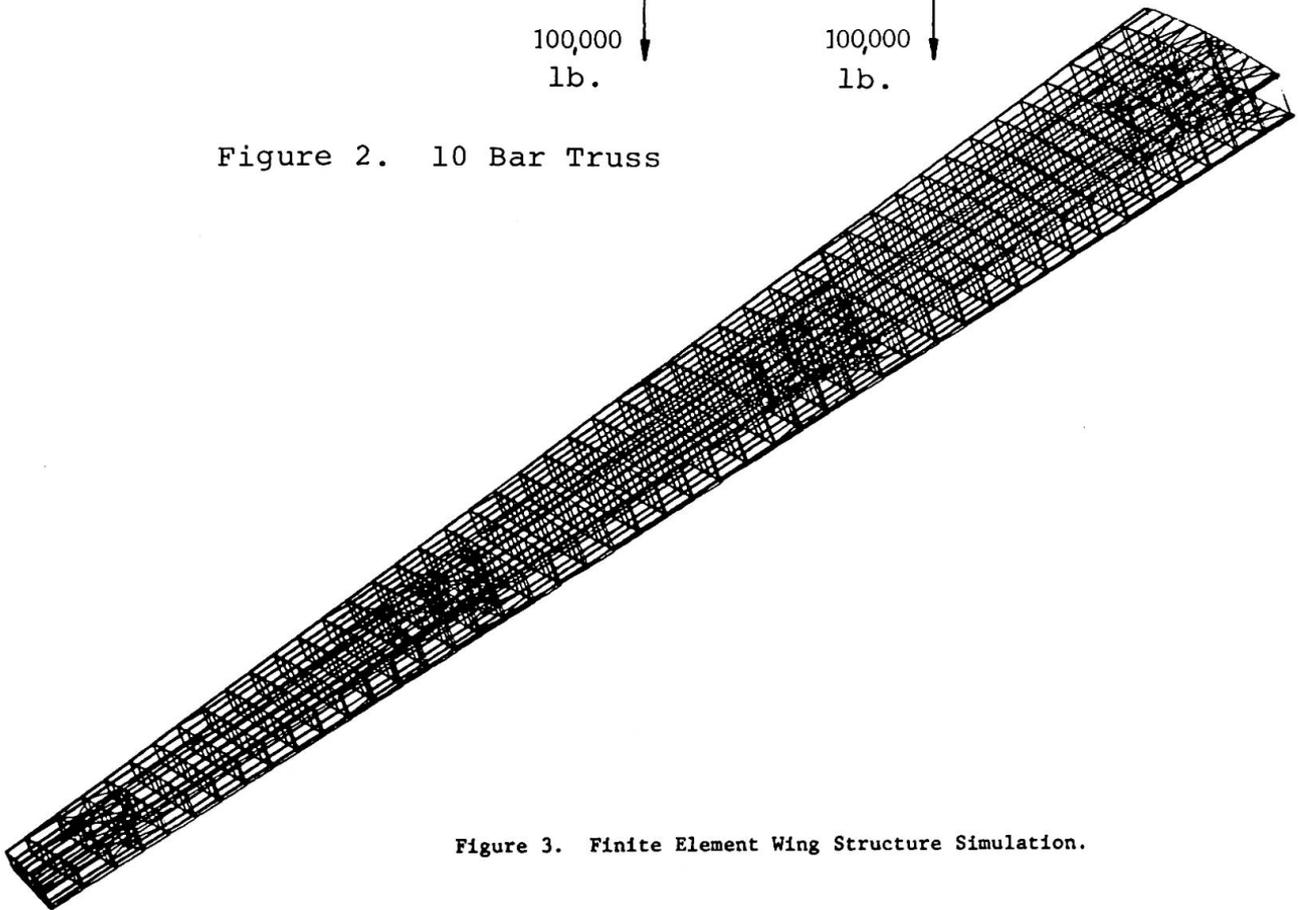

Figure 2.   10 Bar Truss

Figure 3. Finite Element Wing Structure Simulation.

is written and differentiated with respect to A_i to yield

$$\frac{\partial W^*}{\partial A_i} = \frac{\partial W}{\partial A_i} - \lambda \frac{\partial F}{\partial A_i} = 0 \tag{2}$$

Equation (2) is then the necessary condition for the optimum system, or the optimality criterion. For specific classes of problems, it can be proven that this condition is also sufficient for global or local optima.

Equation (2) can be rewritten in the more revealing form

$$\frac{\partial W / \partial A_i}{\partial F / \partial A_i} = \lambda = \text{constant, for all i} \tag{3}$$

Written in this form, there can be seen the valuable and relevant information that, in an optimal structure, the change in the measure of the behavior (displacement) for a unit change in the measure of merit is the same for every free variable. That is, the cost of improvement in the design is the same for every member in the optimal system. This statement is quite general and applies to the optimization of a structure for any type of merit function (weight, cost, etc.) and for any type of constraint which is characteristic of the structure as a whole. Thus, not only displacement constraints can be considered, but also overall buckling, dynamic response, flutter and any other phenomena which are indicative of total structural response.

By the same token, strength constraints do not satisfy the criterion of Equation (3), since they are, of necessity, individual characteristics of the constituent members and not of the structure in toto.

In order to translate Equation (3) into a working procedure for the stiffness optimization of a structure represented by an assemblage of finite elements, some particularization of the general definitions used previously is necessary. It is assumed that both merit and stiffness of the system are linear functions of the design variables A_i. These specializations are not necessary; they are made only to simplify the expressions for the cost and constraint function derivatives for a concise presentation. Other functional relationships are possible. One additional specification is crucial to the derivation of the final simple numerical procedure. This requirement, which is generally satisfied by most analytical methods, is that both the total cost and total stiffness be sums of individual members contributions. As a result, the simultaneous equations implied by Equation (3) uncouple for each value of i and can hence be solved in an extremely expedient manner using simple recursion formulae. The not very widely recognized importance of these key considerations, satisfied fortuitously by finite element analysis techniques, is that they remove obstacles which hitherto existed to the use of classical Lagrangian multiplier formulations for structural optimization. It is assumed in the following brief development that the complete

behavior of the structure is analyzed using the finite element displacement method. In accordance with the above definitions, a merit function (weight) is written

$$W(A_i) = \Sigma w_i = \Sigma \bar{w}_i \, A_i \tag{4}$$

Similarly the stiffness behavior is written

$$F(A_i) = \Sigma e_i = \Sigma \bar{e}_{i/A_i} \tag{5}$$

Equations (4) & (5) merely express the linear summations discussed previously. $\bar{w}_i$ & $\bar{e}_i$ are the contributions of individual unit-sized elements to the total weight and stiffness of the system. For a simple bar element with A_i as the cross-sectional area

$$\bar{w}_i = L_i \, \rho_i \tag{6}$$

where L_i is the bar length

and ρ_i is unit material cost (density).

