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Elastic and Inelastic Analyses of Pretensioned Cable Networks

Analyses élastiques et non-élastiques de couvertures précontraintes suspendues

Elastische und nichtelastische Analysen von vorgespannten Seilnetzen

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1. INTRODUCTION

In recent years several studies [1] have been published on cable roofs. Siev and Eidelman [2] developed a procedure for determining the initial shape of a cable roof. They also described [3] an approximate method of analysis of prestressed roofs, neglecting the horizontal displacements of the joints; Siev [4] presented a general linear method of analysis accounting for horizontal displacements and introducing a correction for nonlinearity by means of an iterative procedure. These analyses were in general for orthogonal nets where the angle between the two sets of cables was assumed to be a right angle.

Thornton and Birnstiel [5] derived nonlinear equations for a three-dimensional suspension structure; an influence coefficient method was used by Krishna and Sparkes [6] for the solution of the nonlinear equations with the principle of superposition assumed in a limited way to analyze pretensioned cable systems consisting of two cables of reverse curvature, pretensioned together by means of a set of vertical hangers; Buchholdt [7] employed a theory based on the minimization of the total potential energy and presented a solution by the method of steepest descent. Bathish [8] utilized the membrane theory to analyze cable roofs. Siev [9] analyzed an orthogonal roof bounded by main cables and compared his results with experimental findings.

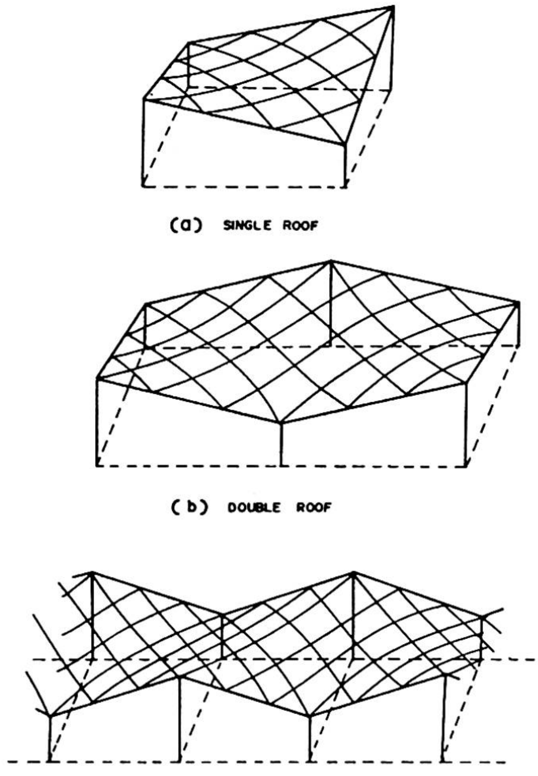
In this study, nonlinear displacement equations are derived for general nonorthogonal cable networks. The solution is substantiated by experimental results from tests conducted on models of cable roofs.

2. THEORETICAL STUDY

The displacement equations for a general nonorthogonal cable net were derived with the following assumptions: The cables are weightless and the applied load acts at the joint between cables; the cables are straight between joints and have constant cross-sectional area; the joints are perfectly smooth; and, the cables do not carry any compressive or bending loads.

The Newton-Raphson method was suitably adapted to provide a convenient numerical solution of these equations. The behaviour of nonorthogonal hyperbolic paraboloid nets under various modes of loading and temperature changes were studied. The effects of changes in nonorthogonality of the cables, initial pretension and the slope of the roof were also examined. In addition to taking into account the geometric nonlinearity, material nonlinearity was also considered by assuming an appropriate theoretical model for the stress-strain curve of the cable and hence the ultimate load capacities of the roofs were determined; the stress-strain curve of the cable was assumed to be a second-degree parabola between the proportional limit and the ultimate strength.