For other types of elements A_i & L_i must be appropriately defined, but the general form of Eq. (6) still holds.

The stiffness of a structure under an actual loading system (P) is computed by imposing a virtual unit load system (Q) in the direction of displacement required, and computing the virtual work of system. The contribution of each element is given by

$$e_i = \delta_i^{Pt} \, K_i \, \delta_i^{Q} \tag{7}$$

where $\delta_i^{P,Q}$ are the vectors of the nodal displacement of ith element due to the actual and virtual loading systems, $K_i = k_i \, A_i$ is the stiffness matrix of the element and k_i is the unitized element stiffness matrix.

For other types of stiffness related constraints, such as buckling, vibrational response, etc., corresponding relationships to Eq. (7) can be derived and used in the subsequent development of a suitable redesign algorithm. Examples of buckling and dynamic response constraint formulations can be found in References 23 and 24. Substituting the above relationships into Eq. (2) and after some algebraic manipulations the recursion relationship is obtained

$$A_i^{\nu+1} = \frac{A_i^{\nu}}{C^*} \sqrt{\frac{\delta_i^{Pt} \, k_i \, \delta_i^{Q}}{L_i \, \rho_i}} \sum_j A_j^{\nu} \, L_j \, \rho_j \sqrt{\frac{\delta_j^{Pt} \, k_j \, \delta_j^{Q}}{L_j \, \rho_j}} \tag{8}$$

where the superscripts ν, $\nu+1$ indicate the values of A_i at successive iterations and C^* is the prescribed value of the stiffness.

Eq. (8) is then the redesign algorithm for a single stiffness equality constraint. In order to generalize the algorithm for multiple inequality constraints, the recursion relationship is applied to each constraint in turn and then the dominant values of A_i are selected for each member. The redesign process is iterative at each stage and a procedure for partitioning design variables into active and passive groups is used to select which members are effectively design by which constraints. This algorithm, known as the envelope method, also permits the simultaneous consideration of strength and minimum member sizes. The envelope method is an obvious simplifying approximation and does not strictly satisfy the correct optimality criteria for multiple constraints. It basically disregards the sizing given by one constraint when satisfying another.

Thus, analogous to the case of f.s.d., a procedure has been obtained for stiffness redesign based on an approximate criterion which has the merit of great simplicity and good general behavior. Experience has shown that the solutions for stiffness constrained problems obtained using the envelope method usually compare very favorably with more rigorous solutions obtained otherwise at much greater computational cost. The convergence characteristics of the envelope method are similar to those of f.s.d. with usually rapid convergence in a very small number of iterations, apparently independent of problem size.

A number of computer programs using optimality criteria algorithms have been developed. The program OPTIM II[6] is a large scale program which contains eight different finite elements in its basic library and is capable of application to a considerable variety of large scale problems. The elements include bars, beams and plates of various types. The program also contains a number of special features such as provision for linking elements, plate buckling computations and other capabilities intended to simplify the analyst's work.

The capabilities of such optimization programs can be best illustrated by a few example problems. These problems are generally small scale, but are intended to demonstrate the potentialities of the programs rather than to overwhelm by sheer size of problem alone. The programs themselves are only really limited by available computer size and the price (in terms of numbers of analyses) which the designer is prepared to pay.

The first example (Fig. 3) is of a simple four-level tower structure, composed of 72 primary members. The tower is subjected to two loading conditions as indicated. For obvious reasons it is desired to maintain the double symmetry of the structure, although the loading itself is nonsymmetric. The automatic linking feature is used to tie together elements where necessary. There are stiffness constraints to ensure that the tower does not sway too much under load. Figure 4 indicates that only four analyses were required for convergence. The efficient redesign logic at each stage requires only 10-15% of the analysis time.
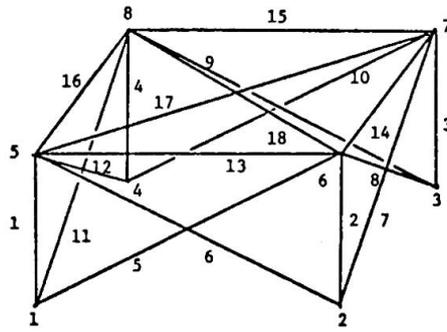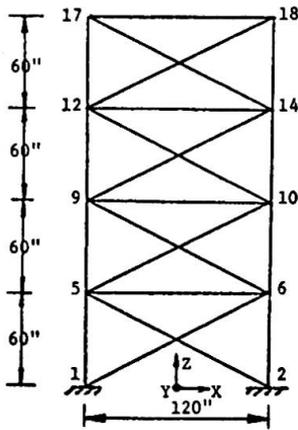
Typical Element Numbering
for First Level

$E = 10^7 PSI$; $\rho = .1 \ lbs/in^3$; $\sigma_{ALL} = \pm 25 \ KSI$; $A_{MIN} = .1 \ in^2$

Displacement Limit = .25 in (All Displacements)

LOAD CONDITION	NODE	DIRECTION		
		X	Y	Z
1	17	5,000	5,000	-5,000
	17	0	0	-5,000
	18	0	0	-5,000
2	19	0	0	-5,000
	20	0	0	-5,000

(a)   Geometry and Loading
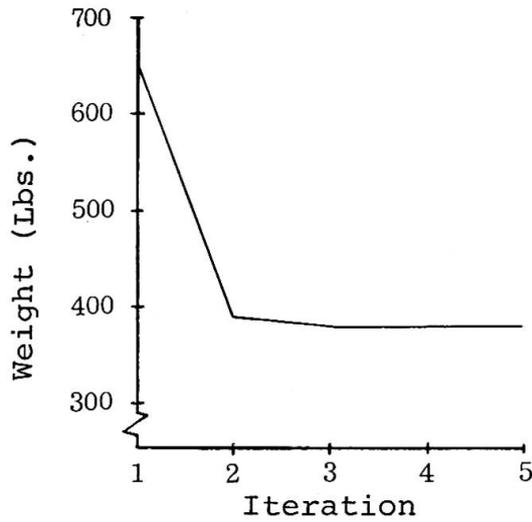
(b)   Iteration History

Figure 4.   72 Bar Four Level Tower

The second example is the geodesic dome of Figure 5, designed
for both strength and stiffness constraints. In this problem
involving 156 elements, the dome was subject to a uniform vertical
load and the vertical displacement of the central point was limited.
This problem was studied using various optimization programs avail-
able and full details of the results may be found in Reference 22.

Figure 6 represents the idealization of a wing carry-through
structure on a large heavy swing-wing aircraft. The loading arises
from operation with the wing in two different positions. The load-
ings on the pivot points were then principally flexural for the
wing in a forward, unswept position and torsional with the wing
fully swept back. In order to maintain the aerodynamic character-
istics of the wing, the rigidity of this structure must be very
high. Severe limits are therefore placed on the allowable dis-
placements and rotations of the pivot points. Initially a strength
only optimization was performed yielding a weight of 5035 lb in
50 iterations. This is a very slow convergence but it should be
noted that a weight of 5049 lb (0.3% heavier) was reached by
iteration 18. The structure was then reoptimized with both
strength and displacement constraints. The least weight of
6159 lb was reached at 50 iterations, with the same slow con-
vergence, but 6216 lb (1% heavier) was obtained at iteration 14.

If all members of the initially obtained strength-limited
design had been directly scaled to reduce the displacements of
that design to meet the specified stiffness constraints, the
structure would have weighed 7961 lb, over 29% heavier than the
actually optimized structure. This indicates the redistribution
of material effected by the optimization algorithm.

In this example, a bar idealization has been used for sim-
plicity, but in the actual structure, plates and shear webs would
be used. This raises an important point in structural optimiza-
tion regarding the influence of the idealization on the optimal
system. All redesign logic, for both stress and stiffness con-
straints is eventually predicated upon the detailed internal
stresses in the individual elements. Finite elements, or indeed
any other numerical analysis techniques, by their very nature
introduce a certain degree of approximation into a solution.
Finite elements are a piecewise representation of a continuum
and certain approximating assumptions are essential to their
basic derivation. The actual errors introduced into a given
analysis using finite elements is usually very small and hence
the results obtained are perfectly satisfactory for an engineer-
ing analysis. The widespread use and acceptance of finite element
methods is a testimony to their validity.

For optimization, where many analyses may be performed and
each redesign is dependent upon an erroneous analysis, the effect
of the inaccuracies may be cumulative. This does not imply that
the final system will be unsafe, but merely that the optimization
of a structure modelled by two slightly different idealizations
could result in two radically different designs. Care must be
exercised in the development of optimization programs to ensure
that only the most accurate analysis techniques are used. In
finite element analyses, bar elements are exact and involve no

$E = 10^7 \text{psi}$
Stress Limit = 75,000 psi
Specific wt = 0.1

Figure 5. Geodesic Dome

$E = 16 \times 10^3$ ksi
$\rho = .16 \text{lbs/in}^3$
$\sigma_{ALL} = \pm 100.$ ksi
$A_{MIN} = .01 \text{ in}^2$

Figure 6. 63 Bar Wing Carry-Through Box

a) 50 Bar Truss Column

b) Comparison of Chord Areas
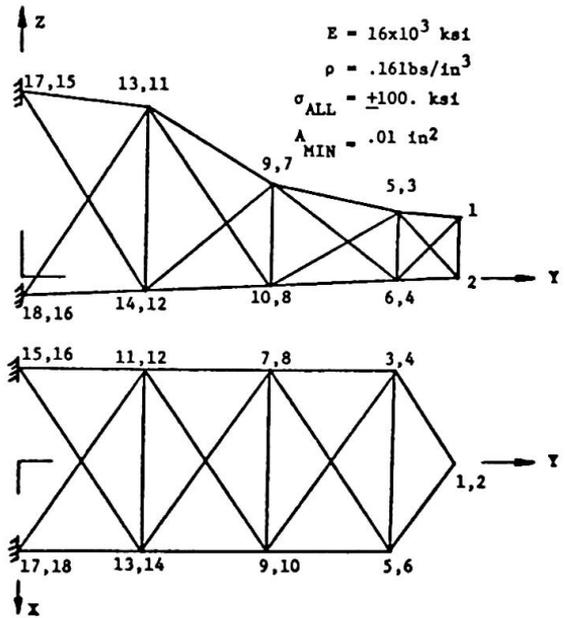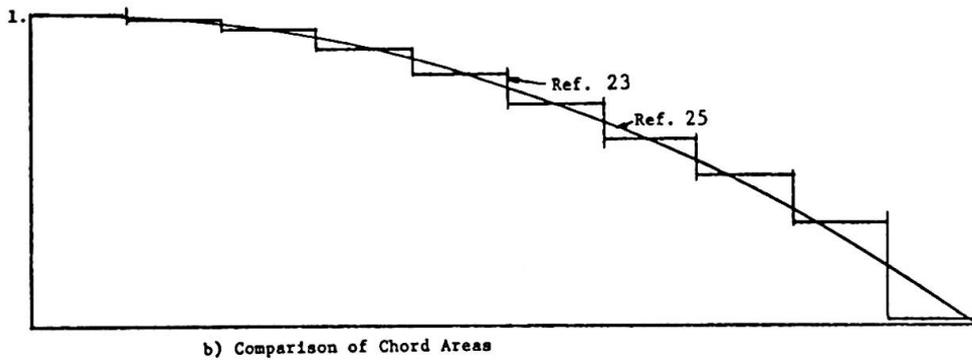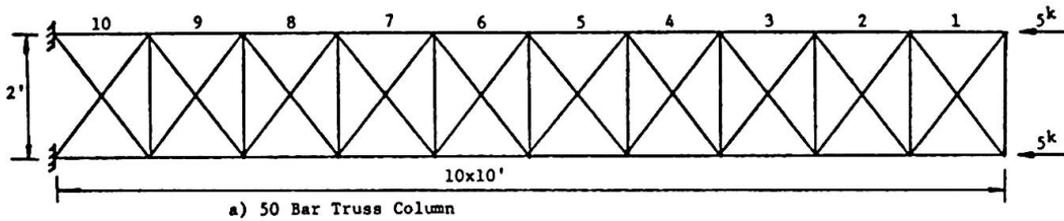
Figure 7   50 Bar Truss Column Problem

approximations. They are therefore frequently used for demonstration problems since they invite direct comparison of optimization solutions obtained by other methods by eliminating idealization errors.