Numerical and experimental studies were carried out on two types of roofs: (i) a common saddle-shaped hyperbolic paraboloid roof consisting of two nonorthogonal sets of cables; this is referred to as the 'single roof' herein; (ii) a compound shape consisting of two hyperbolic paraboloids connected together; this is referred to herein as the 'double roof'. The single and the double roofs are shown in Fig. 1 (a) and (b) respectively. The double roof may also be extended to form a continuous multi roof with a series of hyperbolic paraboloids as shown in Fig. 1 (c). The two roofs used in the numerical analysis were 120 ft. x 240 ft. in plan with a difference in heights of 12 ft. between adjacent corners. The single roof had a total of 61 joints while the double roof had only 28 joints.



(a) SINGLE ROOF

(b) DOUBLE ROOF

(c) CONTINUOUS MULTIROOF

Fig. 1

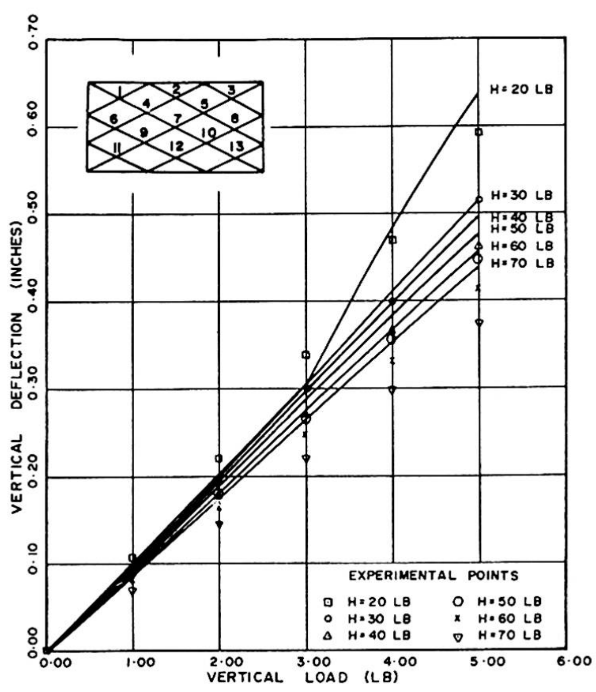
When a uniform load was applied on these roofs, the deflections were found to be more nonlinear than the tension changes with deviations of about 40% and 15% respectively from the corresponding linear solutions. For concentrated loads, the tension changes behaved more nonlinearly than the deflections with corresponding deviations of 25 - 30% and 10% respectively. The nonlinear solution was underestimated in some cases and overestimated in others by the linear solution. This behaviour was found to be related to the slope of the roof. The effect of changing the nonorthogonality of the cables on the deflections and tension changes was also examined. The deflections were found to increase as the nonorthogonality of the cables increased but the tension changes were practically unaffected by any change in nonorthogonality. When the cable pretensions were increased, the deflections and tension changes decreased as expected. The nonlinearity was also reduced at the same time since the stiffness of the roof increased. The final cable tensions increased with the pretension but at higher loads this increase became smaller. Thus it is advantageous to use a high pretension to avoid large deflections without appreciably increasing the final cable tensions.

It was revealed that it is beneficial to use a higher pretension in the prestressing cables than in the load carrying cables. It is possible to find an optimum ratio of pretensions at which the maximum cable tension produced is least and the variation in cable tensions is a minimum.