The final example presented is that of the buckling of a simple laced column (Figure 7). The column has 50 bar elements and was optimized using a stiffness representation of the eigenvalue buckling problem.[23] The areas obtained for the chord (axial) members are shown plotted in Figure 7 in comparison with the exact solution obtained for the face sheets of a similar sandwich column.[25] The comparison is very encouraging.

III. OPTIMIZATION USING SIEVE-SEARCH

The selection of a truly optimal design to satisfy a particular set of engineering requirements is a complex process which strictly involves the consideration of all the classes of variables discussed in Section I. The approach presented in Section II deals with a more limited design problem in which geometry, material and construction are assumed to have been fully defined. A major question indeed must be on what basis will these governing design characteristics have been selected.

While it is true, that many psuedo-design parameters such as materials and construction cannot be treated as continuous variables and hence cannot be incorporated into any standard mathematical programming search technique, other considerations do enter into the picture. For the vast majority of engineering systems, only a limited number of materials really come into consideration. For civil engineering primary structures, titanium or boron-reinforced plastics, for example would have little or no applicability. Similarly reinforced concrete is seldom to be found in aerospace structures. Thus although there may be a potentially large number of possible materials and construction types, engineering practice and experience will indeed limit these to a finite set, which may be considered discretely. In a similar manner, although some aspects of the structural configuration, as defined by the arrangement and location of the principal structural members, are parameters to be selected by the designer, certain configurational characteristics will be absolutely defined by the service requirements of the structure. In addition, aesthetics and engineering codes will probably place some restrictions on other variables. The net result again is the specification of a finite set of configurational parameters. Finally the detailed design of individual structural components is governed by the critical loading which they experience locally. This critical loading may either arise from overall structural loading or may be a purely localized loading system which has little influence on the structure as a whole. Thus the optimum design can be generated for a given component under a specific loading system in isolation. Extending this concept, a range of optimal members can be predesigned in some suitable manner for appropriate ranges of applied loadings and sizes. This then is a so-called data bank. An example of a data bank is a structural handbook, which specifies appropriate code sizes of beams, columns, etc., for given applied loadings. It is well recognized that internal loading distributions are not strongly influenced by small variations in member

properties. Hence, except for highly pathological problems of the
type discussed in the previous section, it is assumed that only a
very limited number of redesign iterations is required for a satis-
factory degree of convergence. If the critical loading is purely
local, convergence is achieved almost immediately.

With the above considerations as guides, an approximate
optimization procedure for large structures was developed.[9] The
guiding philosophy in this sieve-search approach is that the
optimum system is an optimal arrangement of pre-optimized
components.

Individual components are optimized initially under local
loading conditions and the potential designs stored in a data bank.
A program is then set up which cycles sequentially through all the
finite combinations of the major variables. For each configuration
so defined, or segment thereof, an optimum design is generated
using the data banks and compared with the best design available
at that point. The best design is retained and the cycling is
continued.

The efficiency of this process is then highly dependent upon
the data banks available. These banks contain properties of
optimized components generated either by classical methods of
optimization or selected from standard structural codes. An
additional, but nonetheless important facet of the preset tech-
nique is the use of simplified engineering analysis methods where-
ever possible during the iterative phases of the redesign cycles.
Herein lies the efficiency of the sieve-search technique whereby
literally hundreds of redesigns are rapidly made for selected
configurational variables from which the optimum is obtained.

As a prime example of the sieve-search technique its applica-
tion to the design of class of surface effect vehicles (SEV) is
considered initially. The extension of the procedure to other
structural systems is discussed later with particular emphasis on
bridge structures.

Figure 8 is an actual photograph of a surface effect vehicle
which is prototypical of an extensive class of high speed cargo
vessels. Although operating in a marine environment, SEV are
essentially aircraft-type structures which must be supported on
a cushion of air. The development of least weight structures is
therefore of prime importance in the design of such vehicles since
the economic viability of SEV are dependent on low structural
weight.

Before initiating the design process consideration must be
given to the classes of parameters which would realistically be
regarded as variables in performing the actual design. Thus
external envelopes would be fixed by hydrodynamic and performance
requirements - although some trade-off studies between configura-
tions and performance might be desired. Figure 9 indicates the
general form of the external craft envelope.

Constructional materials and modes may be fixed or may be selectable from a limited class of candidates. Environmental considerations will narrow the number of available materials and for each material only a very small number of constructional modes is technically feasible.

The internal arrangement of longitudinal and transverse beams and bulkheads will have been fixed in an overall sense, but the individual spacings and sizes will be treatable as free variables. The only possible restrictions being dictated by internal storage requirements. This then selects the classes of potential variables - material and construction modes, configurational variables and component sizes. In a sieve-search procedure, an attempt is made to consider all three classes.

In the particular case of SEV existing experience has indicated that a major portion of the structural design is governed by local hydrostatic and hydrodynamic pressure loadings. In addition, the requirement for internal cargo containers has a profound influence on the ranges of beam and bulkhead spacings which can be reasonably used in the ship design.

With these considerations, the design for minimum weight can be conducted on the basis of optimizing the structure for normal pressure loading and subsequently checking the resulting design for strength due to overall bending, shear and torsion loads. Plating (panel) thicknesses and beam cap areas are then increased to ensure the overall integrity of the structure. This approach led to two main procedural items - overall ship weight minimization and plating optimization. These led naturally to definition of the following variables:

a) Construction module, including both material and constructional characteristics. Figure 10 presents sixteen combinations of materials and constructions which were considered feasible for this type of system.

b) Configurational Variables (Figure 9)

   1) Longitudinal bulkhead spacing, l_{LB}

   2) Transverse bulkhead spacing, l_{TB}

c) Dimensional Variables

   1) Plating - Panel Skin Thickness and Stiffener Dimensions

A finite number of longitudinal and transverse bulkheads and transverse frame spacings are specified and these configurational variables are optimized for minimum weight. Optimization of the dimensional variables results in generation of the data banks which store pre-optimized dimensional variables of structural components. In the present application, panels of the type shown in Figure 11 were optimized for minimum weight on the basis of normal pressure. A penalty function formulation with a Rosenbrock[26] search procedure was used. Geometric programming

Figure 8. Prototype SEV
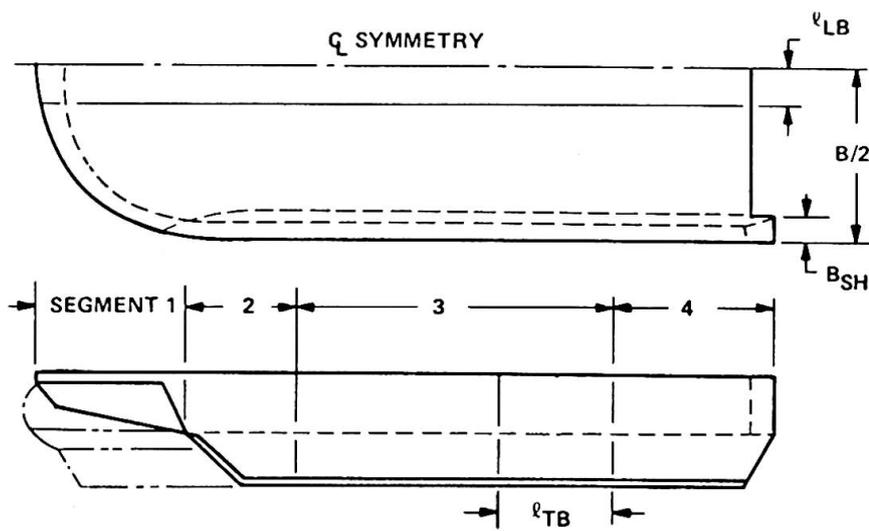

Figure 9. SEV Structural Envelope

methods for structural optimization[27] also appear extremely promising for use in component design where constraint and merit functions are expressible as nonlinear polynomials. The governing equations for the strength and stability of the panels under the action of uniaxial compression and in-plane shearing were also derived for use in the sieve-search method. For purposes of simplification, the panels were assumed infinitely wide and all critical conditions were expressed in terms of panel length, normal pressure, material characteristics and panel cross-sectional dimensions. The optimizations were then performed using the panel cross-sectional dimensions as variables. In addition to strength and stability constraints, consideration was also given to fabricational limitations for the various types of sections optimized.

Sixteen data banks consisting of eight basic geometric configurations with four materials namely, aluminum, steel, titanium, glass reinforced plastics were calculated and labeled construction modules. For these, all practical "failure" modes were derived analytically in five basic categories: material strength, overall buckling, local buckling, deformation limits and fabrication limits. Actually deformation and fabrication limits are not failure modes, but rather design specification modes which in many cases determined the optimum panel design.

When performing the optimization procedure, all of these critical conditions were expressed as inequality constraints. The fabricational constraints were based upon:

1)   Considerations of practical sections, for example, no overlap of flanges, and

2)   Data on the range of extruded sections which could be manufactured using existing dies and presses.

The deformation constraints were based upon the specific maximum allowable panel deflections.

The data banks are entered during the sieve-search process using the current spacing, L, and panel pressure, p, as shown by the dashed line on Figure 11. The resultant minimum weight, w, and cross-sectional geometry is stored for subsequent weight calculations.

A flow chart for the sieve-search program proper is shown in Figure 12. For application of the method, the vehicle was broken down into the four segments shown in Figure 9. These segments were defined in the present case by variations in the pressure loadings acting on the hull. Other forms of segmentation could have been selected to suit any arbitrary conditions. Within each segment certain configurational parameters were kept constant, although varying from segment to segment. The location of the longitudinal bulkheads was common to all segments. Each segment was further broken down into smaller zones such as deck, sidehull, etc. Each zone is then designed separately and combined to form the design of a segment.

Alloys	Aluminum		Steel		Titanium		GRP	
	5086	6061	HY-130	ALMAR-362	6Al-2Cb-1Ta-1Mo	6Al-4V	Polyester, Fiberglass Reinforced	Epoxy, Fiberglass Reinforced
Temper or Condition	H-111 Extrusion H-34 Sheet H-117 Plate	T6 { Sheet Plate Extrusion	Q & T } Plate Extrusion	Q & T } Plate Extrusion	Annealed { Sheet Plate Rolled-Forms Extrusion	Annealed { Sheet Plate Rolled-Forms Extrusion	Mechanical-Hand Lay-up	Mechanical-Hand Lay-up
Type of Construction	Stringers and Plate Integral Extrusion Welded Assy.	Stringers and Plate Integral Extrusion Riveted Assy.	Plate and Extrusion Welded Welded Assy.	Plate and Extrusion Welded Welded Assy.	Plate and Extrusion Welded Welded Assy.	Multiple Frames Plate and Rolled Forms Welded Assy.	Box Core Bonded Assy.	Box Core Bonded Assy.
Module	1(a)	1(b)	4(a)	4(b)	4(c)	5(b)	8(a)	8(b)
Type of Construction	Formed Stringers and Frame - Sheet Welded Assy.	Extruded Web-Core Riveted Assy.	Formed Stringers and Frame - Sheet Welded Assy.	Multiple Frames Plate and Rolled Forms Welded Assy.	Formed Stringers and Frame - Sheet Welded Assy.	Sheet and Rolled Forms Resistance Welded Assy.	Stringers - Frames and Skin Bonded Assy.	Stringers - Frames and Skin Bonded Assy.
Module	2(a)	3	2(b)	5(a)	2(c)	6	7(a)	7(b)

Figure 10. Construction Modules

OPTIMIZATION PARAMETERS: p, L

OPTIMIZATION VARIABLES: w, t_s, t_w, t_f b_s, b_w, b_f

OPTIMIZATION CONSTRAINTS: STRESSES DEFLECTION AND FABRICATIONAL

Figure 11. SEV Data Bank

R.A. GELLATLY — D.M. DUPREE
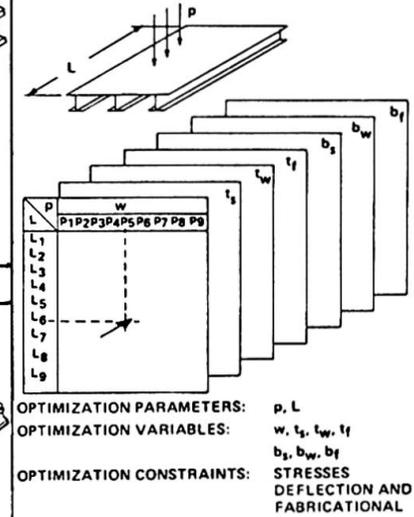
97

The cyclic nature of the design process is apparent from the —
flow chart (Figure 12). It can be seen to be essentially a series
of iterative looping operations which indeed permit the sequential
consideration of all feasible possibilities.