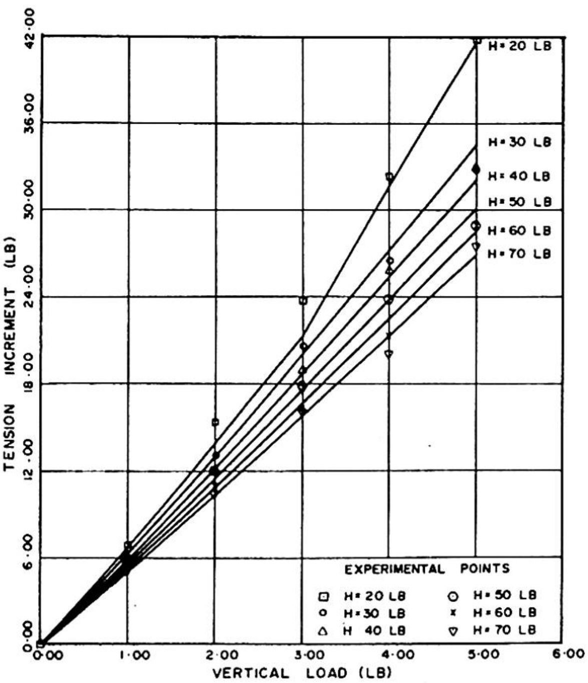
When the behaviour of roofs with different heights was examined, it was found that the deflections decreased with increase in the roof-height. The tension increment was found to be a maximum at a particular roof height which was defined as the critical roof-height. Based on this definition of critical height, roofs can be classified as flat and steep roofs. Steep roofs tend to weaken under increased load with the linear solution underestimating the actual values; while flat roofs tend to increase in strength under load with the linear solution overestimating the true solution. The ultimate load capacity of cable roofs are also affected by the slope of the roof. The ultimate capacity is highest for flat roofs and lowest for steep roofs with an intermediate value at the critical height.

3. EXPERIMENTAL STUDY

Experimental investigations were carried out on test models, to verify the validity of the theoretical solution. A nonorthogonal single roof model and an orthogonal double roof model having dimensions of 36 in. x 72 in. in plan and a height of 9 in., were tested. Both models consisted of five 3/64 inch diameter stainless steel wire ropes of 7 x 7 construction in each direction. Tension measurements were made with precalibrated load cells connected at the ends of the wire ropes. Deflections were measured by displacement transducers. Tests were carried out with various values of initial pretension, the horizontal component of which was kept constant in all the cables in both directions. The models were subjected to equal loads at all the joints and in different tests, concentrated loads at specific joints.



DEFLECTION AT JOINT 7 OF SINGLE ROOF MODEL
(UNIFORM LOADING)
Fig. 2



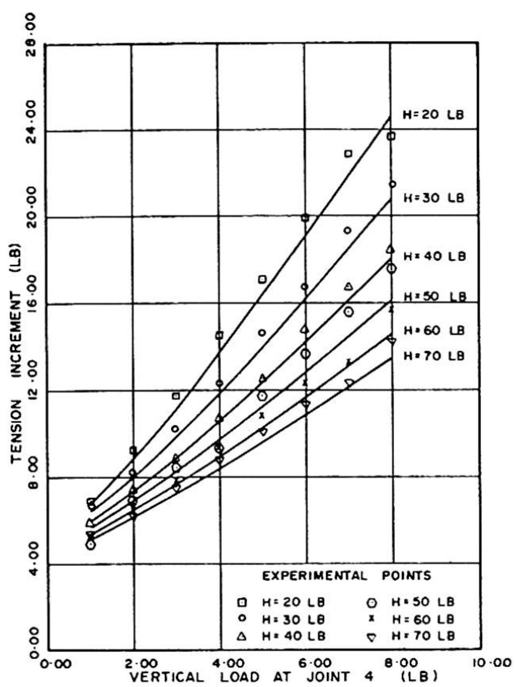
MAX. TENSION INCREMENT IN SINGLE ROOF MODEL
(UNIFORM LOADING)
Fig. 3

The deflection at the centre of the single roof model under a uniformly applied load is shown in Fig. 2. Each theoretical line is for a specific value of H, the horizontal component of the initial tension, and the corresponding experimental points are shown. The measured deflections are generally lower than the theoretical values, with a maximum difference of 12%.