The design process begins by selection of the appropriate
SEV gross weight and construction module. The next choice is of
the longitudinal bulkhead spacing from a list of allowable spacings.
In SEV's cargo container size provides a lower bound on bulkhead
spacing. For the specific longitudinal bulkhead spacing, allowable
ranges of transverse bulkhead spacings are defined for each seg-
ment. In each segment the geometry is fully defined. Using the
known local pressure loadings, the data banks are accessed for
appropriate loads and geometry for each zone. The weight of a
segment is computed and compared with that obtained for other
transverse bulkhead spacings. This is repeated for each segment
yielding the minimum weight design for the specified longitudinal
bulkhead spacing. The entire looping is then carried out again
for the next longitudinal bulkhead spacing and repeated to obtain
the minimum weight craft.

Final checks on strength are performed using engineering
analyses and where necessary incremental material is provided.
For the ship system costing data is also computed.

The program then automatically cycles to the next construc-
tion module and SEV configuration, and repeats the entire process.

The above program was used extensively in the design of a
range of SEV's varying from 500 to 10,000 tons gross weight.

Out of a total possible number of 232 ship designs, 173 were
obtained. Designs for the remaining 59 configurations were not
obtained due to the non-existence of minimum weight data for
certain pressure/length combinations in the data banks. The
availability of such data is directly dependent on the constraints
placed on panel deflection, stress, and geometry in the process of
generating the data banks. The constraints will yield, at times,
nonfeasible panel designs and these appear as blanks in the data
banks. If some of the constraints used in the component design
are considered to be artificially severe, they may be modified.
Using these less stringent criteria, additional ships designs
would have been obtained.

Computational time was as low as 20 cpu seconds per ship
design on an IBM 360/65 computer. The resulting output gave a
very full description of the proposed structure including all
scantlings, frame spacings and cost data.

As a second example of the use of the sieve-search procedure
in a structural design process, its potential application to a
bridge design problem is briefly considered.

For the purposes of a design study, a complete bridge
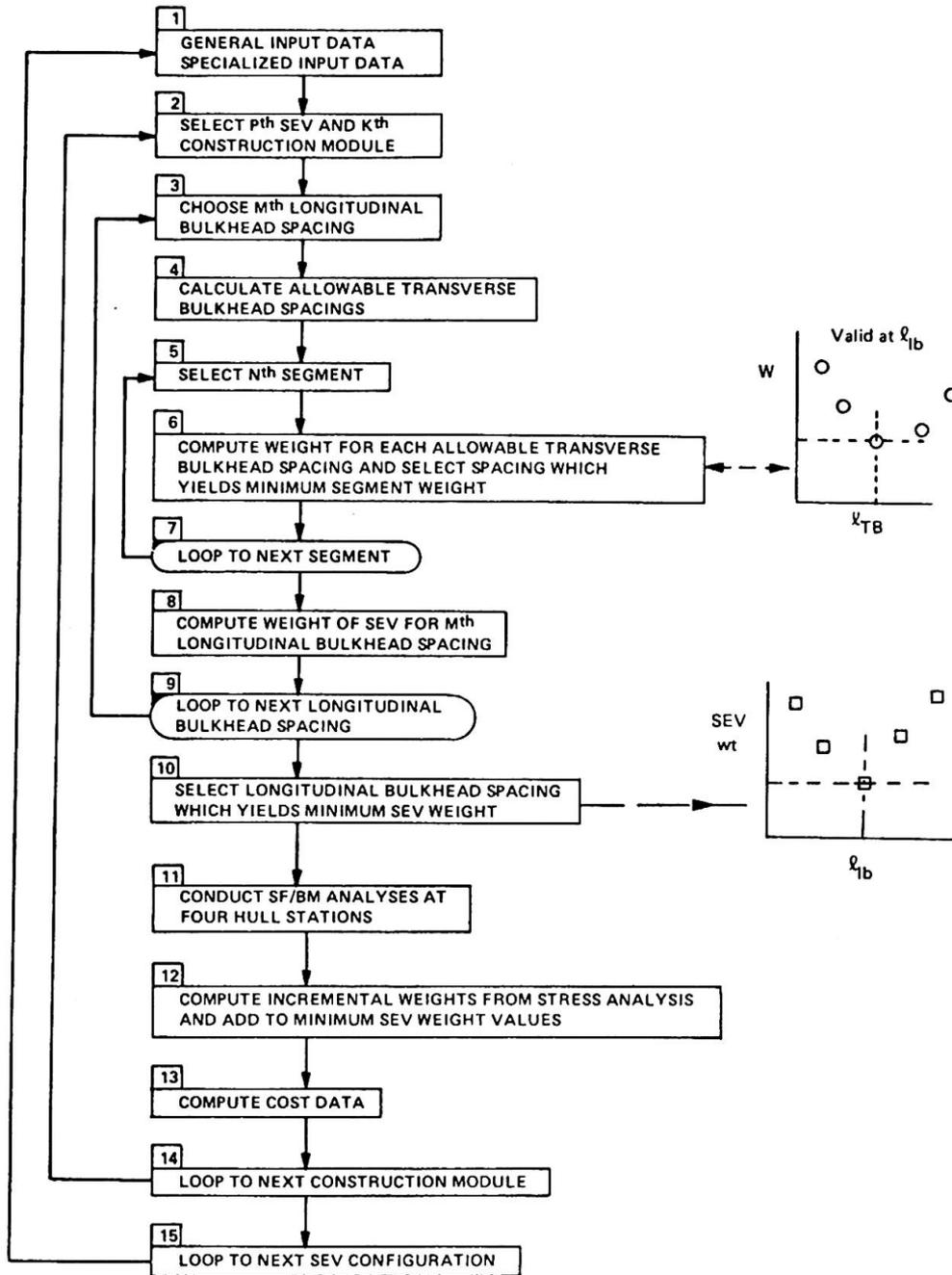structure may be broken down into the three major subdivisions,

FIGURE 12.   Sieve-Search for Minimum SEV Weight

(i)   Deck

(ii)   Primary structure spanning between piers and
       supporting the deck

(iii) Substructure

The use of a sieve-search procedure for the optimal design of the
deck and primary structure is outlined in a flow chart (Figure 13).

The essential characteristics of multiple levels of itera-
tions with detailed design performed through the use of data banks
is retained from the previous example, although the actual opera-
tions performed at each iterative stage may be totally different.
For the bridge example the use of multiple data banks is deemed
necessary.

The data banks for a bridge structure may contain a variety
of different construction modules such as deck panels, plate or
tubular girders, precast concrete beams, steel wide-flange beams
with cover plates, cable arrangements or steel towers and concrete
columns. All such potential bridge structural components may be
pre-optimized on any suitable merit basis for suitable ranges of
critical loadings and span lengths. The optimized data is then
stored in banks readily accessible at the appropriate stage of
the sieve-search program.

In selecting the bridge configuration a number of choices
may exist and each may be programmed according to its intrinsic
shape. Table I from Reference 28 indicates that for various spans
alternate configurations may be possible, but engineering judgment
and/or environmental conditions as well as other factors may
narrow the choice of feasible designs.

For the deck construction, the most commonly used construc-
tions are in-situ concrete, precast concrete and steel. Also
experiencing growing popularity is the so-called orthotropic steel
deck consisting of deck plate stiffened by parallel stringers.
Some typical cross-sections may be found in Reference 28. In
order to choose an appropriate deck, the following prime factors
must be considered,

    1. Strength, longitudinal and transverse
    2. Dead weight
    3. Cost

An efficient design includes the deck as part of the primary struc-
ture for load transferal and the true economic evaluation of the
above three items may be successfully achieved when and only when
the total bridge design is considered. For example, an ortho-
tropic steel deck if viewed only as a slab will not compete in
cost with reinforced concrete but the steel deck may be competi-
tive if its axial force capacity and reduced dead load effects
are considered through the complete superstructure and sub-
structure designs.

The comparisons of all typical deck sections in context with
the complete bridge structure are ideally suited for an automatic

1
General Input Data

2
Select P^{th} Bridge Configuration

3
Choose Deck Module

4
Choose M^{th} Longitudinal Beam Spacing

5
Calculate Pier/Tower Stations (Fixed-Variable) for each Segment

6
Select N^{th} Segment (Pier to Pier)

7/1
Specialized Input Optimized Beam and Deck Data, Segment Live Loads

7
Compute Weight for Each Pier Spacing and Select Spacing which Yields Min. Weight (include transverse grid variations)

7/0
OUTPUT - Segment Min Wt. breakdown and costs for N^{th} segment includes beam and deck geometries

Loop to next Segment

8/1
Specialized Input "As-Built Factors"

8
Compute Bridge Weight for M^{th} Longitudinal Beam Spacing

8/0
OUTPUT Structural Weight-Deck and Framing. Pier/Tower/ Arch Loads

Loop to Next Longitudinal Beam Spacing

9
Select Longitudinal Beam Spacing for Minimum Wt.

9/0
OUTPUT (1_{LB}) min. and minimum weight (same quantities as block 8/0 except for min. weight)

10/1
Specialized Input Optimized Box Beams, Tubular Ribs, Cable Construction, etc.

10
Superstructure Computations (Depending on Configuration Arch + Verticals, Towers + Cables, etc.) for Minimum Wt. L.B. Spacing

10/0
OUTPUT Superstructure Details

11
Overall Analyses Performed for External Conditions such as Wind Loads. Incremental Adjustments Applied if Necessary

12/1
Specialized Input Cost Data
    Material Costs
    Construction Costs
    Maintenance Costs

12
Compute Costs

12/0
OUTPUT Cost Breakdown

Loop to Next Deck Module

13
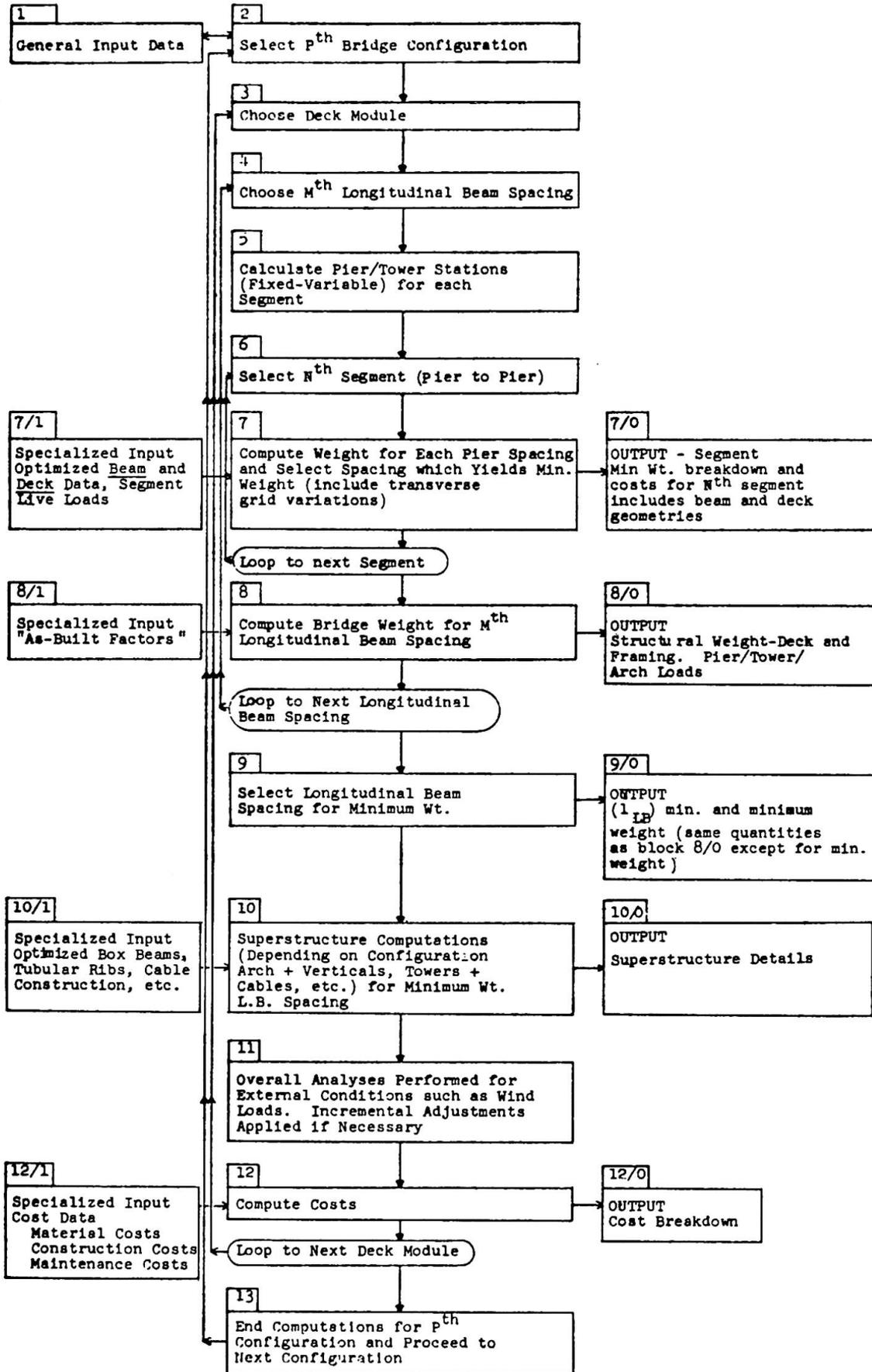End Computations for P^{th} Configuration and Proceed to Next Configuration