The maximum tension increment produced by the uniform load on the single roof model has been plotted against applied load in Fig. 3. The experimental values are within 4% of the theoretical values. The discontinuity in the line at H = 20 lbs. is due to the fact that some prestressing cables became slack as the load is increased beyond 3 lb/joint. The theory takes such discontinuity into account and the experimental results substantiate this.

Fig. 4 shows the maximum tension increment produced by a concentrated load applied in addition to a uniform load of 1 lb/joint. Here the nonlinearity is clearly demonstrated. The experimental values are within 5% of the theoretical values in almost all cases.

Similar tests were carried out on the double roof model. Curves of maximum deflection versus load for the double roof model under a uniform load are presented in Fig. 5. It can be observed that the nonlinearity is more marked here than that in the single roof model. The experimental deflections are again within 10% of the theoretical results in most cases with a maximum difference of 13%. The corresponding tension increments under uniform load is shown in Fig. 6. The experimental values are quite close to the theoretical values with a maximum difference of 4%. Fig.7 shows the maximum tension increment vs. load when a concentrated load is applied on the double roof model in addition to a uniform load of 1 lb/joint. The agreement between the experimental and theoretical values is within 5%.

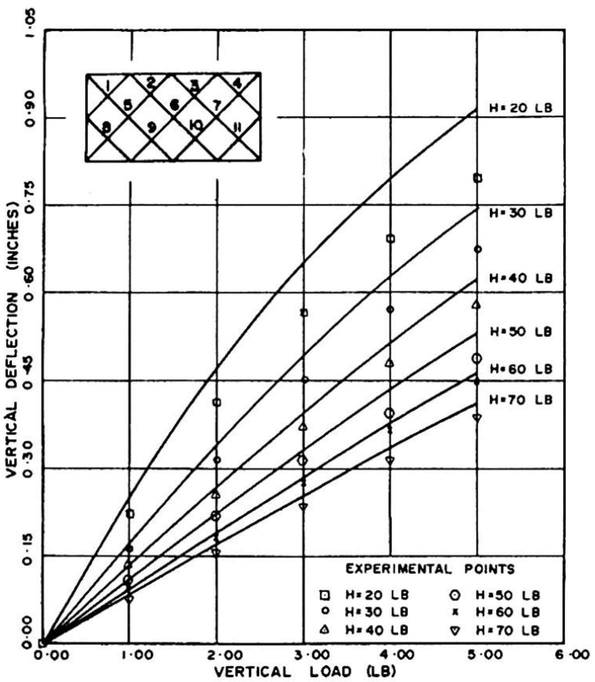


MAX. TENSION INCREMENT IN SINGLE ROOF MODEL
(UNIFORM LOAD OF 1 LB/JT + CONC. LOAD AT THE JT)
Fig. 4

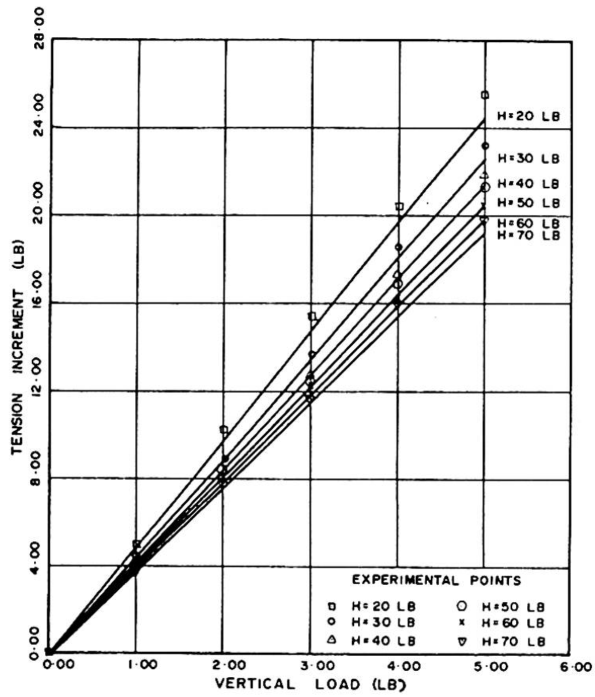
The linear and nonlinear theoretical solutions for the tension change in a prestressing cable of the single roof model and the corresponding experimental values are presented in Fig. 8. The experimental values are within 5% of the nonlinear solution while the linear solution overestimates it by as much as 90%.