Figure 13. SIEVE-SEARCH FLOW CHART FOR BRIDGE DESIGN

sieve-search procedure. Data banks for each of the candidate deck cross-sections may be established. These files can be as sophisticated as desired wherein a range of span lengths together with a range of critical loads may be applied to each typical section. Figure 11 illustrates this and associated with each minimum weight (live + deal load), W_i, is a unit cost factor and optimal cross section geometries. The minimum weight and/or cost is evaluated under such constraints as deflection, strength, buckling, torsion, web crippling, etc. The definition here may be either working-load or ultimate. Fabricational limitations, code specifications and cost penalty factors may be included as well.

In the sieve-search, a predetermined table of acceptable longitudinal beam spacings may be specified, along with appropriate transverse spacings. The program will cycle through all the defined grids in its search for the optimum design. The configuration is also controlled by combination of fixed and variable lengths between abutments and piers. Each of the variable spans would be designated as a semi-independent segment for which a detailed design would be performed. For each segment, deck module and beam arrangement, the appropriate specialized data banks would be accessed to generate the local design which would then be compared with the previously stored optimal design. All segmental data is then assembled for the evaluation of the total design for a given longitudinal beam spacing. Specialized input, labeled "as-built" factors are provided to account for nonstructural items such as expansion joints, catwalks, railings, wearing surfaces, protective coatings, etc. After all potential longitudinal beam spacings have been considered, an interim optimal design is obtained. For this configuration, the superstructure is designed, again using appropriate specialized data banks. At this point a complete deck and superstructure have been designed and final check analyses should be performed. Some incremental adjustments on component sizing may be necessary. Consideration may even be given to the use of some suitable form of optimality-criteria optimization to refine a design, if this is felt to be appropriate.

Finally, the program would generate complete cost data for the selected design, including maintenance. The program is then repeated for other deck modules and configurations until the final design is rendered.

The preceding discussion has not been based upon an existing program but has been intended to indicate the possible extension of the sieve-search procedure to a civil engineering structure.

IV.   CONCLUDING REMARKS

Two distinctly different approaches to the optimal design of structures have been presented. In both cases, the greatest possible emphasis has been placed on the practical aspects of the design problem in an attempt to produce a workable tool for the designer.

The optimality criteria approach is gaining acceptance by designers because of its fortuitous combination of simplicity and effectiveness. Computer programs based thereon are being used simply because no other method exists at this time that can cope with the very large number of variables encountered in finite element representations of real structures.

The use of the sieve-search procedure is a direct contrast in approach. The results obtained from the SEV design studies for an extremely modest expenditure of computer time, have indicated that this method is also an efficient cost-effective approach to automated optimal design. The ideal solution would possibly appear to be a combination of the two approaches, whereby the sieve-search defines configuration and noncontinuous variables and the optimality criteria method is used for refinement of the design. The extension of the procedures to other classes of design offers a considerable potential for overall system optimization.

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SUMMARY

Examples are presented of two approaches to the optimal design of complex
structural systems. The first approach, based upon the use of optimality crite-
ria is capable of optimizing finite element representations of large scale,
complex structures with prescribed geometry. Both strength and stiffness con-
straints are considered. The second procedure is labeled sieve-search and is
used for the overall optimization of structures. The method permits the full
variation of construction method, materials and configuration as well as com-
ponent sizing.

RESUME

Des exemples de calcul d'optimisation pour des systèmes de structures
complexes sont présentées selon deux approches. La première, basée sur le cri-
tère d'optimisation, permet de résoudre des ensembles de grande dimension d'élé-
ments finis, ou des structures complexes à géométrie donnée. Les contraintes de
résistance et de raideur sont prises en considération. La seconde méthode, dite
"sieve-search" (tamiser-chercher), sert à l'optimisation globale des structures.
La méthode permet une complète variation de la méthode de construction, des ma-
tériaux, de la forme et des dimensions.

ZUSAMMENFASSUNG

Beispiele des Computer-Einsatzes bei der Optimierung von komplizierten Trag-
werken sind nach zwei Methoden aufgeteilt. Die erste Methode wird das Optimierungs-
kriterium benützen, und erlaubt die Optimierung von komplexen Tragwerken mit einer
bestimmten Geometrie, durch mächtigen Darstellungen finiten Elementen. Die zweite
Methode, die"sieve-search" (sieben-suchen) heisst, wird für die globale Optimierung
von Tragwerken benützt. Sie erlaubt eine totale Bearbeitung der Baumethode, der Ma-
terialien, der Form und der Abmessungen.

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