4. CONCLUDING REMARKS

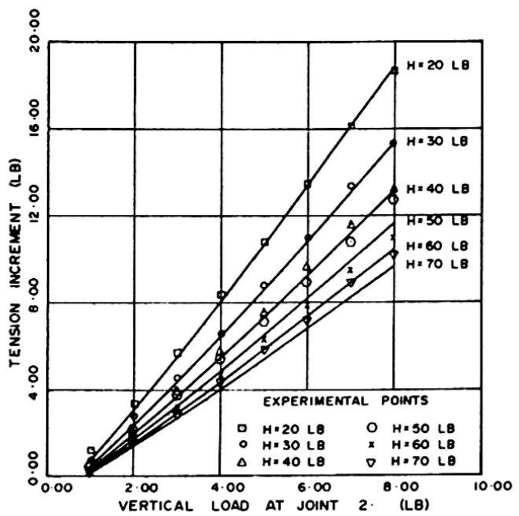
The equations and the method of solution developed in this study could be



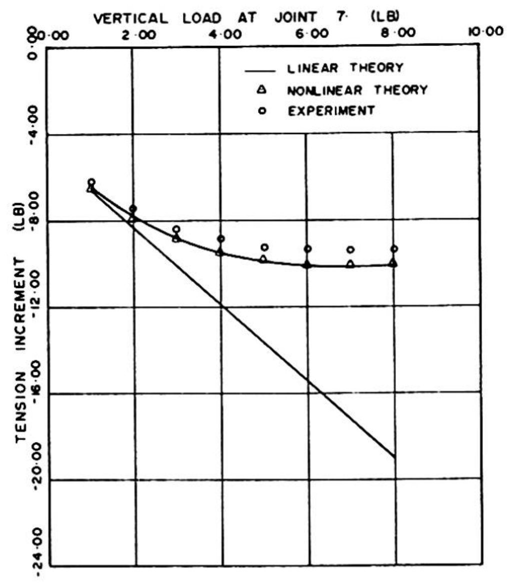
DEFLECTION AT JOINT 6 OF DOUBLE ROOF MODEL
(UNIFORM LOADING)
Fig. 5



MAX. TENSION INCREMENT IN DOUBLE ROOF MODEL
(UNIFORM LOADING)
Fig. 6



MAX· TENSION INCREMENT IN DOUBLE ROOF MODEL
(UNIFORM LOAD OF 1 LB/JT + CONC· LOAD AT THE JT)
Fig. 7



MAX· TENSION DECREMENT IN SINGLE ROOF MODEL
(H=20 LB· UNIFORM LOAD OF 1 LB/JT + CONC· LOAD AT JOINT 7)
Fig. 8

used to predict the nonlinear behaviour of hyperbolic paraboloid cable roofs with any degree of nonorthogonality. This is established by the good agreement between theory and experiment.

In a practical design, the choice of roof slope should not be based purely on aesthetic considerations. Careful attention should be given to strength and performance since the curvature considerably influences the behaviour of the roof. Noting the fact that the factor of safety against failure, based on a working load corresponding to the proportional limit, is excessively high for all slopes, it would seem advantageous to use a steep slope with an increased working load and smaller deflections in exchange for a reduced ultimate capacity.

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SUMMARY

General equations describing the nonlinear behaviour of nonorthogonal cable networks, and their solution, are developed. The influence of initial cable tension, degree of cable nonorthogonality, and slope on deflection and load-carrying capacity of cable roofs are studied. The discrepancies in the linear solution of such structures are examined. The theoretical solution is verified by test results.